A SCHEMATIC THEORY OF NUCLEAR REACTIONS †*

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RESUMEN

In the present paper a description of a many channel nuclear reaction is given on the basis of the formalism of boundary conditions introduced some years ago by Mosbinsky. It is supposed that the interaction between the particles takes place at the point of contact. As a result of the interaction, they can form a compound particle, which is the analogous of the compound nucleus in a nuclear reaction. The behavior of the wave function describing the relative motion of the particles in the different channels, is determined by a set of conditions they must satisfy at the point of contact. In this formalism we have also the possibility of a direct interaction, i.e., reactions which proceed without the formation of a com-

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pound particle. We derive the general form of the S-Matrix, which includes both the compound nucleus and the direct interaction contributions. We give an interpretation of the modulation of the reduced widths of the resonances associated with the compound particles, modulation that was explained by the optical nuclear model. Finally, we discuss the behavior of the transfer cross section when the reaction takes place through the direct interaction mechanism.

I. Introduction

In the past few years, several theories have been formulated to determine the scattering matrix for nuclear reactions, starting from the formal solution of the Hamiltonian associated with a system of nucleons[1]. Although this procedure seems to be the most natural one, it would be possible to follow a different way, starting not from the Hamiltonian of the system, but from other general principles of quantum mechanics. A possible form of realizing this idea is found in Moshinsky's description of nuclear reactions by means of boundary conditions[2], which we shall consider here in some detail.

In order to set up the scheme underlying this description, it is convenient first to consider briefly the physical situation we are faced with in a nuclear reaction. We deal here with a system consisting of a definite number of nucleons, originally grouped in two separate particles. After a reaction has taken place between these particles, we can find the system in one of a given number of different final states. Every possible state of the system, initial or final, wich we shall call a channel, is characterized by the interna quantum numbers of the initial or final particles such as energy and spin. In the following analysis we shall restrict ourselves to the case in which only two particles are present in the final state. The mechanisms followed by the reaction are very complicated. A possible one is the formation of a compound nucleus, wich after certain time disintegrates, leading the system into one of the possible channels.

We idealize this situation as follows: we consider a system of two separated point particles with definite mass and no spin or internal structure. After the reaction takes place, the system is found in one of the different possible channels,

each one characterized by the masses of the resultant pair of particles. The formation of a compound particle is, in our scheme, the analogue of the compound nucleus formation for the actual process. Thus, we can find the system either in the form of two separated particles, or in the form of a compound particle. We shall designate each of these possibilities as a stage of the system. Further, we shall assume free motion for the particles in every channel, the interaction taking place only at the point of contact. The essential feature in the derivation of the scattering matrix associated with this problem is to demand that the total probability of finding the system in any of its stages, no matter which, is independent of time. This requirement leads to a set of boundary conditions at the point of contact for the wave functions of the particles relative motion in the different channels. The elements of the scattering matrix can be expressed in terms of these boundary conditions.

II. The Scattering Matrix

Let $m_{1\rho}$ and $m_{2\rho}$ be the masses of the two particles in the channel ρ and $\phi_\rho(\bar{r}, t)$ the wave function of their relative motion, where $\bar{r}$ denotes the vector distance between the two particles and t the time. For the compound particle μ, we call M_μ its mass and $\psi_\mu(t)$ its wave function. We assume that there are n_1 possible channels, $\rho = 1, 2, \ldots, n_1$, and n_2 compound particles can be formed, $\mu = 1, 2, \ldots, n_2$. Thus the system can be found in $n = n_1 + n_2$ different stages.

Let us first consider the case in which the different stages of the system are not coupled. The wave function describing the relative motion of the particles in the channel ρ, in the centre-of-mass system, satisfies the time-dependent Schrödinger equation for the free particle.

$$-\frac{\hbar^2}{2m_\rho}\Delta\phi_\rho + (m_{1\rho}+m_{2\rho})\phi_\rho = i\hbar\frac{\partial\phi_\rho}{\partial t}\,, \qquad (1)$$

where m_ρ denotes the reduced mass of the particles in the channel ρ. We have included in the Schrödinger equation the energy corresponding to the rest masses

of the particles, since we are dealing with non-elastic processes too. We choose our system of units in such a form that the velocity of light is equal to one.

Also, in the absence of coupling with other stages of the system, the wave function of the compound particle μ satisfies the equation

$$M_\mu \psi_\mu = i\hbar \frac{\partial \psi_\mu}{\partial t} \quad . \tag{2}$$

If there is any coupling, equation (1) is no longer valid for $r = 0$, and (2) is not valid at all. Instead, not only must ϕ_ρ satisfy appropriate boundary condition at $r = 0$, but (2) must be susbstituted by another equation, in such a way the probability of finding the system in any of its stages,

$$P(t) = \sum_\rho \int d\tau \phi^*_\rho \phi_\rho + \sum_\mu \psi^*_\mu \psi_\mu \, , \tag{3}$$

is independent of time. The integral appearing in (3) extends over all points $\vec{r}$. Excluding the origin from this integration, and using (1), we obtain

$$0 = \frac{\hbar}{i} \frac{dP(t)}{dt} = \sum_\rho \lim_{a \to 0} \int d\tau \frac{b^2}{2m_\rho} [\phi_\rho \frac{\partial \phi^*_\rho}{\partial r} - \phi^*_\rho \frac{\partial \phi_\rho}{\partial r}]_{r=a} r^2$$

$$+ \sum_\mu [\psi^*_\mu (\frac{\hbar}{i} \frac{\partial \psi_\mu}{\partial t} + M_\mu \psi_\mu) - \psi_\mu (\frac{\hbar}{i} \frac{\partial \psi_\mu}{\partial t} + M_\mu \psi_\mu)^*] . \tag{4}$$

In what follows, we shall restrict ourselves to the case of zero relative angular momentum.

If (4) is satisfied by the two sets of wave functions $[\phi_\rho, \psi_\mu]$ and $[\phi'_\rho, \psi'_\mu]$, $\rho = 1 \ldots, n_1$, $\mu = 1, \ldots, n_2$, is also satisfied by a linear combination of them. This requirement leads us to the relation[2]

$$\sum_{\rho} \frac{2\pi\hbar^2}{m_\rho} \left[\left(\frac{\partial r\,\phi_\rho}{\partial r}\right)^* (r\,\phi'_\rho) - (r\,\phi_\rho)^* \left(\frac{\partial r\,\phi'_\rho}{\partial r}\right) \right]_{r=0} +$$

$$+ \sum_{\mu} \left[\psi^*_\mu \left(\frac{\hbar}{i}\frac{\partial \psi'_\mu}{\partial t} + M_\mu \psi'_\mu\right) - \left(\frac{\hbar}{i}\frac{\partial \psi_\mu}{\partial t} + M_\mu \psi_\mu\right)^* \psi'_\mu \right] = 0 \tag{5}$$

Defining the $2n = 2(n_1 + n_2)$ dimensional vector $\vec{X}$

$$X_\rho = \frac{2\pi\hbar^2}{m_\rho}\left(\frac{\partial r\ \phi_\rho}{\partial r}\right)_{r=0}, \quad X_{n_1+\mu} = \psi_\mu,$$

$$X_{n+\rho} = (r\phi_\rho)_{r=0} \;\; , \quad X_{n+n_1+\mu} = \frac{\hbar}{i}\frac{\partial \psi_\mu}{\partial t} + M_\mu \psi_\mu , \tag{6}$$

and a similar vector $\vec{X}'$ for the primed quantities, then it follows from (5) that

$$\sum_{s=1}^{n} \left(X^*_{n+s} X'_s - X^*_s X'_{n+s}\right) = 0 \tag{7}$$

The linear relations between the components of the vector defined by (6), which satisfy the set of equations (7), are

$$X_{n+s} = \sum_{s'=1}^{n} C_{s\,s'}\, X_{s'}, \tag{8}$$

where $\| C_{SS'} \|$ is a constant Hermitian matrix[2]. With the help of (6), we can write these relations in full detail as

$$(r\,\phi_\rho)_{r=0} = \sum_{\rho'} C_{\rho\rho'} \frac{2\pi\hbar^2}{m_{\rho'}} \left(\frac{\partial r\,\phi_{\rho'}}{\partial r}\right)_{r=0} + \sum_{\mu'} C_{\rho\,\mu'}\, \psi_{\mu'}, \tag{9a}$$

$$-i\hbar\frac{\partial\psi_\mu}{\partial t} + M_\mu\psi_\mu = \sum_{\rho'} C_{\mu\rho'}\,\frac{2\pi\hbar^2}{m_{\rho'}}\left(\frac{\partial r\,\phi_{\rho'}}{\partial r}\right)_{r=0} + \sum_{\mu'} C_{\mu\mu'}\psi_{\mu'} \qquad (9b).$$

These relations and the Schrödinger equation (1) determine completely the behaviour of the system. In particular (9b) takes the place of (2) when there is coupling between the different stages of the system.

From (9a) and (9b) it is clear that $C_{ss'}$ gives the coupling between the stages s and s' of the system. For this reason we shall refer to the matrix $||C_{ss'}||$ as the coupling matrix.

It should be noted that $||C_{ss'}||$ will be considered to be subdivided in the form

$$\left\|\begin{array}{cc} C_{\rho\rho'} & C_{\rho\mu'} \\ C_{\mu\rho'} & C_{\mu\mu'} \end{array}\right\|$$

where ρ, ρ' go from 1 to n_1 and μ, μ' from 1 to n_2.

We expect that, when there is no coupling between the different stages of the system, the particles in the channel ρ move, even at $r = 0$, as free particles, and that (9b) reduces to (2). This leads us to set all $C_{\rho\rho}$ and $C_{\mu\mu}$ equal to zero.

Let us consider now the stationary state of the system with total energy E. For this case, (9b) becomes

$$(M_\mu - E)\,\psi_\mu = \sum_{\rho'} C_{\mu\rho'}\,\frac{2\pi\hbar^2}{m_{\rho'}}\left(\frac{\partial r\phi_{\rho'}}{\partial r}\right)_{r=0} + \sum_{\mu'} C_{\mu\mu'}\psi_{\mu'}. \qquad (10)$$

In order to obtain the elements of the scattering matrix, we must eliminate from (9a) the explicit appearance of the wave functions of the compound particles. To do this, consider the matrix

$$C_c = ||G_{\mu\mu'} - M_\mu\,\delta_{\mu\mu'}||, \qquad (11)$$

which is an Hermitian matrix and can allways be diagonalized by a similarity_transformation with a unitary matrix $\mathcal{U}$:

$$(\mathcal{U}\, C_c\, \mathcal{U}^+)_{\mu\mu'} = -E_\mu \delta_{\mu\mu'} . \tag{12}$$

Here all E_μ are real. With the help of this relation, from (10) we obtain that.

$$\psi_\mu = \sum_\nu \mathcal{U}^*_{\nu\mu} \frac{1}{E_\nu - E} \sum_{\rho'} C'_{\nu\rho'} \frac{2\pi b^2}{m_{\rho'}} \left(\frac{\partial_r \phi_{\rho'}}{\partial r}\right)_{r=0} \tag{13}$$

where

$$C'_{\nu\rho} = \sum_{\nu'} \mathcal{U}_{\nu\nu'} C_{\nu'\rho} = (C'_{\rho\nu})^* . \tag{14}$$

Using (13), the set of boundary conditions (9a) takes the form

$$(r\phi_\rho)_{r=0} = \sum_{\rho'} [R'(E)_{\rho\rho'} + C_{\rho\rho'}] \frac{2\pi b^2}{m_{\rho'}} \left(\frac{\partial_r \phi_{\rho'}}{\partial r}\right)_{r=0} , \tag{15}$$

with

$$R'(E)_{\rho\rho'} = \sum_\mu \frac{C'_{\rho\mu} C'_{\mu\rho'}}{E_\mu - E} . \tag{16}$$

As the $C_{\mu\rho}$ are energy independent, it follows from (12) and (14), that both $C'_{\mu\rho}$ and E_μ are also independet of the total energy of the system, being totally determined by the intrinsic properties of the system. Besides, the quantities $R'(E)_{\rho\rho'}$ are the elements of a Hermitian matrix, since $C'^*_{\mu\rho} = C'_{\rho\mu}$.

If the reaction starts in the channel α, the wave function for the relative motion of the particles in the channel ρ, is given by[3]

$$\phi_\rho = \sqrt{\frac{m_\rho}{4\pi b k_\rho}} [\delta_{\alpha\rho} e^{-ik\rho r} - S_{\alpha\rho} e^{+ik\rho r}] \frac{1}{r} . \tag{17}$$

Here $k_\rho = \sqrt{\frac{2m_\rho}{\hbar^2}}\,[E - (m_{1\rho} + m_{2\rho})]$ is the wave number of the relative motion of the two particles in the channel ρ. The normalization of (17) is such that the flux associated with the ingoing-wave part of ϕ_α is equal to unity and the outgoing flux in channel ρ is $|S_{\alpha\rho}|^2$.

From (17) and the boundary conditions (15), we obtain the following set of equations for the elements of the scattering matrix:

$$\delta_{\rho\alpha} + i\sqrt{\frac{2\pi\hbar^2 k_\rho}{m_\rho}}\,[R'(E)_{\rho\alpha} + C_{\rho\alpha}]\sqrt{\frac{2\pi\hbar^2 k_\alpha}{m_\alpha}} =$$

$$= \sum_{\rho'}\{\delta_{\rho\rho'} - i\sqrt{\frac{2\pi\hbar^2 k_\rho}{m_\rho}}\,[R'(E)_{\rho\rho'} + C_{\rho\rho'}]\sqrt{\frac{2\pi\hbar^2 k_{\rho'}}{m_{\rho'}}}\}S_{\rho'\alpha}$$

Introducing the matrices $\mathfrak{A}$, $\mathfrak{B}$ and C_d, whose elements are

$$\mathfrak{A}_{\rho\rho'} = \delta_{\rho\rho'}\ ,\quad \mathfrak{B}_{\rho\rho'} = \sqrt{2\pi\hbar^2 k_\rho/m_\rho}\;\delta_{\rho\rho'}\ ,\quad (C_d)_{\rho\rho'} = C_{\rho\rho'}\ , \tag{18}$$

the scattering matrix may be written in the form

$$\mathfrak{S} = \frac{\mathfrak{A} + i\mathfrak{B}[R'(E) + C_d]\mathfrak{B}}{\mathfrak{A} - i\mathfrak{B}[R'(E) + C_d]\mathfrak{B}}\ . \tag{19}$$

It is to be noted that except for the terms that take into account the direct reaction mechanism, our expression for the S-Matrix is very similar to that obtained by the R-Matrix formalism of Wigner[1].

The transfer cross section from channel α into channel ρ can be expressed as

$$\sigma_{\alpha\rho} = \frac{\pi}{k_\alpha^2}\,|\delta_{\alpha\rho} - S_{\alpha\rho}|^2\ . \tag{20}$$

In the scattering matrix (19), the contribution to the cross section due to processes that lead to the formation of compound particles is contained in $R'(E)$, and the contribution of those that do not lead to the formation of a compound particle or as we shall call them, direct processes, is contained in C_d.

Finally, we would like to obtain the Schrödinger equation for the relative motion of the particles in channel ρ valid for all values of r. In the stationary case, from (1) we have

$$\Delta \phi_\rho + k_\rho^2 \phi_\rho = 0 \quad , \quad r = 0 \ , \tag{21}$$

whose solution is given by (17). We can extend this equation to include also the point of interaction by observing that

$$(\Delta + k^2) \frac{e^{\pm ikr}}{r} = -4\pi\delta(\vec{r})$$

and thus the interaction appears in the equation as a pseudo potential

$$(\Delta + k_\rho^2)\,\phi_\rho = -4\pi(r\phi_\rho)\,\delta(\vec{r}) = -\frac{4\pi i\,\delta_{\alpha\rho} - S_{\alpha\rho}}{k_\rho\,\delta_{\alpha\rho} + S_{\alpha\rho}}\left(\frac{\partial_r \phi_\rho}{\partial r}\right)\delta(\vec{r}). \tag{22}$$

The behaviour of the system in its stationary state is completely determined by this equation.

III. The Coupling Matrix

We would like to analyse the physical meaning of the different elements of the coupling matrix. For this purpose, we shall study in detail some simple situations.

a) Elastic scattering.

Let us consider the case in which the possible stages of the system are the entrance channel α and n_2 composite particles. This situation corresponds physi-

cally to elastic scattering through the formation of a compound nucleus. The elements $C_{\alpha\rho}$ and $C_{\mu\rho}$, $(\rho \neq \alpha)$, are all zero.

The significance of the coupling constants $C_{\alpha\mu}$ becomes clearer if we suppose first that there is no coupling between the different compound particles, i.e., $C_{\mu\mu'} = 0$. Then, the amplitude of the outgoing in the entrance channel is given by

$$S_{\alpha\alpha} = \frac{1 + i k_\alpha (2\pi\hbar^2/m_\alpha) R(E)_{\alpha\alpha}}{1 - i k_\alpha (2\pi\hbar^2/m_\alpha) R(E)_{\alpha\alpha}} , \tag{23}$$

where

$$R(E)_{\alpha\alpha} = \sum_\mu \frac{|C_{\alpha\mu}|^2}{M_\mu - E} \tag{24}$$

The elastic-scattering cross-section $\sigma_{\alpha\alpha}$, which can be calculated from (20), shows, as a function of the energy of the incoming particles, a resonance structure: to each compound particle there is associated a resonance; the position of its maximum value and its width are given, to a first approximation, by M_μ and $\Gamma_{\alpha\mu} = (\frac{2\pi\hbar^2}{m_\alpha}) 2 k_\alpha |C_{\alpha\mu}|^2$, respectively, where k_α is to be calculated at the energy of the maximum. Thus the square of the modulus of the coupling constant $C_{\alpha\mu}$ between the entrance channel α and the compound particles μ is seen to be proportional to the width of the resonance. We obtain a well-defined resonance spectrum in the cross section if the width of each resonance is much smaller than the distance to the two nearby resonances, a situation which we suppose to be realized in the present discussion. For convenience, we define the square modulus of this coupling constant as the reduced width of the resonance, i.e.,

$$\Gamma_{\alpha\mu}/2k_\alpha(2\pi\hbar^2/m_\alpha) = |C_{\alpha\mu}|^2 .$$

Let us consider now what happens if, maintaining the values of the coupling constants $C_{\alpha\mu}$, we allow for some coupling between the stages of the different

composite particles μ: the constants $C_{\mu\mu'}$ are now different from zero. After diagonalizing the matrix C_c given by (11), we obtain for the amplitude of the outgoing wave in the entrance channel the same form as (23) but with $R'(E)_{\alpha\alpha'}$ given by (16), substituted for $R(E)_{\alpha\alpha'}$ The behaviour of the elastic-scattering cross-section has changed in two points compared with the behaviour we had in the former case. Firstly, there has been a shift from M_μ to E_μ in the position of the maximum value of the resonance associated with the compound particle μ, and, secondly, the reduced width of the resonance has changed its value from $|C_{a\mu}|^2$ to $|C'_{a\mu}|^2$. From (12) and (14), we can derive two general relations which must be satisfied by the energy shifts and by change in the value of the reduced widths. Setting $\mu = \mu'$ in (12), summation over all μ leads to

$$\sum_\mu (M_\mu - E_\mu) = 0 \tag{25}$$

i.e., the sum of the shifts associated with each resonance must be equal to zero. Further, since U is a unitary matrix, it follows from (14) that

$$\sum_\mu |C'_{a\mu}|^2 = \sum_\mu |C_{a\mu}|^2 , \tag{26}$$

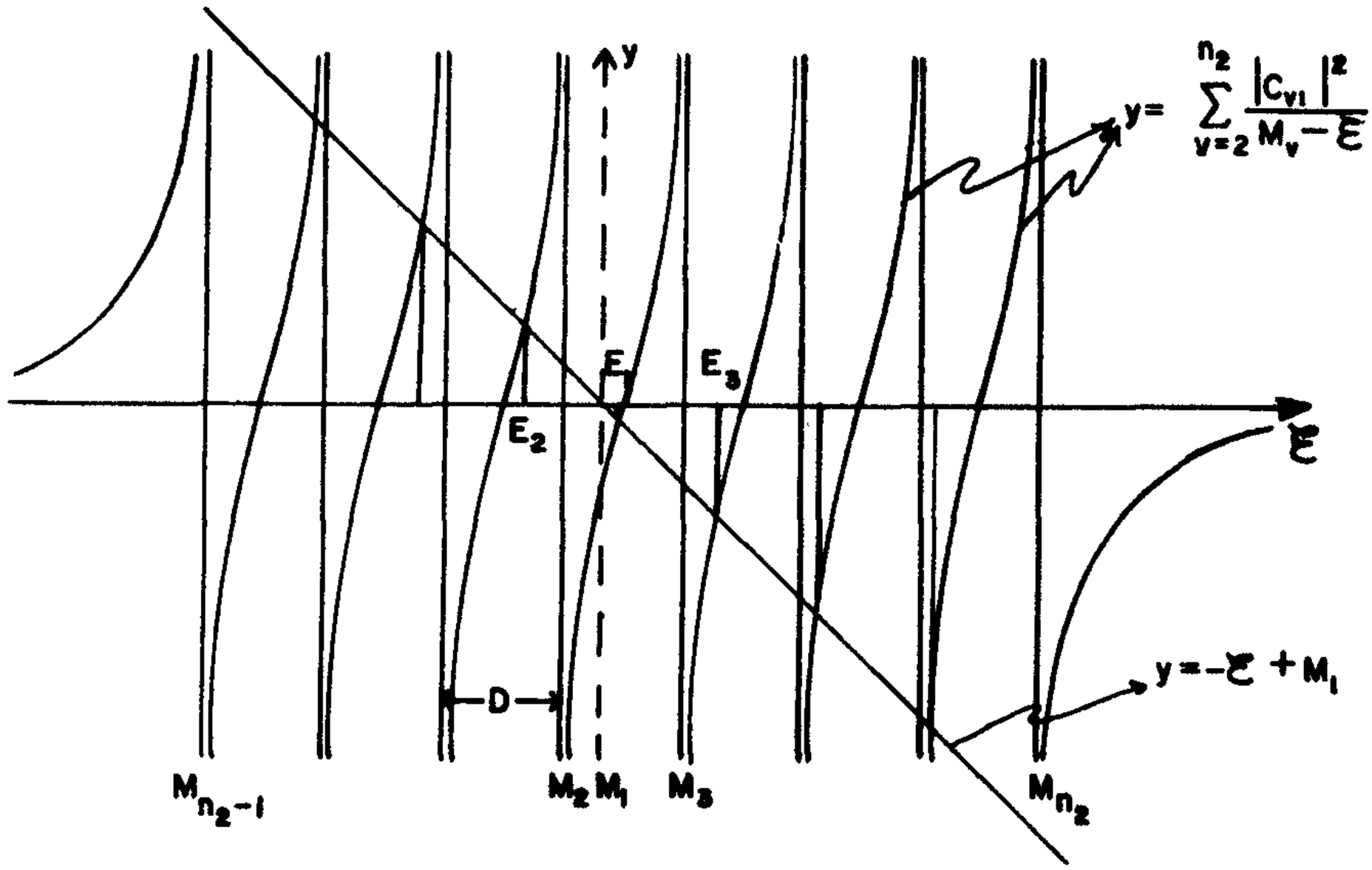
i.e., the sum of the reduced widths of all resonances is independent of the strength of the coupling between the compound particles.

To illustrate these facts, let us consider a simple example in which the entrance channel a is coupled to a single compound particle, say the particle $\mu = 1$, and this in turn, is coupled to a set of other particles $\mu = 2, \ldots, n_2$, while there is no coupling between these last particles. We have thus $C_{a1} \neq 0$, $C_{1\mu} \neq 0$, $C_{a\mu} = C_{\mu\mu'} = 0$ for $\mu, \mu' = 2, \ldots, n_2$. In this case, the diagonalization of the C_c matrix, given by (11), is straightforward. From the secular equation associated with this matrix, one obtains the position of the resonances, wich are given by the n_2 roots of the equation

$$(M_1 - E_\mu) = \sum_{\nu=2}^{n_2} \frac{|C_{1r}|^2}{M_\nu - E_\mu} . \tag{27}$$

That these roots are all different, can be seen directly from Fig. 1.

Fig 1



From (14), the reduced widths of the resonances are given by

$$|C'_{\alpha\mu}|^2 = |C_{\alpha 1}|^2 \, |U_{\mu 1}|^2 , \quad \mu = 1, \ldots, n_2 , \tag{28}$$

since all $C_{\alpha\mu} = 0$ except for $\mu = 1$. The value of $U_{\mu 1}$ is the first component of the unit eigenvector of C_c associated with its μ-th eigenvalue. A straightforward calculation leads to

$$|C'_{\alpha\mu}|^2 = \frac{|C_{\alpha 1}|^2}{1 + \sum_{\nu=2}^{n_2} \frac{|C_{\nu 1}|^2}{(M_\nu - E_\mu)^2}} \tag{29}$$

To obtain a better insight into the expression (29), let us assume that both $|C_{1\nu}|^2 = C^2$ and the distance D between two consecutive M_μ are independent of μ.

We assume, further, that the number of compound particles, n_2, is very great. So we can set

$$\sum_{\nu=2}^{n_2} \frac{|C_{1\nu}|^2}{M_\nu - \mathcal{E}} = \frac{\pi C^2}{D} \cot\left[\frac{\pi}{D}(M_2 - \mathcal{E})\right] .$$

It is to be noted that the expression (30) is accurate only if $\mathcal{E}$ is in the vicinity of M_1, since the right hand side of (30) does not take the edge effects properly into account. With the help of the last expression, we can write the sum appearing in the denominator of (29) in a closed form

$$\sum_{\nu=2}^{n_2} \frac{|C_{1\nu}|^2}{(M_\nu - E_\mu)^2} = \frac{\pi^2 C^2}{D^2}\left\{1 + \cot^2\left[\frac{\pi}{D}(M_2 - E_\mu)\right]\right\} . \qquad (31)$$

This can be evaluated with the help of (27) and (30). The expression for the reduced widths takes finally the form

$$|C'_{\alpha\mu}|^2 = \frac{C^2 |C_{\alpha 1}|^2}{(M_1 - E_\mu)^2 + C^2\left(1 + \left(\frac{C\pi}{D}\right)^2\right)} \qquad (32)$$

The reduced widths of the compound nucleus resonances in reactions with neutrons show a behaviour similar to that we describe in the present example and which has been accounted for by the optical model[4]. Our results, however, are formally identical to those found, for example, in the analysis of the excitation of collective modes of the nucleus by coupling of the incoming neutron with the nuclear surface[5,6]. In the derivation of the behaviour of the reduced widths of the compound nucleus resonances given by Bohr and Mottelson,[5] for instance, it is supposed that incoming neutron and the target nucleus couple only at the nuclear surface and form a compound nucleus. Inside this surface, the neutron moves under the action of a real square-well potential. If the strength of the coupling between the initial state of the system - target nucleus and neutron moving under the influence of the square-well potential - and the compound nucleus has the same value for all compound states, the reduced widths of the resonances associated with the

compound nucleus show a behaviour equal to that given by (32), if we restrict ourselves to the neighbourhood of the resonance of the neutron in the potential well, that is, the single-particle resonance. Comparing this result with ours, M_1 determines the position of the single-particle resonance and $|C_{1\,a}|^2$ gives its reduced width. This is much larger then the reduced width $|C'_{a\mu}|^2$ of the compound nucleus resonances. Furthermore, C^2 is essentially the square modulus of the matrix element of the surface coupling between the initial state of the system an the compound nucleus.

If we interpret the fact of the modulation of the reduced widths of the compound nucleus resonances in reactions with neutrons in the light of our formalism, the compound states are formed and disintegrate through the formation of a single particle state, which we have identified with the compound particle $\mu = 1$ The compound states whose energy is near to the single-particle resonance have a shorter mean life a larger reduced width - because the single-particle can be easily realized at this energy; as a result, the compound state can easily disintegrate into the single-particle state and this, in turn, disintegrates into the entrance channel.

b) Direct interactions.

Finally, we would like to consider another very simple example, in which the reaction proceeds from the entrance channel α into the reaction channel ρ without the formation of a compound particle, i.e., the reaction takes place through a direct mechanism. The system can be found only in two stages, corresponding to the entrance and the reaction channels, and the coupling matrix has only two elements different from zero, $C_{\alpha\rho}$ and $C_{\rho\alpha}$. From (19), the amplitude of the outgoing wave in the reaction channel ρ is given by

$$S_{\alpha\rho} = \frac{2i\sqrt{(4\pi^2\hbar^4/m_\alpha m_\rho)\,k_\alpha k_\rho}\;C_{\alpha\rho}}{1 + (4\pi^2\hbar^4/m_\alpha m_\rho)\,k_\alpha k_\rho\,|C_{\alpha\rho}|^2}\quad , \tag{33}$$

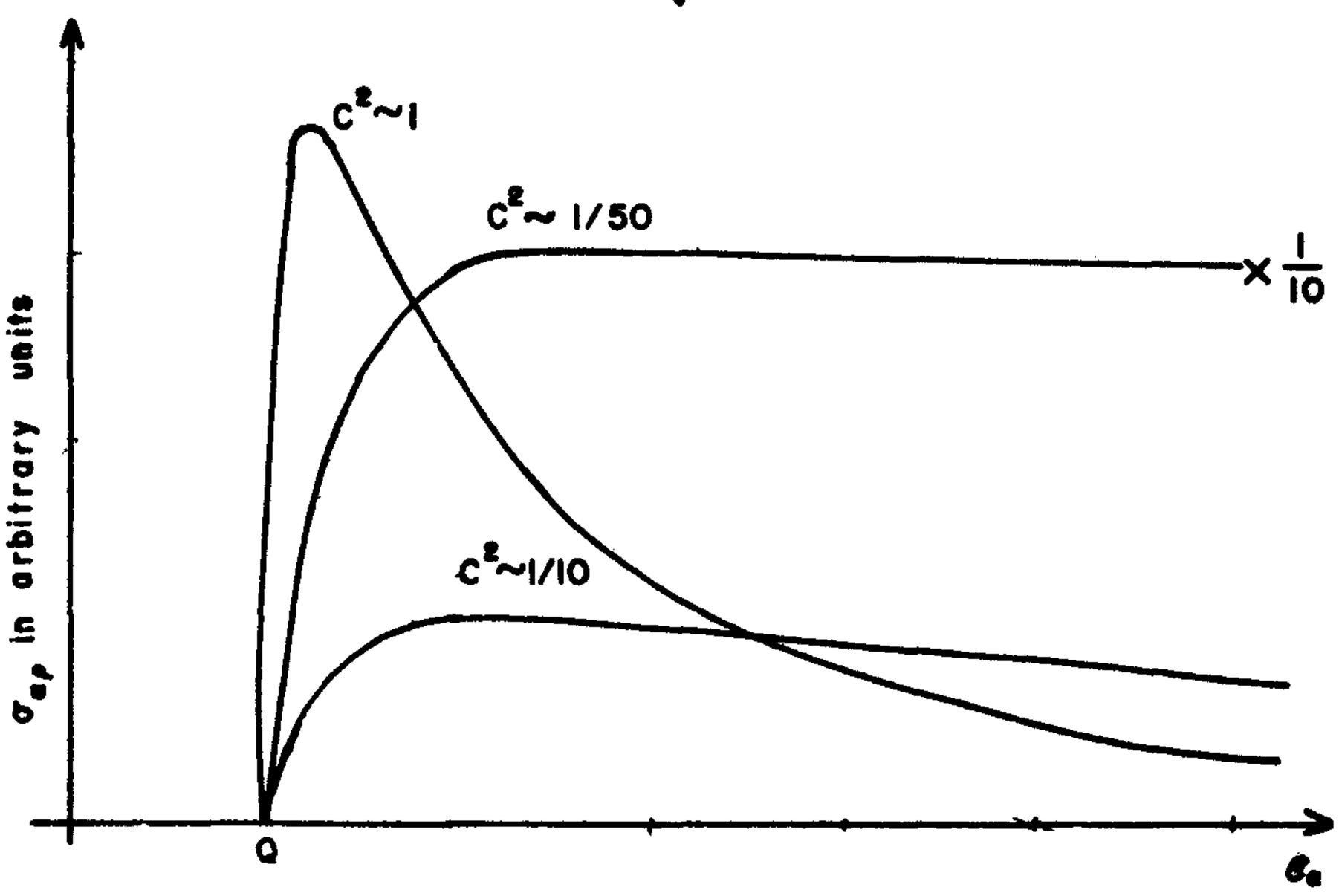
and the reaction cross-section takes the form

$$\sigma_{a\rho} = \frac{4\pi}{k_a^2} \frac{A_{a\rho}\sqrt{\epsilon_a(\epsilon_a+Q_{a\rho})}\ |C_{a\rho}|^2}{(1+A_{a\rho}\sqrt{\epsilon_a(\epsilon_a+Q_{a\rho})}\ |C_{a\rho}|^2)^2} \qquad (34)$$

where $A_{a\rho} = 8\pi^2\,\hbar^2/\sqrt{m_a m_\rho}$, $\epsilon_a = \hbar^2\,k_a^2/2\,m_a$ is the kinetic energy of the relative motion of the particles in the entrance channel and $Q_{a\rho} = (m_{1a}+m_{2a}) - (m_{1\rho}+m_{2\rho})$ is the Q-value of the reaction.

This cross section starts from zero at $\epsilon_a = -Q_{a\rho}$ and rises as $(\epsilon_a + Q_{a\rho})^{1/2}$. Since $C_{a\rho}$ is independent of the kinetic energy of the incoming particles, the cross section reaches a single maximum and tends to zero as ϵ_a^{-2} for large values of the kinetic energy.

Fig.2



In Fig. 2 we show the bahaviour of the cross section (34) for a fixed Q-valued and for several values of the coupling constant. These cross sections behave rather like those for the formation of a single excited state of the target

nucleus by inelastic scattering of neutrons[7]. We do not look for agreement with the experimental data, because in our formalism we have not taken into account higher relative angular momenta nor the spin of the particles, elements which are essential in this kind of analysis.

A time dependent description of this kind of process would probably show that when the particles enter into contact, the reaction products appear in the reaction channel without any delay. That this is true has been demonstrated for the case $Q = 0$, otherwise the mathematical problem associated with this time-dependent description has not yet been solved[8]. Thus there is a sharp distinction between this type of process and that in which the reaction takes place trough the formation of a compound particle, because in the latter case the particles appear in the reaction channel with some delay, which must be of the order of magnitude of the mean life of the compound particle.

III. Conclusions

In the present derivation of the scattering matrix, all the physics of the problem is contained in the coupling matrix, whose elements give the strength of the coupling between the different stages of the system. This matrix has two important properties: it is Hermitian and it is energy independent. The first of these properties makes the scattering matrix a unitary matrix, and the second permits writing in an explicit form its energy dependence.

Furthermore, according to our scheme, any nuclear reaction can take place through the compound-particle mechanism as well as through the direct one, the former being the only one that gives a resonance behaviour to the cross sections.

Since the magnitudes of the coupling constants between any channel and the compound particles are not determined by the formalism, we can identify the resonances associated with these particles with the experimental compound-nucleus resonances or with the single-particle resonances, i.e., those associated with the scattering by a potential well whose range is of the order of magnitude of the nuclear radius.

Another very interesting result is the allowance for coupling between those

stages of the system that correspond to compound particles. In terms of this coupling we were able to derive in a simple way the behaviour of the reduced widths of the compound-nucleus resonances in the case of reactions with neutrons. It would be also possible, by a proper choice of the coupling constants, to give an analysis of the deuteron stripping reactions, which is currently done in terms of the distorted-wave method.

The non-resonant or direct processes are probably also encountered in reality, when in a reaction participates, not the whole system, but only a small number of nucleons. Here there would be neither the formation of a compound nucleus nor the scattering by the averaged nuclear potential, processes which lead to resonances in the cross sections.

We believe that by taking properly into account the nuclear radius, the spin for all particles and arbitrary angular momenta for the motion in the different channels, this formalism would be useful in.analyzing a great variety of experimental data and in obtaining a better understanding of the mechanism though which a nuclear reaction takes place.

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