Thomson scattering revisited

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We review Thomson scattering, as the low energy limit of Compton scattering, for target particles of spin 0, 1/2, 1, and 3/2 . We show how, at the scattering amplitude level, Thomson result emerges; then, the computation of the cross section is quite simple.

Keywords: Scattering; photons; particle spin.

Calculamos la dispersión Thomson, como el límite de bajas energías de la dispersión Compton, para partículas blanco de espines 0, 1/2, 1, and 3/2. Mostramos como el resultado de Thomson se obtiene al nivel de la amplitud de dispersión, de donde el cálculo de la sección de dispersión resulta más simple.

Descriptores: Dispersión; fotones; espín de partícula

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1. Introduction

The scattering of an electromagnetic wave by a free charged particle was first studied by J.J. Thomson in 1904 [1], and turn out to be the classical version of the relativistic effect studied by A. H. Compton in 1923. The relativistic quantum mechanical calculation of this cross section is known as the Klein-Nishina formula.

In many books on Relativistic Quantum Mechanics, the procedure to calculate Compton scattering is through the use of Feynman rules obtained from the Quantum Electrodynamics Theory [2]. This is done for the scattering of a photon, the electromagnetic wave, off a free electron, a particle of spin 1/2 or from a spin-0 particle. In Figs. 1(a)-(c) we show the Feynman diagrams contributing to Compton scattering, where the first two corresponds to the case of a target of spin-1/2 and spin 3/2. For the case of a target of spin-0 and spin-1 the three of them contributes to scattering. After the differential or total cross-section is computed, the low energy limit is performed to get Thomson's result, which is, as expected, independent of target spin effects. In fact, the classical angular differential cross section for the scattering of an incident wave off an electron of mass m and charge e, is given by [3]

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{m}\right)^2 |\varepsilon^*(k')\cdot\varepsilon(k)|^2 , \tag{1}$$

where $\varepsilon(k)$ and $\varepsilon(k')$ are the polarization three-vectors of the incident and scattered waves, and the condition $\omega \ll m$ for the energy ω of the incident wave is fulfilled.. Thus, Thomson squared amplitude is proportional to $|\varepsilon^*(k')\cdot\varepsilon(k)|^2$.

In the next sections we show how to obtain Eq.(1) as the low energy limit of expressions at the level of scattering amplitudes. Then, we calculate the differential cross-section for targets of spins 0, 1/2, 1, and 3/2.

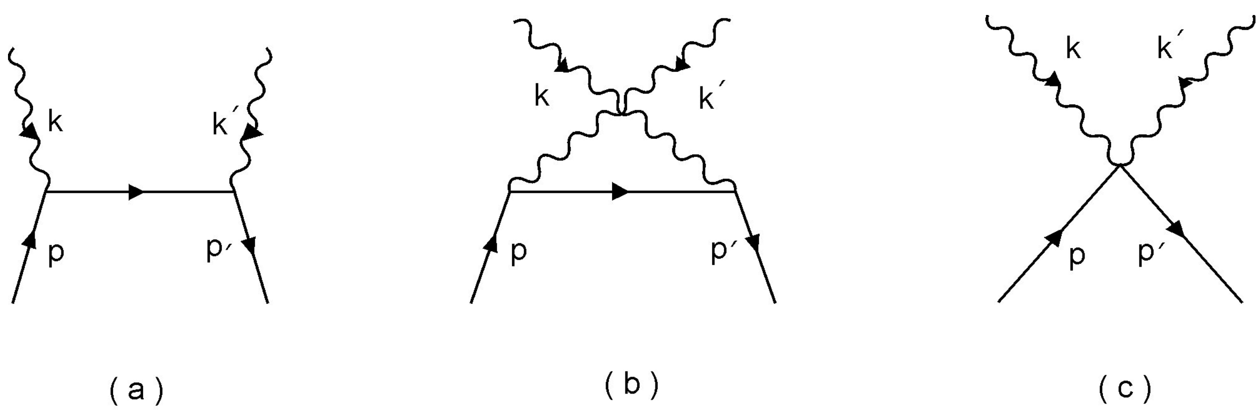

FIGURE 1 Feynman diagrams involved in Compton scattering for a spin-cero and spin-one target (a)-(c), and spin-1/2 and spin-3/2 target (a)-(b).

2. Spin-0 target

Electrodynamics of a spin-zero charged field ϕ is described by the Klein-Gordon Lagrangian density

$$L_{KG} = (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi, \tag{2}$$

where $D_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative, with A_μ the electromagnetic 4-vector. L_{KG} leads to the following Feynman rules [4]

$$\begin{aligned} V_\mu(p,p') &= -ie(p+p')_\mu, \\ V_{\mu\nu} &= 2ie^2 g_{\mu\nu}, \\ \Delta(q) &= \frac{i}{q^2 - m^2 + i\epsilon}, \end{aligned} \tag{3}$$

where the first line is the interaction vertex of the photon, index μ, the incident particle of 4-momentum p, and scattered particle of momentum p'. The second line is the vertex with two photons, the incident and the scattered one, and the two particles, and the last line is the spin-zero propagator. Adding the three amplitudes in Fig.1, and performing some simplifications, we obtain

$$\mathcal{M} = -2ie^2 \left(\frac{p_\nu p'_\mu}{p \cdot k} - \frac{p_\mu p'_\nu}{p \cdot k'} - g_{\mu\nu} \right) \varepsilon^{*\mu}(k') \varepsilon^\nu(k). \tag{4}$$

Polarization vectors of the incident and the scattered photons are denoted by $\varepsilon^\nu(k)$ and $\varepsilon^\mu(k')$, respectively. This amplitude is gauge invariant, as can be checked by making the substitutions $\varepsilon^{*\mu}(k') \to k'^\mu$, $\varepsilon^\nu(k) \to k^\nu$, and taking into account the relations $p+k = p'+k'$ and $k^2 = k'^2 = 0$. Now, Thomson limit means we have to do $k' \to k$ and $p' \to p$, for which Eq.(3) reduces to

$$\mathcal{M} \approx 2ie^2 g_{\mu\nu} \varepsilon^{*\mu}(k') \varepsilon^\nu(k). \tag{5}$$

At this point we have to remember that the angular differential cross-section in the elastic scattering $A+B \to A+B$, considering no target recoil, is

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{2M_B} \right)^2 \left\langle |\mathcal{M}|^2 \right\rangle, \tag{6}$$

where $\langle \, \rangle$ implies an average over spin states. For Eq.(4) the average is over photon polarization states, then

$$\frac{d\sigma}{d\Omega} \approx \left(\frac{e^2}{m} \right)^2 |\varepsilon^*(k') \cdot \varepsilon(k)|^2. \tag{7}$$

In laboratory system we can choose $\varepsilon^\mu(k) = (0, \varepsilon(k))$ and $\varepsilon^\mu(k') = (0, \varepsilon(k'))$, then Eq.(7) reduces to the expression in Eq.(1).

3. Spin-1/2 target

Electrodynamics of a spin $\frac{1}{2}$ field ψ is described by the Dirac lagrangian density

$$L_D = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi, \tag{8}$$

where the γ_μ are Dirac matrices. The corresponding Feynman rules are

$$\begin{aligned} V_\mu &= -ie\gamma_\mu, \\ D(p) &= \frac{i}{p \cdot \gamma - m + i\epsilon}. \end{aligned} \tag{9}$$

For this case we only have two diagrams, depicted in Fig. 1 (a) - (b). The total amplitude is

$$\begin{aligned} \mathcal{M} = ie^2 \overline{u}(p', s') \Bigg[& \frac{\gamma_\nu (p \cdot \gamma - k' \cdot \gamma + m) \gamma_\mu}{2p \cdot k'} \\ & - \frac{\gamma_\mu (p \cdot \gamma + k \cdot \gamma + m) \gamma_\nu}{2p \cdot k} \Bigg] u(p,s) \varepsilon^{*\mu}(k') \varepsilon^\nu(k), \end{aligned} \tag{10}$$

where $u(p,s)$ is particle spinor of momentum p and spin s;

again the amplitude is gauge invariant. In the limit $k' \to k$ and $p' \to p$, Eq.(10) reduces to

$$\begin{aligned} \mathcal{M} = & \frac{ie^2}{2p \cdot k} \overline{u}(p', s') \left(\gamma_\mu \gamma \cdot k \gamma_\nu + \gamma_\nu \gamma \cdot k \gamma_\mu \right) \\ & \times u(p,s) \varepsilon^{*\mu}(k') \varepsilon^\nu(k). \end{aligned} \tag{11}$$

Using some identities of the gamma matrices we obtain

$$\begin{aligned} \mathcal{M} &= \frac{ie^2}{p \cdot k} \overline{u}(p', s') \gamma \cdot k u(p,s) \varepsilon^*(k') \cdot \varepsilon(k) \\ &= \frac{ie^2}{p \cdot k} k^\mu \Lambda_\mu \varepsilon^*(k') \cdot \varepsilon(k), \end{aligned} \tag{12}$$

with $\Lambda_\mu = \overline{u}(p', s') \gamma_\mu u(p,s)$. Now, to calculate $\left\langle |\mathcal{M}|^2 \right\rangle$ we have to use some properties of the traces of gamma matrices and of the spin projector operator. After this is done, we arrive to

$$\left\langle |\mathcal{M}|^2 \right\rangle = 4e^2 |\varepsilon^*(k') \cdot \varepsilon(k)|^2. \tag{13}$$

From this equation we obtain, again, Eq. (1).

The preceding two cases are usually worked examples in books as those cited previously [2]. The less known cases of spin-1 and spin-3/2 are described in the next sections.

4. Spin-1 target

Electrodynamics of a spin-1 vector field ϕ_μ is described by the Proca lagrangian density

$$L_P = -\frac{1}{2} \phi^*_{\mu\nu} \phi^{\mu\nu} + m^2 \phi^*_\mu \phi^\mu, \tag{14}$$

with $\phi^{\mu\nu} = D^\mu \phi^\nu - D^\nu \phi^\mu$, and the Feynman rules are

$$V_{\mu\alpha\beta}(p_1,p_2) = -ie\left[(p_1+p_2)_\mu g_{\alpha\beta} - p_{2\alpha} g_{\beta\mu} - p_1 g_{\alpha\mu}\right],$$
$$V_{\mu\nu\alpha\beta} = -ie^2\left(2g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right),$$
$$D_{\mu\nu}(p) = -i\frac{g_{\mu\nu} - p_\mu p_\nu/m^2}{p^2 - m^2 + i\epsilon}. \quad (15)$$

Here, we have three diagrams as in the spin-0 case, with all the lines attached to vector particles. The total scattering amplitude is [5]

$$\mathcal{M} = -ie^2\left[\frac{\Lambda^{\alpha\beta}_{(a)}}{2p\cdot k} - \frac{\Lambda^{\alpha\beta}_{(b)}}{2p\cdot k} + \Lambda^{\alpha\beta}_{(c)}\right]\varepsilon_\alpha(p)\varepsilon_\beta(p'), \quad (16)$$

where $\varepsilon_\alpha(p)$ and $\varepsilon_\beta(p')$ are the polarization vectors of the spin-1 target particle. The second range tensors in Eq.(16) are

$$\Lambda^{\alpha\beta}_{(a)} = 4\left(p\cdot\varepsilon_1\right)\left(p_2\cdot\varepsilon_2^*\right)g^{\alpha\beta} - 2\left(p\cdot\varepsilon_1\right)k_2^\beta\varepsilon_2^{*\alpha} - 2\left(p\cdot\varepsilon_1\right)p'^\alpha\varepsilon_2^{*\beta} - 2\left(p'\cdot\varepsilon_1\right)k_1^\alpha\varepsilon_1^\beta + \left(\varepsilon_1\cdot\varepsilon_2^*\right)k_1^\alpha k_2^\beta$$
$$+\left(p'\cdot\varepsilon_1\right)k_1^\alpha\varepsilon_2^{*\beta} - 2\left(p'\cdot\varepsilon_2^*\right)p^\beta\varepsilon_1^\alpha + \left(p\cdot\varepsilon_2^*\right)k_2^\beta\varepsilon_1^\alpha + \left(p\cdot p'\right)\varepsilon_1^\alpha\varepsilon_2^{*\beta} + \frac{1}{m^2}\left(p\cdot\varepsilon_1\right)\left(q_1\cdot p'\right)k_1^\alpha\varepsilon_2^{*\beta}$$
$$-\frac{1}{m^2}\left(p\cdot\varepsilon_1\right)\left(p'\cdot\varepsilon_2^*\right)k_1^\alpha k_2^\beta + \frac{1}{m^2}\left(p\cdot q_1\right)\left(p'\cdot\varepsilon_2^*\right)\varepsilon_1^\alpha k_2^\beta - \frac{1}{m^2}\left(p\cdot q_1\right)\left(p'\cdot q_1\right)\varepsilon_1^\alpha\varepsilon_2^{*\beta}, \quad (17)$$

for $\Lambda^{\alpha\beta}_{(b)}$ we have to make the substitutions $\varepsilon_1 \longleftrightarrow \varepsilon_2$, $k_1 \longleftrightarrow -k_2$, and

$$\Lambda^{\alpha\beta}_{(c)} = -2\varepsilon_1\cdot\varepsilon_2^* g^{\alpha\beta} + \varepsilon_1^\alpha\varepsilon_2^{*\beta} + \varepsilon_1^\beta\varepsilon_2^{*\alpha}. \quad (18)$$

Verification of gauge invariance is more involved, as is the taking of the low energy limit. For Eq.(17) we obtain

$$\Lambda^{\alpha\beta}_{(a)} \approx 4\left(p\cdot\varepsilon_1\right)\left(p\cdot\varepsilon_2^*\right)g^{\alpha\beta} - 2\left(p\cdot\varepsilon_1\right)k_1^\beta\varepsilon_2^{*\alpha} - 2\left(p\cdot\varepsilon_2^*\right)k_1^\alpha\varepsilon_1^\beta$$
$$+\left(\varepsilon_1\cdot\varepsilon_2^*\right)k_1^\alpha k_1^\beta + 2\left(p\cdot\varepsilon_1\right)k_1^\alpha\varepsilon_2^{*\beta} + 2\left(p\cdot\varepsilon_2^*\right)k_1^\beta\varepsilon_1^\alpha - 2\left(p\cdot k_1\right)\varepsilon_1^\alpha\varepsilon_2^{*\beta} - \frac{1}{m^2}\left(p\cdot\varepsilon_1\right)\left(p\cdot\varepsilon_2^*\right)k_1^\alpha k_1^\beta - \frac{1}{m^2}\left(p\cdot k_1\right)^2\varepsilon_1^\alpha\varepsilon_2^{*\beta}, \quad (19)$$

similarly,

$$\Lambda^{\alpha\beta}_{(b)} \approx 4\left(p\cdot\varepsilon_1\right)\left(p\cdot\varepsilon_2^*\right)g^{\alpha\beta} + 2\left(p\cdot\varepsilon_2^*\right)k_1^\beta\varepsilon_1^\alpha - 2\left(p\cdot\varepsilon_1\right)k_1^\alpha\varepsilon_2^{*\beta}$$
$$+\left(\varepsilon_1\cdot\varepsilon_2^*\right)k_1^\alpha k_1^\beta + 2\left(p\cdot\varepsilon_2^*\right)k_1^\alpha\varepsilon_1^\beta - 2\left(p\cdot\varepsilon_1\right)k_1^\beta\varepsilon_2^{*\alpha} + 2\left(p\cdot k_1\right)\varepsilon_2^{*\alpha}\varepsilon_1^\beta - \frac{1}{m^2}\left(p\cdot\varepsilon_1\right)\left(p\cdot\varepsilon_2^*\right)k_1^\alpha k_1^\beta - \frac{1}{m^2}\left(p\cdot k_1\right)^2\varepsilon_2^{*\alpha}\varepsilon_1^\beta, \quad (20)$$

and $\Lambda^{\alpha\beta}_{(c)}$ remains unchanged. Substituting these expressions in Eq.(16) we find

$$\mathcal{M} = -2ie^2\left(\varepsilon_1\cdot\varepsilon_2^*\right)g^{\alpha\beta}\varepsilon_\alpha(p_1)\varepsilon_\beta^*(p_2). \quad (21)$$

In laboratory system, where in the spheric basis the components of the polarization vector $\varepsilon^\alpha(p_1)$ are

$$\varepsilon^\alpha(m,\pm) = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0),$$
$$\varepsilon^\alpha(m,0) = (0,0,0,1),$$

and for $\varepsilon^\beta(p_2)$ are

$$\varepsilon^\beta(p_2,\pm) = \mp\frac{1}{\sqrt{2}}(0,\cos\theta_2,\pm i,-\sin\theta_2),$$
$$\varepsilon^\beta(p_2,0) = \frac{1}{m}(p_2,E_2\sin\theta_2,0,E_2\cos\theta_2),$$

the product $g^{\alpha\beta}\varepsilon_\alpha(p_1)\varepsilon_\beta^*(p_2) = -1$, and Eq.(21) takes the form

$$\mathcal{M} = 2ie^2\left(\varepsilon_1\cdot\varepsilon_2^*\right). \quad (22)$$

Now we have to compute $\left\langle|\mathcal{M}|^2\right\rangle$, for which we sum over the massive vector boson polarizations,

$$\sum_{s_1,s_2}\varepsilon_\alpha(p,s_1)\varepsilon_\beta^*(p,s_2) = -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2}. \quad (23)$$

Then, from Eq.(21) we finally obtain

$$\left\langle|\mathcal{M}|^2\right\rangle = 4e^4\left|\varepsilon_1\cdot\varepsilon_2\right|^2. \quad (24)$$

5. Spin-3/2 target

Electrodynamics of a spin-3/2 field Ψ_μ is described by the Rarita-Schwinger Lagrangian density

$$L_{RS} = \overline{\Psi}^\mu \left[g_{\mu\nu}(i\gamma_\alpha D^\alpha - m) + \frac{i}{3}\gamma_\mu(\gamma_\alpha D^\alpha)\gamma_\nu - \frac{i}{3}(\gamma_\mu D_\nu + \gamma_\nu D_\mu) + \frac{m}{3}\gamma_\mu\gamma_\nu \right] \Psi^\nu. \quad (25)$$

The Rarita-Schwinger spin-vector Ψ_μ is restricted to supplementary conditions

$$\partial^\mu \Psi_\mu = 0, \qquad \gamma^\mu \Psi_\mu = 0, \quad (26)$$

in order to eliminate the spin-1/2 components, which arise as a consequence of the making of Ψ_μ through the product of a spin-1/2 and a spin-1 fields. The Lagrangian L_{RS} is invariant under contact transformations

$$\Psi_\mu \to R_{\mu\nu}\Psi^\nu, \quad (27)$$

where

$$R_{\mu\nu}(A) = g_{\mu\nu} + \frac{3A+1}{2}\gamma_\mu\gamma_\nu, \quad (28)$$

and the parameter $A \neq -\frac{1}{2}$ is otherwise arbitrary. After a contact transformation, Eq.(25) transforms to

$$L_{RS} = \overline{\Psi}^\mu \left[g_{\mu\nu}(i\gamma_\alpha D^\alpha - m) + iB\gamma_\mu(\gamma_\alpha D^\alpha)\gamma_\nu + iA(\gamma_\mu D_\nu + \gamma_\nu D_\mu) + mC\gamma_\mu\gamma_\nu \right] \Psi^\nu, \quad (29)$$

where $B = \frac{3}{2}A^2 + A + \frac{1}{2}$ and $C = 1 + 3A + 3A^2$. From Eq.(29) we can read the Feynman rules for the electromagnetic vertex

$$\Gamma^\alpha_{\beta\sigma}(A) = -iR^\mu_\beta(A)\Gamma^\alpha_{\mu\nu}R^\nu_\sigma(A),$$
$$\Gamma^\alpha_{\mu\nu} = -e\left[g_{\mu\nu}\gamma^\alpha + \frac{1}{3}\gamma_\mu\gamma^\alpha\gamma_\nu - \frac{1}{3}\left(\gamma_\mu g^\alpha_\nu + \gamma_\nu g^\alpha_\mu\right) \right], \quad (30)$$

and the spin-3/2 propagator

$$\Delta_{\mu\nu}(A,p) = -\frac{i}{p^2 - m^2 + i\epsilon}\left[2mS_{\mu\nu}(p) - \frac{1+A}{6(1+2A)}\frac{p^2-m^2}{m} \left(\frac{2}{m}(\gamma_\mu p_\nu + \gamma_\nu p_\mu) - 2\gamma_\mu\gamma_\nu - \frac{1+A}{1+2A}\left(\frac{1}{m}\gamma_\mu\gamma_\alpha p^\alpha\gamma_\nu - 2\gamma_\mu\gamma_\nu\right) \right) \right],$$
$$S_{\mu\nu}(p) = \left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3m}\left(\gamma_\mu p_\nu - \gamma_\nu p_\mu\right) + \frac{2}{3m^2}p_\mu p_\nu \right] \frac{\gamma_\alpha p^\alpha + m}{2m}. \quad (31)$$

Physical quantities, as the differential cross-section, must not depend on the parameter A.

Like in the case of spin-1/2 we have two diagrams only; and the total amplitude is given by [6]

$$\mathcal{M} = \mathcal{M}_{(a)} + \mathcal{M}_{(b)}, \quad (32)$$

with

$$\mathcal{M}_{(a)} = \overline{u}^\nu(p',s')\left(\Gamma^\beta_{\nu\sigma}(A)\varepsilon_\beta(k_2)\right)\Delta^{\sigma\rho}(A,q)\left(\Gamma^\alpha_{\rho\mu}(A)\varepsilon_\alpha(k_1)\right)u^\mu(p,s),$$
$$\mathcal{M}_{(b)} = \overline{u}^\nu(p',s')\left(\Gamma^\alpha_{\nu\sigma}(A)\varepsilon_\alpha(k_1)\right)\Delta^{\sigma\rho}(A,q')\left(\Gamma^\beta_{\rho\mu}(A)\varepsilon_\beta(k_2)\right)u^\mu(p,s).$$

It is straightforward to show that the parameter A in $\mathcal{M}_{(a)}$ and $\mathcal{M}_{(b)}$ disappears using some properties of Dirac gamma matrices and the equation of motion for the Rarita-Schwinger spinor u^μ:

$$R^\eta_\sigma(A)\Delta^{\sigma\rho}(A,q)R^\omega_\rho(A) = \Delta^{\eta\omega}(q),$$

and Eq.(26) to find

$$\mathcal{M}_{(a)} = \overline{u}^\nu(p',s')O^{(a)}_{\mu\nu}u^\mu(p,s),$$
$$O^{(a)}_{\mu\nu} = \frac{ie^2}{2p\cdot k_1}\Big[-g_{\mu\nu}\gamma_\alpha(2p_\beta + k_1\cdot\gamma\gamma_\beta) + \frac{2}{3m^2}\gamma_\alpha(2p_\beta + m\gamma_\beta + k_1\cdot\gamma\gamma_\beta)k_{1\mu}k_{1\nu} - \frac{2}{3m}\left(\gamma_\alpha k_{1\mu}(m + k_1\cdot\gamma)g_{\nu\beta} + g_{\alpha\mu}k_{1\nu}(2p_\beta + k_1\cdot\gamma\gamma_\beta - m\gamma_\beta)\right) - \frac{4}{3}k_1\cdot\gamma g_{\alpha\mu}g_{\beta\nu} - \frac{4}{3m^2}p\cdot k_1\left(g_{\beta\nu}k_{1\mu}\gamma_\alpha - g_{\alpha\mu}k_{1\nu}\gamma_\beta + g_{\alpha\mu}g_{\beta\nu}(k_1\cdot\gamma - m)\right)\Big]\varepsilon^\alpha(k_1)\varepsilon^{*\beta}(k_2), \quad (33)$$

and similarly

$$\mathcal{M}_{(b)} = \overline{u}^{\nu}(p',s')O^{(b)}_{\mu\nu}u^{\mu}(p,s),$$

$$O^{(a)}_{\mu\nu} = -\frac{ie^2}{2p\cdot k_2}\Big[-g_{\mu\nu}\gamma_\beta(2p_\alpha - k_2\cdot\gamma\gamma_\alpha) + \frac{2}{3m^2}\gamma_\beta(2p_\alpha + m\gamma_\alpha - k_2\cdot\gamma\gamma_\alpha)k_{2\mu}k_{2\nu} + \frac{2}{3m}\left(\gamma_\beta k_{2\mu}(m - k_2\cdot\gamma)g_{\nu\alpha} + g_{\beta\mu}k_{2\nu}(2p_\alpha - k_2\cdot\gamma\gamma_\alpha - m\gamma_\alpha)\right) + \frac{4}{3}k_2\cdot\gamma g_{\beta\mu}g_{\alpha\nu} - \frac{4}{3m^2}p\cdot k_2\left(g_{\alpha\nu}k_{2\mu}\gamma_\beta - g_{\beta\mu}k_{2\nu}\gamma_\alpha + g_{\beta\mu}g_{\alpha\nu}(k_2\cdot\gamma + m)\right)\Big]\varepsilon^{\alpha}(k_1)\varepsilon^{*\beta}(k_2). \quad (34)$$

As in previous calculations the total amplitude is gauge invariant. The limit of photon low energy leads to

$$\mathcal{M} = \overline{u}^{\nu}(p',s')O_{\mu\nu}u^{\mu}(p,s),$$

$$O_{\mu\nu} = \frac{ie^2}{2p\cdot k_1}\Big[2g_{\alpha\beta}g_{\mu\nu}\gamma\cdot k + \frac{4}{3}(g_{\alpha\mu}\gamma_\beta + g_{\beta\mu}\gamma_\alpha)k_{1\nu} + \frac{4}{3m}(g_{\alpha\mu}g_{\nu\beta} + g_{\beta\mu}g_{\alpha\nu})(p\cdot k_1 - m\gamma\cdot k\Big] \times\varepsilon^{\alpha}(k_1)\varepsilon^{*\beta}(k_2). \quad (35)$$

In laboratory system $k_1^{\mu} = (\omega,0,0,\omega)$, and $p\cdot\varepsilon(k_1) = 0$, $p\cdot\varepsilon^*(k_2) = 0$, along with the use of Rarita-Schwinger equation, reduces $O_{\mu\nu}$ to

$$O_{\mu\nu} = \frac{ie^2}{2p\cdot k_1}\Big[2g_{\alpha\beta}g_{\mu\nu}\gamma\cdot k + \frac{4}{3}(g_{\alpha\mu}\gamma_\beta + g_{\beta\mu}\gamma_\alpha)k_{1\nu}\Big]\varepsilon^{\alpha}(k_1)\varepsilon^{*\beta}(k_2). \quad (36)$$

Notice that the first term is quite similar to Eq.(12), then the second one must vanishes. This is so when we explicitly consider the contraction of indices. Finally,

$$\mathcal{M} = 2ie^2\left(\varepsilon_1\cdot\varepsilon_2^*\right)$$

6. Conclusions

There exist in the literature a general result for the Compton amplitudes at strictly zero frecuency [7]. If s is the spin of the target particle, the most general form of the zero-energy amplitude matrix is

$$\mathcal{M}(0) = \mathcal{M}_1(0)\varepsilon_1\cdot\varepsilon_2^* + \mathcal{M}_2(0)\mathbf{s}\cdot(\varepsilon_2^*\times\varepsilon_1) + \mathcal{M}_3(0)\mathbf{s}\cdot\left((\varepsilon_1\times\mathbf{n}_1)\times(\varepsilon_2^*\times\mathbf{n}_2)\right), \quad (37)$$

where the amplitudes $\mathcal{M}_i(0)$ are given by

$$\mathcal{M}_1(0) = \frac{e^2}{m}, \quad (38)$$

which is the Thomson limit,

$$\mathcal{M}_2(0) = 0, \quad (39)$$

which is a consequence of crossing symmetry, and

$$\mathcal{M}_3(0) = 0, \quad (40)$$

implying the vanishing of double spin-flip.What we have done in this work is to show how to obtain Thomson scattering formula as the limit of the scattering amplitude of a photon from target particles of spins 0, 1/2, 1, and 3/2 explicitly.

Needless to say that Thomson scattering gives no information on particle structure, for which we need, at least, the next order in perturbation theory, related to particle magnetic dipole moment as stated in Low's theorem [8].

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