A simple connection of the (electroweak) anapole moment with the (electroweak) charge radius of a massless left-handed Dirac neutrino

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Recibido el 22 de mayo de 2000; aceptado el 23 de octubre de 2000

Assuming that the neutrino is a massless left-handed Dirac particle, we show that the neutrino anapole moment a_ν and the neutrino charge radius $\langle r_\nu^2 \rangle$ satisfy the simple relation $a_\nu = \langle r_\nu^2 \rangle/6$, in the context of the Standard Model of the electroweak interactions. We also show that the neutrino electroweak anapole moment $a_{\nu_l}^{\rm EW}$ and the neutrino electroweak charge radius $\langle r_{\nu_l}^2 \rangle^{\rm EW}$, which have been defined through the $\nu_l l'$ scattering at the one-loop level and are physical quantities, also obey the relation $a_{\nu_l}^{\rm EW} = \langle r_{\nu_l}^2 \rangle^{\rm EW}/6$.

Keywords: Neutrino properties; charge radius; anapole moment

Suponiendo que el neutrino es una partícula de Dirac, sin masa y con helicidad izquierda, mostramos que el momento anapolar a_ν y el radio de carga $\langle r_\nu^2 \rangle$ del neutrino satisfacen la relación simple $a_\nu = \langle r_\nu^2 \rangle/6$, en el contexto del Modelo Estándar de las interacciones electrodébiles. Además, mostramos que el momento anapolar electrodébil $a_{\nu_l}^{\rm EW}$ y el radio de carga electrodébil $\langle r_{\nu_l}^2 \rangle^{\rm EW}$ del neutrino, los cuales han sido definidos a través de la dispersión $\nu_l l'$ a nivel de un lazo y que son cantidades físicas, también obedecen la relación $a_{\nu_l}^{\rm EW} = \langle r_{\nu_l}^2 \rangle^{\rm EW}/6$.

Descriptores: Propiedades del neutrino; radio de carga; momento anapolar

PACS: 13.10 +q; 13.15+g

1. Introduction

An important electromagnetic quantity of a chargeless particle is the charge radius. Several authors have discussed the electromagnetic charge radius of the neutrino, $\langle r_\nu^2 \rangle$ [1]. In 1972, Bardeen, Gastmans, and Lautrup calculated $\langle r_\nu^2 \rangle$ in the context of the Standard Model of the electroweak interactions (SM) [2] and using the unitary gauge showed that is infinite and therefore is not a physical quantity in this model [3]. However, even a massless chargeless fermion may develop an charge radius [4]. Further, the fact that neutrinos could be produced electromagnetically might be an indication for the existence of a magnetic moment or a charge distribution for the neutrino [5]. Besides, from 1987 there has been a revival of interest in the concept due to an experimental limit on the neutrino charge radius reported by K. Abe *et al.* $\langle r_\nu^2 \rangle \leq 0.81 \times 10^{-32}$ cm^2, with 90% C. L., and obtained by comparing $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$ with other weak interaction processes not involving neutrinos [6].

On the other hand, also all fermion may develop an anapole moment $\kappa_{\rm anap}$ [4, 7]. In fact, a particle with spin different from zero can have an anapole moment due to radiative corrections with magnitude $\kappa_{\rm anap} \cong 4 \times 10^{-33}$ cm^2 [8]. Moreover, if the neutrino has a charge radius, it is possible to calculate its anapole moment, because the anapole moment is dimensionally a mean-square radius and measures the correlation between spin and charge distribution [9, 10]. In 1987, M. Abak and C. Aydin [10] calculated $\kappa_{\rm anap}$ in the context of the Standard Model of the electroweak interaction (SM) [2], using the 't Hooft-Feynman gauge [11] and conclude that $\kappa_{\rm anap}$ is too small to be measured. Also in 1987, H. Czyz *et al.* [12] discussed the charged lepton $\kappa_{\rm anap}$ in the context of the SM and showed that this quantity is gauge dependent and therefore is not a physical quantity in this model. In 1992, A. Góngora-T and R.G. Stuart, defined the charge radius and anapole moment of a free fermion as being its vector and axial-vector contact interactions with an external electromagnetic current and they got, at one loop in the SM, a finite and gauge invariant expression for these quantities [13].

In this work, we assume that the neutrino is a massless left-handed Dirac particle, and show that the neutrino anapole moment a_ν and the neutrino charge radius $\langle r_\nu^2 \rangle$ satisfy the simple relation $a_\nu = \langle r_\nu^2 \rangle/6$, in the context of the Standard Model of the electroweak interactions, in the general linear R_ξ gauge. We also show that the neutrino electroweak anapole moment $a_\nu^{\rm EW}$ and the neutrino electroweak charge radius $\langle r_\nu^2 \rangle^{\rm EW}$, which have been defined through the $\nu_l l'$ scattering at the one-loop level and are physical quantities, also obey the relation $a_\nu^{\rm EW} = \langle r_\nu^2 \rangle^{\rm EW}/6$.

In Sect. 2, we show the simple connection of the neutrino anapole moment of the ν with the neutrino charge radius $\langle r_\nu^2 \rangle$, in the context of the standard model, using the general linear R_ξ gauge and assuming that the neutrino is a massless left-handed Dirac particle. In Sect. 3, We show that the neutrino electroweak anapole moment $a_{\nu_l}^{\rm EW}$ and the neutrino electroweak charge radius $\langle r_{\nu_l}^2 \rangle^{\rm EW}$, which have been defined through the $\nu_l l'$ scattering at the one-loop level and are physical quantities, also obey a similar simple relation. Finally, in Sect. 4, we discuss our results.

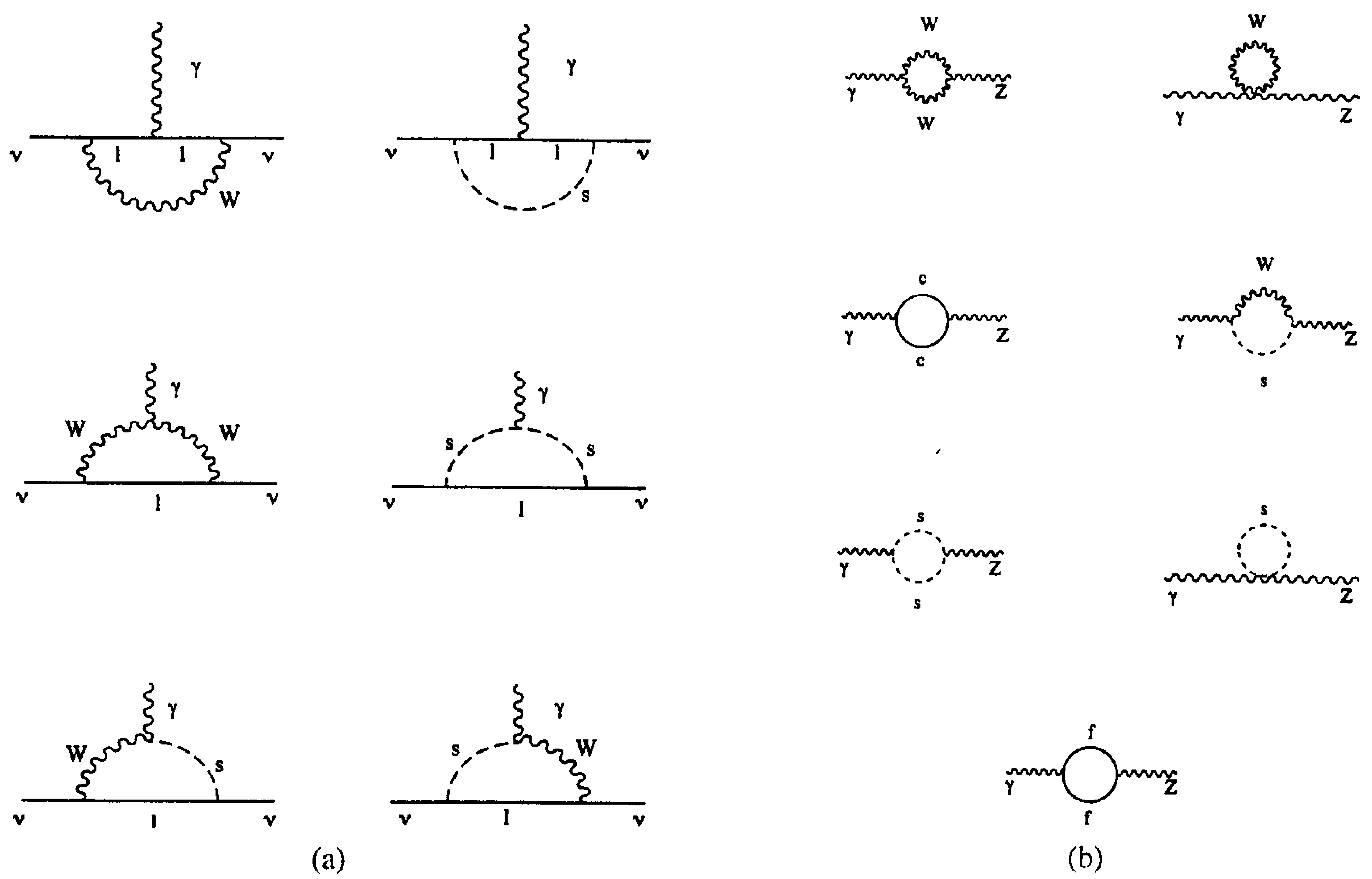

FIGURE 1. (a) Proper vertex and (b) $\gamma - Z$ diagrams which contribute to the neutrino anapole form factor $f_3(q^2)$ at the lowest order in α.

2. The neutrino anapole moment and the neutrino charge radius in the linear R_ξ gauge

For a massless left-handed neutrino the matrix element of the electromagnetic current can be expressed in terms of only one form factor $F(q^2)$ as

$$M_\mu = ieF(q^2)\bar{u}_\nu(p')\gamma_\mu(1-\gamma_5)u_\nu(p). \quad (1)$$

In Figs. 1a and 1b, we show the one-loop diagrams which contribute to the form factor.

For a massless neutrino we can rewrite Eq. (1), as follows:

$$M_\mu = ie\bar{u}_\nu(p')\{\gamma_\mu f_1(q^2) - \gamma_\lambda\gamma_5[g^\lambda{}_\mu q^2 - q^\lambda q_\mu]f_3(q^2)\}u_\nu(p), \quad (2)$$

where [4] $f_1(q^2) = F(q^2)$ and $f_3(q^2) = F(q^2)/q^2$ are the electric and anapole form factor of the neutrino, respectively, with

$$0 = f_1(0), \quad (3)$$

$$\langle r_\nu^2\rangle = 6\frac{\partial f_1(q^2)}{\partial q^2}\,|_{q^2=0} = 6\frac{\partial F(q^2)}{\partial q^2}\,|_{q^2=0}, \quad (4)$$

and

$$a_\nu = f_3(0) = \frac{F(q^2)}{q^2}\,|_{q^2=0}\,. \quad (5)$$

Using the results obtained in the R_ξ-gauge for $F(0)$ [14] and for $\partial F(q^2)/\partial q^2\,|_{q^2=0}$ in Sect. II of Ref. 15, we can write

$$F(q^2) = \frac{\langle r_\nu^2\rangle}{6}\,q^2 + \mathcal{O}(q^4), \quad (6)$$

with $\langle r_\nu^2\rangle$ being an infinite and gauge dependent quantity in the standard model of the electroweak interactions.

Finally, from Eqs. (5) and (6) we obtain

$$a_\nu = \langle r_\nu^2\rangle/6. \quad (7)$$

3. The electroweak anapole moment and the electroweak charge radius of the neutrino

As it is well-known, in the frame of the standard model the neutrino charge radius [3, 15] (and according to Eq. (7) also the neutrino anapole moment) is an infinite and gauge dependent quantity in the linear R_ξ gauge. This means that the neutrino charge radius and the neutrino anapole moment are not static quantities because one cannot measure them with an external field. Therefore, in order to look for a definition of a physical charge radius anapole moment one has to consider other diagrams which contribute to the total amplitude of the physical process $\nu_l l' \to \nu_l l'$.

Marciano and Sirlin [16], making use of the current-algebra formalism of radiative corrections [17] and working in the context of the standard model, showed that for $q^2 \ll M_W^2$ and $q \cdot P \ll M_W^2$ (where q and P are the transferred and initial charged lepton momenta, respectively) one can write the total amplitude of the scattering $\nu_l l'$ in terms solely of the currents l^γ and l^Z which are defined as follows:

$$l_\mu^\gamma = -\bar{u}_f\gamma_\mu u_i \quad (8)$$

and

$$l_\mu^Z = \bar{u}_f\Big[-\frac{1}{4}\gamma_\mu(1-\gamma_5) + \sin^2\theta_W\gamma_\mu\Big]u_i, \quad (9)$$

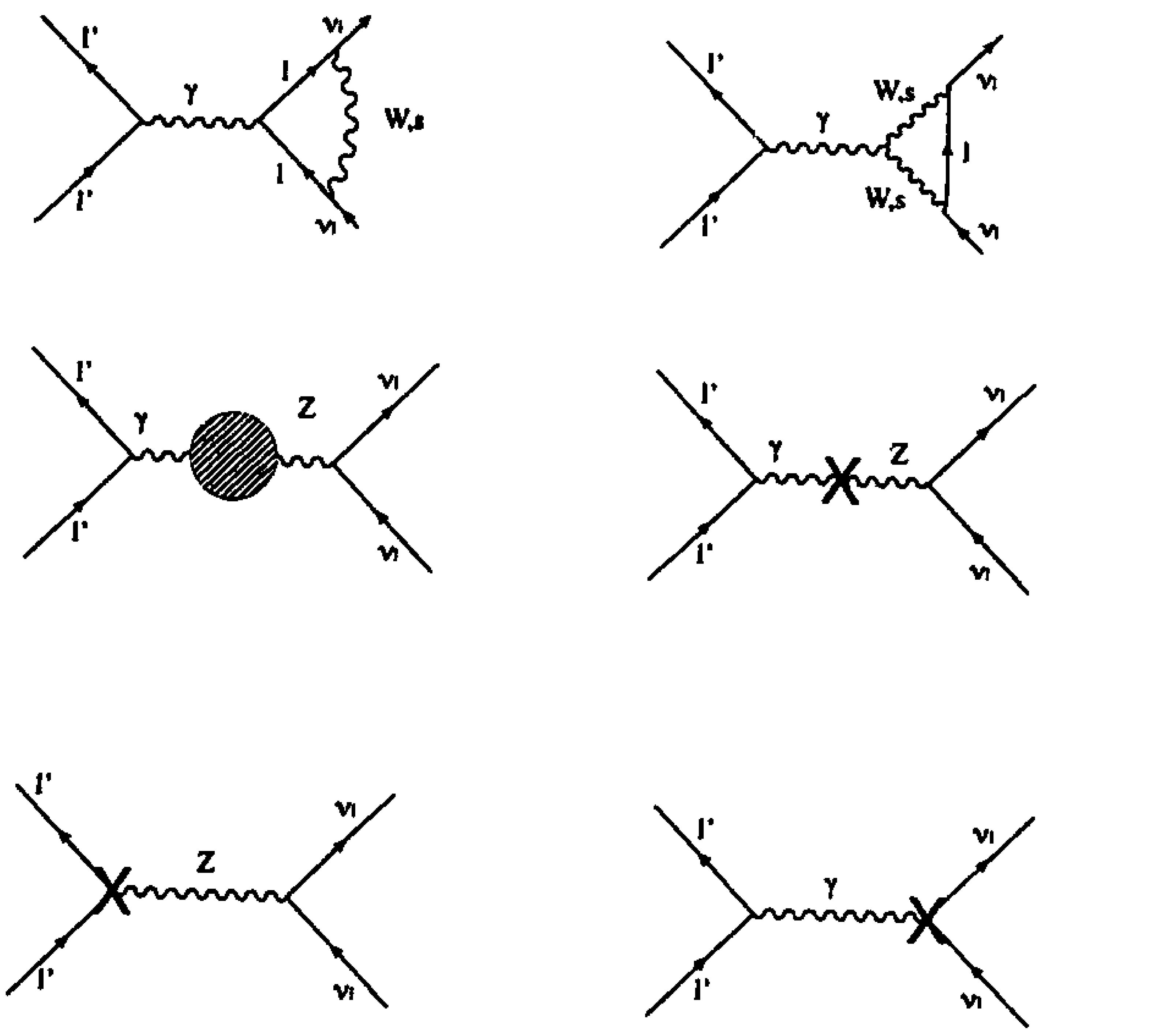

FIGURE 2. Neutrino anapole moment, $\gamma - Z$ mixing, and counterterm diagrams which contribute to the $\nu_l l' \to \nu_l l'$ scattering amplitude.

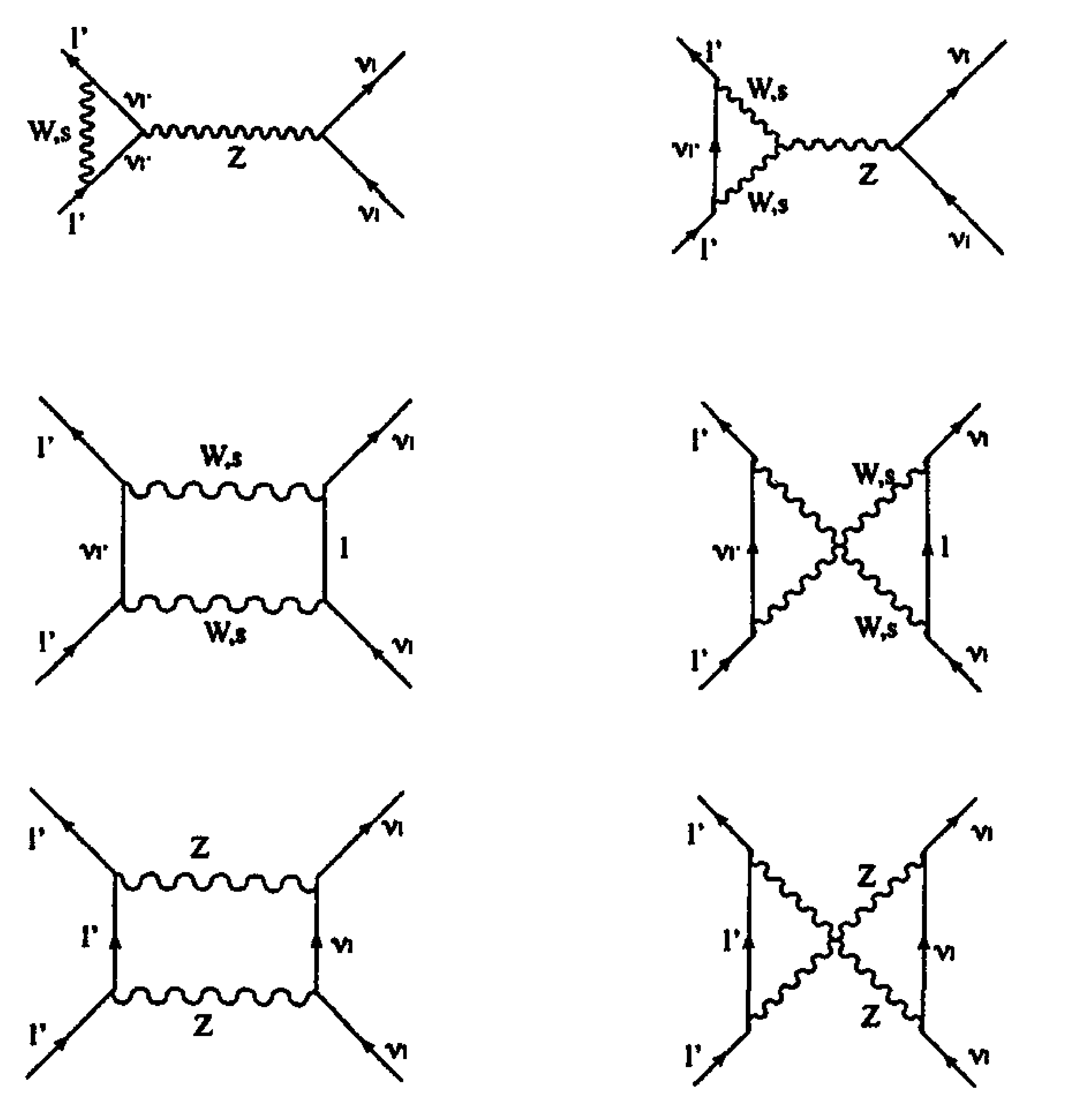

FIGURE 3. (a) Lowest order charged lepton vertex corrections and (b) box diagrams which contribute to the $\nu_l l' \to \nu_l l'$ scattering amplitude.

where u_i and u_f are the spinors of the initial and final charged leptons and θ_W is the electroweak angle; *i.e.*, it is possible to write the total amplitude of the mentioned process in the following form:

$$M = ie^2 \left\{ \frac{F_\gamma(q^2)}{q^2} l^\gamma_\mu + \frac{F_Z(q^2)}{q^2 - M_Z^2} l^Z_\mu \right\} \bar{\nu}_f \gamma^\mu (1-\gamma_5)\nu_i, \quad (10)$$

where $F_\gamma(q^2)$ and $F_Z(q^2)$ are finite and gauge independent functions separately. ν_i and ν_f are the spinors of the initial and final neutrino. $F_\gamma(q^2)$ gets contributions from the proper neutrino electromagnetic vertex, from the γZ self-energy, and from part of the box diagrams (see Figs. 2 and 3).

For a massless neutrino we can rewrite Eq. (10), as follows:

$$M = ie^2 \left(\frac{l^\gamma_\mu}{q^2}\right) \bar{\nu}_f \left[\gamma^\mu F_\gamma(q^2) - \frac{\gamma^\lambda\gamma_5(g^\mu_\lambda q^2 - q_\lambda q^\mu)F_\gamma(q^2)}{q^2}\right]\nu_i + ie^2 \left(\frac{l^Z_\mu}{q^2 - M_Z^2}\right) \bar{\nu}_f \left[\gamma^\mu F_Z(q^2) - \frac{\gamma^\lambda\gamma_5(g^\mu_\lambda q^2 - q_\lambda q^\mu)F_Z(q^2)}{q^2}\right]\nu_i. \quad (11)$$

In Ref. 15, it was introduced the electroweak charge radius of the neutrino as

$$\langle r^2_{\nu_l}\rangle^{\rm EW} = 6\frac{\partial F_\gamma(q^2)}{\partial q^2}\,|_{q^2=0}\,. \quad (12)$$

In a similar way, it can be introduced the electroweak anapole moment of the neutrino as

$$a^{\rm EW}_{\nu_l} = \frac{F_\gamma(q^2)}{q^2}\,|_{q^2=0}\,. \quad (13)$$

Evaluating explicitly the electroweak charge radius, in the 't Hooft-Feynman gauge, by using the results of Ref. 16, leads (taking $\sin^2\theta_W = 0.23$ and neglecting the term proportional to $1 - 4\sin^2\theta_W$) to

$$\langle r^2_{\nu_l}\rangle^{\rm EW} \approx \frac{\alpha}{4\pi M_W^2 \sin^2\theta_W}\left\{\ln\frac{M_W^2}{m_l^2} + 3.57 + \frac{3}{2\sin^2\theta_W}\left[H(\rho) - \cos^2\theta_W H\left(\frac{\rho}{\cos^2\theta_W}\right)\right]\right\}, \quad (14)$$

where $l = e, \mu, \tau$,

$$H(\rho) = \int_0^1 dx \left[1 - \frac{x^2}{2} - \frac{\rho}{2}(1-x)\right]\ln[x^2 + \rho(1-x)] + \frac{\rho}{4}\left(\ln\rho - \frac{1}{2}\right), \quad (15)$$

and $\rho = M_H^2/M_Z^2$ (M_H is the mass of the physical Higgs scalar).

According to Ref. 16, $F_\gamma(0) = 0$ and hence, using Eq. (12), we can write

$$F_\gamma(q^2) = \frac{\langle r^2_{\nu_l}\rangle^{\rm EW}}{6} q^2 + \mathcal{O}(q^4), \qquad (16)$$

with $\langle r^2_{\nu_l}\rangle^{\rm EW}$ being a finite and gauge independent quantity, which does not depend on the properties of the lepton l' used to define it, in the standard model of the electroweak interactions [15]. Finally, using Eq. (13) we obtain

$$a^{\rm EW}_{\nu_l} = \frac{\langle r^2_{\nu_l}\rangle^{\rm EW}}{6}. \qquad (17)$$

All the diagrams which contribute to the neutrino charge radius (*i.e.* to the neutrino anapole moment) (Fig. 2) are proportional to l^γ_μ. The corrections to the charged lepton vertex and the box diagrams (Fig. 3) also contribute to $F_\gamma(q^2)$, because these diagrams contain a part proportional to l^γ_μ [15].

In Table I, we present our results of $\langle r^2_{\nu_l}\rangle^{\rm EW}$, taking $M_W = 80.4\, GeV$, $m_e = 0.51$ MeV, $m_\mu = 105$ MeV, and $m_\tau = 1,777$ MeV, and for some values of the physical Higgs boson M_H [18].

TABLE I. $\langle r^2_{\nu_l}\rangle^{\rm EW}$ ($l = e, \mu, \tau$.) for different values of M_H.

M_H (GeV)	$\langle r^2_{\nu_e}\rangle^{\rm EW}$ (10^{-33} cm^2)	$\langle r^2_{\nu_\mu}\rangle^{\rm EW}$ (10^{-33} cm^2)	$\langle r^2_{\nu_\tau}\rangle^{\rm EW}$ (10^{-33} cm^2)
77.5	4.3	2.5	1.6
127	4.4	2.6	1.7
450	4.8	3.0	2.1
740	4.9	3.2	2.3

4. Conclusion

We have assumed that the neutrino is a massless left-handed Dirac particle and we have shown that the neutrino anapole moment and the neutrino charge radius satisfy the simple relation

$$a_\nu = \frac{\langle r^2_\nu\rangle}{6}$$

in the context of the Standard Model of the electroweak interactions and working in the general linear R_ξ guage.

We have also shown that the neutrino electroweak anapole moment and the neutrino electroweak charge radius, which have been defined through the $\nu_l l'$ scattering at the one-loop level and are physical quantities, also obey the same simple relation

$$a^{\rm EW}_\nu = \frac{\langle r^2_\nu\rangle^{\rm EW}}{6}.$$

in the frame of the Standard Model of the electroweak interactions.

Acknowledgments

This work was supported in part by the *Consejo Nacional de Ciencia y Tecnología* (CONACyT)(*Proyecto: 26168-E*) and *Sistema Nacional de Investigadores (SNI) de México*.

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