On Casimir forces for media with arbitrary dielectric properties

W. L. Mochán
Centro de Ciencias Físicas, Universidad Nacional Autónoma de México
Av. Universidad S/N, Cuernavaca, Morelos 62210, México.

C. Villarreal and R.Esquivel-Sirvent
Instituto de Física, Universidad Nacional Autónoma de México
Ciudad Universitaria, D.F. 04510, México.

Recibido el 19 de febrero de 2002; aceptado el 1 de abril de 2002

We derive an expression for the Casimir force between slabs with arbitrary dielectric properties characterized by their reflection coefficients. The formalism presented here is applicable to media with a local or a non-local dielectric response, an infinite or a finite width, inhomogeneous dissipative, etc. Our results reduce to the Lifshitz formula for the force between semi-infinite dielectric slabs by replacing the reflection coefficients by the Fresnel amplitudes.

Keywords: Casimir forces; dielectrics; Lifshitz formula.

Se presenta una deducción para la expresión de la fuerza de Casimir entre placas con propiedades dieléctricas arbitrarias caracterizadas por sus coeficientes de reflección. El formalismo que presentamos es válido para medios con una respuesta dieléctrica local, no local, placas de ancho finito o semi–infinito, inhomogéneos, disipativos, etc. Nuestros resultados se reducen a la fórmula de Lifshitz para la fuerza entre placas dieléctricas semi–infinitas substituyendo los coeficientes de reflección por las amplitudes de Fresnel.

Descriptores: Fuerzas de Casimir; dieléctricos; fórmula de Lifshitz.

PACS: 12.20.D.s; 03.70.+k; 77.55.+f; 78.67.-n

1. Introduction

Even though Casimir [1] predicted in 1948 an attractive force between perfectly conducting plates placed in quantum vacuum, it is only in recent years that experimental studies of Casimir forces have reached the necessary accuracy to test in detail theoretical predictions. The first measurements were done by Derjaguin *et al*. [2] in 1951 using dielectric materials. In the following decades, a number of experiments to measure Casimir interactions between dielectric or conducting materials were performed, however involving large relative errors in the measured forces [3]. It was until 1997 that Lamoreaux [4] performed measurements with a precision of the order of 5 % by using an electromechanical system based on a torsion balance. Other experiments were made taking advantage of the sensitivity of atomic force microscopes achieving precisions close to 1% [6, 7]. Additional measurements have been made by Chan *et al*. [8] using a micro torsional balance. This experiment is representative of the effects that Casimir forces have in micromechanical systems as was theoretically shown by Serry and Maclay [9]. Applications to nanostructures have also been considered [10–12]. This has boosted investigations in which the detailed properties of the materials such as absorptivity, rugosity, or finite temperature effects are taken into account in the theoretical calculations of the Casimir forces [13].

The standard approach to study vacuum forces between imperfect conductors is the macroscopic theory proposed by Lifshitz [14] in 1956 for semi-infinite dielectric materials. In this theory, the dissipative effects associated to the radiation reaction of the elementary atomic dipoles composing the dielectric is balanced by the fluctuating vacuum field in accordance with the fluctuation-dissipation theorem.

For a configuration of two semi-infinite slabs, with dielectric permitivities ϵ_1 and ϵ_2, separated by a gap of width L and permitivity ϵ_3, the Lifshitz formula for force per unit area is

$$F(L) = -\frac{\hbar}{2\pi^2 c^3}\int_1^\infty dp p^2 \int_0^\infty d\xi \xi^3 \epsilon_3^{3/2} \times \left[G_1(\xi,p)^{-1} + G_2(\xi,p)^{-1}\right], \quad (1)$$

with

$$G_1(\xi,p) = \frac{\epsilon_3 s_1 + \epsilon_1 p}{\epsilon_3 s_1 - \epsilon_1 p}\frac{\epsilon_3 s_2 + \epsilon_2 p}{\epsilon_3 s_2 - \epsilon_2 p} \times e^{2\xi p\sqrt{\epsilon_3}L/c} - 1, \quad (2)$$

and

$$G_2(\xi,p) = \frac{s_1 + p}{s_1 - p}\frac{s_2 + p}{s_2 - p} \times e^{2\xi p\sqrt{\epsilon_3}L/c} - 1, \quad (3)$$

where $\xi = i\omega$ is an imaginary frequency, p and s_1, s_2 are defined in terms of the momenta parallel and perpendicular to the slabs k, and $K_i^2 = k^2 + \epsilon_i(i\xi)\xi^2/c^2$, respectively: $K_3 = \sqrt{\epsilon_3}\xi p/c$ and $K_{1,2}^2 = \epsilon_3 \xi^2 s_{1,2}^2/c^2$. Lifshitz theory has been succesfully employed in a number of experimental situations, and it yields the Casimir force for perfect conductors. However, as pointed out by Barash and Ginzburg [15], it is not clear how to generalize the theory to more complex problems, such as nonplanar surfaces, multilayer systems, anisotropic media, etc. Thus, they proposed an alternative approach to Lifshitz formula. As a dissipative system does not posses well defined natural frequencies of oscillation, they introduced an auxiliary system in which the dielectric permitivity depended only parametrically on the frequency. This procedure enabled them to calculate the free energy of the field as a sum over allowed states of the system. In turn, the force per unit area was obtained as the derivative of the free energy. Kats [22] re-elaborated the formalism of Barash and Ginzburg by writing the dispersion law for surface electromagnetic waves in terms of the reflection amplitudes r^s and r^p of the media for s and p polarizations. Approximating the reflection coefficients in terms of the frequency and wavevector dependent surface impedance $Z(\omega, Q)$ he was able to obtain approximate non-local corrections to the Casimir force for good conductors. Kats remarked that dielectrics materials required alternative formulations, as their reflection coefficients cannot be expressed merely in terms of surface impedances. This is not necessarily correct, as an exact relation between surface impedance and reflection coefficients may indeed be introduced for arbitrary systems [25]. Jaekel and Reynaud [26] also rederived Lifshitz formula in terms of reflection coefficients for partially transmitting mirrors. Their expression reduces to that obtained by Barash and Ginzburg [15]. However, their derivation is not valid when dissipation is included.

Other approaches have been used to study the vacuum fluctuations in the presence of dielectrics [16–18]. More recently, the problem of quantization in absorbing media and its applications to Casimir forces has been considered by Kupiszewska [19, 20] and also by Matloob [21] in the one-dimensional case. Interestingly enough, when temperature effects are neglected the expression for the Casimir force in absorbing and non-absorbing materials has the same functional form. This fact suggests that it is possible to obtain the Casimir force between two dispersive and absorbing slabs without the need of quantizing in an absorbing medium as was shown by Reynaud *et al*. [26] using a scattering matrix formalism.

Within the framework of the above discussion, it seems valuable to present an alternative, very simple derivation of the Casimir force, valid for materials with arbitrary dielectric properties. This is the purpose of the present paper.

2. Formalism

Consider two slabs $a = 1, 2$ parallel to the xy plane within free space and separated by a distance L along the z-direction, with inner boundaries at $z_1 = 0$ and $z_2 = L$ as shown in Fig. 1. We assume that the slabs are non-chiral, translational invariant and isotropic within the xy plane, but otherwise they may be arbitrary; they could be identical to each other or different, they might have a local or a non-local dielectric response, an infinite or a finite width, they may be opaque or transparent, dissipative, inhomogeneous, etc.

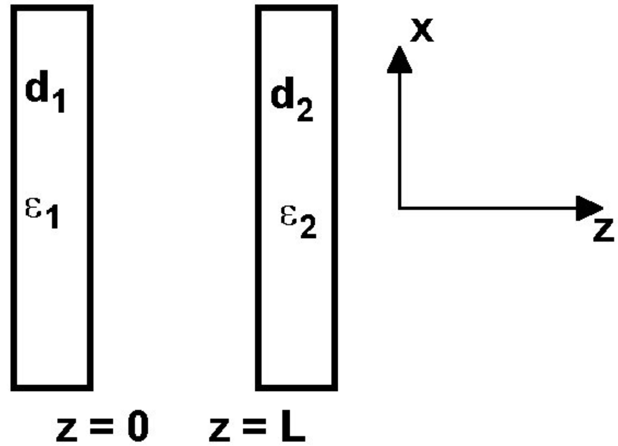

FIGURE 1. The system consists of two parallel slabs characterized by their thickness d_1 amd d_2 and a dielectric function ϵ_1 and ϵ_2.

We want to describe the electromagnetic field only within vacuum, so we hide all the details of the field-matter interaction within the slabs in their reflection amplitudes r_a^s and r_a^p ($a = 1, 2$). The reflection coefficients r_a^α are determined by the generalized surface impedances Z_a^α ($\alpha = s, p$) through

$$r_a^\alpha = \frac{Z_a^\alpha - Z_0^\alpha}{Z_a^\alpha + Z_0^\alpha}. \tag{4}$$

Here Z_a^α is *defined* as the quotient $E_{a\|}^\alpha / H_{a\|}^\alpha$ of the components $E_{a\|}^\alpha$ and $H_{a\|}^\alpha$ of the α-polarized electric and magnetic fields evaluated at the a-th interface for outgoing boundary conditions beyond z_a, taken along appropriately chosen directions parallel to the surface, $Z_0^s = q/k$ and $Z_0^p = k/q$ are the surface impedances of vacuum, $\vec{q}_\pm = (\vec{Q}, \pm k)$ are the vacuum wavevectors with projection $\vec{Q}$ parallel to the surface and components $\pm k$ normal to the surface, $q \equiv \sqrt{q_\pm^2} = \omega/c$, where ω is the frequency and c the speed of light. The sign of k is chosen so that $\vec{q}_+$ propagates (or decays) as z increases. Upon each reflection, $\vec{Q}$ and ω are conserved while the sign of $\pm k$ is reversed. Notice that Eq. (4) *is exact* and that no approximation is involved by the use of our generalized surface impedances, unlike other works [23, 24] that use an inappropiate definition of surface impedance. It should be noticed that for local homogeneous semiinfinite media, $Z_a^s = q/k_a$ and $Z_a^p = k_a/(\epsilon_a q)$, where k_a is the component of the wavevector normal to the surface within medium a with local dielectric response $\epsilon_a(\omega)$, and Eq. (4) yields the well known Fresnel amplitudes. However, Eq. (4) is much more general [25].

The density of states within vacuum may be obtained from the Green's functions of the system. To this end, we study first the case of s-polarized waves choosing $x - z$ as the plane of incidence. With that choice, $\vec{E} = (0, E_y, 0)$, $\vec{B} = (B_x, 0, B_z)$, $\vec{q}_\pm = (Q, 0, \pm k)$ and the boundary conditions for $\vec{E}$ become $iqE_y(0^+) = -Z_1^s \partial_z E_y(0^+)$, and $iqE_y(L^-) = Z_2^s \partial_z E_y(L^-)$, with $\partial_z \equiv \partial/\partial z$. The electric Green's function is

$$G_{k^2}^E(z, z') = \frac{E_y^<(z_<) E_y^>(z_>)}{W}, \tag{5}$$

where $z_<$ and $z_>$ are the smaller and larger of z and z' respectively,

$$E_y^<(z) = e^{-ikz} + r_1^s e^{ikz} \tag{6}$$

and

$$E_y^>(z) = e^{ik(z-L)} + r_2^s e^{-ik(z-L)} \tag{7}$$

are solutions of the scalar 1D wave equation with wavenumber k obeying the appropriate boundary conditions at $z = 0^+$ and L^- respectively, and W is their Wronskian. Analogously, $\vec{B}$ obeys $iqB_x(0^+) = -(Z_0^s)^2 \partial_z B_x(0^+)/Z_1^s$, and $iqB_x(L^-) = (Z_0^s)^2 \partial_z B_x(L^-)/Z_2^s$, so that the magnetic Green's function is obtained by replacing $E_y \to B_x$ and $r_a^s \to -r_a^s$ in Eqs. (5)-(7). We do not consider B_z separately, as it is simply proportional to E_y.

For each $\vec{Q}$, the local density of states per unit k^2 is given by [27]:

$$\rho_{k^2}^s(z) = -\frac{1}{2\pi} \mathrm{Im}\left(G_{\tilde{k}^2}^E(z,z) + G_{\tilde{k}^2}^B(z,z)\right), \qquad (\tilde{k} \equiv k + i0^+) \tag{8}$$

so that by substituting Eqs. (5)-(7) and its magnetic analogues we obtain

$$\rho_{k^2}^s = \frac{1}{2\pi\tilde{k}} \mathrm{Re}\left[\frac{1 + r_1^s r_2^s e^{2i\tilde{k}L}}{1 - r_1^s r_2^s e^{2i\tilde{k}L}}\right], \tag{9}$$

independent of z. The density $\rho_{k^2}^p$ corresponding to p polarization may be derived similarly, and is simply given by Eq. (9) after replacing all the superscripts $s \to p$. Finally, the total density of states is $\rho_{k^2} = \rho_{k^2}^s + \rho_{k^2}^p$.

A photon in a state characterized by $\alpha, \vec{Q}$ and k^2 has momentum $\pm\hbar k$ and moves with velocity $\pm ck/q$ along the z direction, so that its contribution to the momentum flux is $\hbar c k^2/q$. Multiplying this by the photon occupation number, integrating over k^2 with the weight function $\rho_{k^2}^\alpha$ and adding the contributions from all values of $\alpha = s, p$ and $\vec{Q}$, with the usual replacement $\sum_{\vec{Q}} \ldots \to A/2\pi \int Q dQ \ldots$, we obtain the momentum flux from the vacuum gap into slab 2. There is a similar contribution coming from the semiinfinite vacuum on the other side of the slab, obtained by substituting $r_2^\alpha \to 0$ above and reversing the flux direction $z \to -z$. The total force per unit area is obtained by subtracting the contributions from either side [28], that is:

$$F(L) = \mathcal{A}\frac{\hbar c}{2\pi^2} \int_0^\infty dQ Q \int_{q \geq 0} dk \frac{\tilde{k}^2}{q} \times \mathrm{Re}\left[\frac{r_1^s r_2^s e^{2i\tilde{k}L}}{1 - r_1^s r_2^s e^{2i\tilde{k}L}} + \frac{r_1^p r_2^p e^{2i\tilde{k}L}}{1 - r_1^p r_2^p e^{2i\tilde{k}L}}\right]. \tag{10}$$

The integral over k runs from iQ to 0 and then to ∞, so that q remains real and positive. It is easy to show that for perfect mirrors ($r_a^\alpha = \pm 1$) Eq. (10) yields the expected Casimir force. Furthermore, simple substitution of the Fresnel amplitudes

$$r_a^s = \frac{k - k_a}{k + k_a}, \tag{11}$$

$$r_a^p = \frac{k_a - \epsilon_a k}{k_a + \epsilon_a k}, \tag{12}$$

and manipulation of the integration contours in Eq. (10) leads to the formula of Lifshitz [Eq.(1)].

3. Conclusions

We have derived a general expression for Casimir forces between slabs with arbitrary dielectric properties characterized by the reflection coefficients of the material. This procedure avoids complications related to the quantization of the electromagnetic field in dispersive media. The expression we obtain for the Casimir force is convenient for calculations since the reflection coefficients can be obtained straightforwardly in theoretical computations or through experimental studies. Our approach is based on the exact definitions of surface impedance and yields the Lifshitz formula for semi-infinite slabs when the reflection coefficients are replaced by the Fresnel amplitudes. This contradicts the results obtained by Mostepanenko and Trunov [23] and also by Bezerra *et al.* [24] that claim that the use of surface impedances is only an approximation valid for small transverse wave vector **Q**. However, this conclusion arises from considering an approximate expression of the surface impedance. In our work this limitation is not present, since we do take into account that the surface impedances for the p and s polarized waves are different. Therefore, the results that we obtain are valid for any value of the wave vector **Q**. The expresion for the Casimir force in Eq. (10) can be used to calculate accurately the force between non-homogeneous systems. In fact, current experimental setups for measuring Casimir forces consist of two metallic surfaces with a high reflectivity in the frequency range of interest $\omega \sim c/a$, coated by thin (~ 8 nm) Au/Pd layer to avoid Al oxidation. The analysis of the experimental data is performed by giving arguments on the transparency of the Au/Pd layer, which allows the use of the Lifshitz expression for semi-infinite homogeneous media with semi-empirical corrections for the presence of that layer. However, within our formalism, such assumptions are unecessary. In a future work, we will present exact results for Casimir forces between heterostructured media, generalizing our previous one-dimensional analysis [10].

Acknowledgments

This work was partially supported by NASA Breakthrough Propulsion Physics Project, and by DGAPA-UNAM Project IN-110999.

1. H. B. G. Casimir, *Proc. Kon. Ned. Akad. Wet.* **51** (1948) 793.
2. B. V. Derjaguin, I. I. Abrikosova, *Vestnik Akad. Nauk. SSSR* **6** (1951) 125.
3. P. W. Milonni and Mei-Li Shih, *Contemporary Physics* **33** (1992) 313.
4. S. K. Lamoreaux, *Phys. Rev. Lett.* **78** (1997) 5.
5. H. B. Chan, V. A. Aksyuk, R. N. Kliman, D. J. Bishop and F. Capasso, *Science* **291** (2001) 1942.
6. U. Mohideen and Anushree Roy, *Phys. Rev. Lett.* **81** (1998) 4549.
7. B. W. Harris, F. Chen, and U. Mohideen, *Phys. Rev. A* **62** (2000) 052109.
8. H. B. Chan, V. A. Aksyuk, R. N. Kliman, D. J. Bishop and F. Capasso, *Science* **291** (2001) 1942.
9. F. M. Serry, D. Walliser and G. J. Maclay, *J. Appl. Phys.* **84** (1998) 2501.
10. R. Esquivel-Sirvent, C. Villarreal y G. H. Cocoletzi, *Phys. Rev. A* **64** (2001) 052108 .
11. C. Villarreal, R. Esquivel-Sirvent and G. H. Cocoletzi, *Int. J. Mod. Phys. A* (in press).
12. R. Esquivel-Sirvent, C. Villarreal, G. H Cocoletzi and W. L. Mochán, *Phys. Stat. Sol.* (in press).
13. M. Bordag, B.Geyer, G.L. Klimchitskaya, and V.M. Mostepanenko, *Phys. Rev. Lett.* **85** (2000) 503; G.L. Klimchitskaya, and V.M. Mostepanenko, *Phys. Rev.* **63** (2001) 062108.
14. E. M. Lifshitz, *Sov. Phys. JETP* **2** (1956) 73.
15. Yu. S. Barash and V.L. Ginzburg, *Sov. Phys.-Usp.*, **18** (1975) 305.
16. N. G. Van Kampen, B. R. A. Nijboer, and K. Schram, *Phys. Lett.* **26A** (1968) 307.
17. P. Candelas, *Ann. Phys.* (N.Y.) **143** (1982) 241.
18. R. Matloob, *Phys. Rev. A* **60** (1999) 50.
19. D. Kupiszewska and J. Mostowski, *Phys. Rev. A* **41** (1990) 4636.
20. D. Kupiszewska, *Phys. Rev. A* **46** (1992) 2286.
21. R. Matloob, A. Keshavaraz, and D. Sedighi, *Phys. Rev. A*, **60** (1999) 3410.
22. E. I. Kats, *Sov. Phys. JETP* **46** (1977) 109.
23. V. M. Mostepanenko and N. N. Trunov, *Sov. J. Nucl. Physics* **42** (1985) 818.
24. V. B. Bezerra, G. L. Klimchitskaya and C. Romero, *Phys. Rev. A* **65** (2001) 012111.
25. See the review books *Photonic Probes of Surfaces*, edited by P. Halevi (Elsevier, Amsterdam, 1995) and *Spatial Dispersion in Solids and Plasmas, Electromagnetic Waves*, Vol. 1, ed. by P. Halevi (North-Holland, Amsterdam, 1992). For an example of a nontrivial application of the surface impedance to non-local excitonic semiconductor superlattices see G. H. Cocoletzi and W. Luis Mochán, *Phys. Rev. B* **39** (1989) 8403.
26. M. T. Jaekel and S. Reynaud, *J. Phys. I France* **1** (1991) 1395; R. Matloob, A. Keshavaraz, and D. Sedighi, *Phys. Rev. A* **60** (1999) 3410.
27. G. Plunien, B. Müller and W. Greiner, *Phys. Rep.* **134** (1986) 87.
28. A. E. González, *Physica A* **131** (1985) 228.