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On modeling some peculiarities of the tropical atmosphere circulation

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RESUMEN

Las perturbaciones solitarias y los movimientos ondulatorios de diferentes escalas en el flujo general del Este son singularidades características de los procesos atmosféricos en las latitudes bajas. Sistemas relativamente simples de las ecuaciones de hidrotermodinámica permiten, con cierta aproximación, modelar tales singularidades. Un sistema no lineal contiene las soluciones que pertencen a la clase de perturbaciones solitarias en un área limitada; tales soluciones se llaman "modones". El otro sistema, que es lineal, describe prácticamente todas las ondas que se observan en las latitudes bajas, incluso en la zona equatorial. Tres partes del espectro se revelan especialmente claras: la primera parte corresponde al intervalo con un periodo del orden de decenas de minutos, la segunda con un periodo del orden de uno a diez días, y la tercera parte, corresponde a la oscilación quasi-bienal del viento zonal.

ABSTRACT

The solitary disturbances and wave motions of various scales are the peculiarities which appear on the general east flow in the tropical atmosphere. Relatively simple systems of the hydrothermodynamic equations permit the modeling of these peculiarities. A nonlinear system allows to find a solition's type solution in a limited area. Such solution is called "modon". A linear system contains almost all wave motions in the low latitudes including the equatorial belt. Three parts of the spectrum are revealed especially clear: the first part corresponds to the period interval of some dozen minutes, the second one to the order one to ten days, and the third part corresponds to the quasi-biennial oscillation of the zonal wind.

Introduction

The investigation of the tropical circulation is one of the most important aims of the general meteorology and especially of dynamic meteorology. There are many difficulties in attempting to construct the circulation models close to reality. Among them at least two important should be pointed out. Both these difficulties are connected with the peculiarities of atmospheric motions within the equatorial belt. In comparison with the synoptic scale the width of this belt is rather small - some hundreds kilometers. The simple considerations lead to the following formula for the width of the equatorial belt in each hemisphere:

$$L = (2ah)^{1/2}$$

where $a = 6.37 \cdot 10^6$m is the mean radius of the Earth; h is the vertical scale of the atmosphere, say the thickness of the troposphere $\approx$ 16 - 17 km. Thus $L \approx 500$ km. But the length of the belt is 40000 km. Therefore, in this belt even the sub-synoptic scale disturbances are not isotropic. In the equatorial belt the main Coriolis terms $2\omega\sin\varphi \cdot v$ and $2\omega\sin\varphi \cdot u$ are comparable with the usually omitted "small" term $2\omega\cos\varphi \cdot w$. (Here $\omega = 7.29 \cdot 10^{-5}$s, is the angular frequency of the Earth's rotation; u, v, w are the velocity components in the x, y, z directions, respectively; and φ is the latitude.) Thus, in the equatorial belt all components of the Coriolis acceleration should be taken into account. Consequently, barotropic models without vertical velocity and any form of geostrophic approximation are not valid within this belt. It is well known that the presence of the small term $2\omega\cos\varphi \cdot w$ does not permit the use of the conventional Fourier method of variable separation.

The first difficulty arises when we try to construct some kind of dynamical model valid for the whole tropical zone including the narrow equatorial belt. That is why most of the models, for instance β -plane approximation models, may be applied to the tropical atmosphere excluding the equatorial belt.

The second important difficulty concerns the non-linear terms in the momentum equations. These terms are of the same order of magnitude as the linear terms: the Coriolis acceleration terms; pressure gradient components or/and the temperature gradient components. Therefore, to describe the small scale processes in the equatorial belt we have to include the nonlinear terms together with the conventional linear terms. Usually the models of tropical disturbances for the extra-equatorial belt are linear. And it is very difficult to conjugate the nonlinear equatorial model with the linear tropical one.

In this paper two models will be described. The first one is solitary disturbance of finite radius, which is valid to the equatorial troposphere. The second is a linear wave model which to some extent may be applied to the whole tropical zone.

I. The quasi-three-dimensional modons in the equatorial troposphere

The satellite pictures and data analysis of GATE and FGGE clearly show the existence of atmospheric structures of various scales situated more or less parallel to the equator. Cloud streets, cloud stratus, squall lines, ITCZ are the typical representatives of such structures. The cloud pictures in Fig. 1 give the impression of disturbances of various scales and forms stretched out along latitude circles. Each structure may be considered as wave or solitary form disturbance in the easterly flow. As long as the length of a disturbance is larger than any of its diameters in the meridional plane it is possible, as first approximation, to use a zonal model. Such a model will not be pure two dimensional. Although all derivatives along x are zero, a general east flow is taken into account in parametric form.

To model such structures let us assume that:

1. The mean state of the easterly zonal flow is in hydrostatic equilibrium.

2. The disturbance in wind field in meridional plane is due to the temperature disturbance (convection approach).

3. The disturbance is incompressible, adiabatic and nondissipative.

4. The mean flow and disturbance are stationary

Placing the origin of Cartesian coordinates at the equator on the Earth's surface we may write the governing equations in the form detailed by Dobryshman (1980):

$$J(\Psi, \Delta\Psi) = \alpha\frac{\partial\theta}{\partial y} + 2\omega\left(\frac{\partial U}{\partial y} + \frac{y}{a}\frac{\partial U}{\partial z}\right) \tag{1}$$

$$J(\Psi, U) = 0 \tag{2}$$

$$J(\Psi, \theta) = 0 \tag{3}$$

Here Ψ is a stream function defined by $v = -\frac{\partial \Psi}{\partial z}$; $w = \frac{\partial \Psi}{\partial y}$; θ is the disturbance of the potential temperature; $\alpha \approx \frac{1}{30} \frac{m}{s^2 K}$ is the bouyancy parameter; $aU = au + \omega y^2 - 2\omega z$ is the zonal momentum $\Delta \equiv \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is Laplacian; $J(A, B) \equiv \frac{\partial A}{\partial y}\frac{\partial B}{\partial z} - \frac{\partial A}{\partial z}\frac{\partial B}{\partial y}$ is Jacobian.

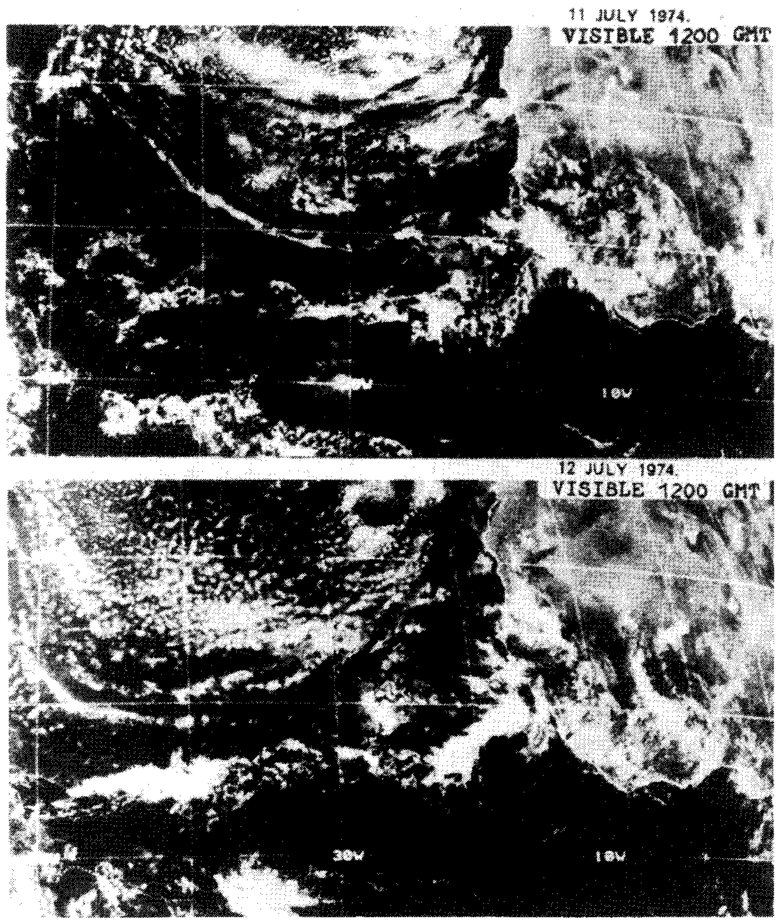

Fig. 1. The examples of various scales cloud structures in the tropical atmosphere, which do not change significantly during the 24 hours (From GATE Report N 17. 1975. ICSU&WMO, Geneva: pp 39, 41).

The momentum equation (2) and thermodynamic equation (3) permit to reduce the problem to one equation for the stream function Ψ:

$$J(\Psi, \Delta\Psi) = \alpha \frac{\partial F}{\partial \Psi}\frac{\partial \Psi}{\partial y} + 2\omega \frac{\partial \phi}{\partial \Psi}\left(\frac{\partial \Psi}{\partial y} + \frac{y}{a}\frac{\partial \Psi}{\partial z}\right),$$

where F and ϕ are arbitrary fuctions of Ψ. It is rather difficult to give strict formulation how to choose these functions. The one of more or less "reasonable" suppositions is that amplitude of three-dimensional disturbance wind field is proportional to the amplitude of the temperature disturbance. In such case F and ϕ are linear functions of Ψ:

$$F(\Psi) = A\Psi; \qquad \phi(\Psi) = U_o + B\Psi; \qquad A, B, U_o \text{ are constants} \tag{4}$$

Under such assumptions the vorticity equation takes the form

$$J(\Psi, \Delta\Psi - \alpha A z - 2\omega B z + \frac{\omega y^2}{a} B) = 0$$

$$\Rightarrow \Delta\Psi = f(\Psi) + (\alpha A + 2\omega B) z - \omega B \frac{y^2}{a} \tag{5}$$

where f is another arbitrary function of Ψ. Again the choice of a linear form of f will only be obvious a posteriori; the solutions will be easy interpreted and in particular cases will lead to the known results. Thus, by writing f in the following form

where $f = M - b^2\Psi$, M and b are constants, we obtain the following linear form of equation (5): *

$$\Delta\Psi + b^2\Psi = M + (\alpha A + 2\omega B) z - \omega B \frac{y^2}{a}. \tag{6}$$

For the purpose of considering the solitary wave disturbance we transform the equation (6) using the cylindrical coordinates:

$$r = [(y - y_o)^2 + (z - z_o)^2]^{1/2}; \qquad \vartheta = \tan^{-1} \frac{z - z_o}{y - y_o},$$

where y_o and z_o are the coordinates of the center of the solitary wave. This gives:

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \vartheta^2} + b^2\Psi = M_o + r\rho \sin(\vartheta - \kappa_o) - \tilde{B} r^2 (1 + \cos 2\vartheta), \tag{6'}$$

where

$$M_o = M + (\alpha A + 2\omega B) z_o - B \frac{\omega}{a} y_o^2; \qquad \tilde{B} = \frac{B\omega}{2a}$$

$$\rho^2 = (\alpha A + 2\omega B)^2 + (2B \frac{\omega}{a} y_o)^2; \qquad \kappa_o = \tan^{-1} \frac{2\tilde{B} y_o}{\alpha A + 2\omega B}. \tag{7}$$

Without loosing the generality, we may put $M = 0$ and write the boundary conditions in the form:

$$r = 0 \qquad \Psi = 0$$

$$r = R \qquad \Psi = 0 \tag{8}$$

where r should be determined from the match conditions: the solitary disturbance must conjugate with the surrounding flow with some degree of smoothness. This implies for instance, the condition

$$\frac{\partial \Psi}{\partial r}\Big|_{r=R} = 0. \tag{9}$$

* The reduction of the non-linear problem to linear one is rather one of the conventional way to find the appropriate solution. There are many examples in soliton theory.

As it will be seen later the desirable match conditions may not always be satisfied.

Below two examples of solution (6') in the form of so called modon, i.e., localized disturbance, will be described. So far the idea and the algorithm of modon construction are quite well described in the papers of Stern (1975); Flierl *et al.* (1980); Verkley (1984); Leith (1985); McSwenson (1986); etc., and we omit some transformation details.

a. Simple modon which does not affect the mean flow

Observing the small-scale disturbances in the tropical atmosphere it seems sometimes that the disturbances live independently on the large scale flow. For studying the sensitivity of the solution to the choice of function for the zonal momentum, we first consider the case $U = U_o =$ constant. Hence $B = 0$ and $\kappa_o = 0$, and the equation (6') will be reduced to:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \vartheta^2} + b^2\Psi = \alpha A r \sin\vartheta. \tag{10}$$

Seeking the solution to (10) in the form

$$\Psi = \Psi_1(r)\sin\vartheta$$

we obtain the Bessel equation for Ψ_1

$$\frac{1}{r}\frac{d}{dr}r\frac{d\Psi_1}{dr} + (b^2 - \frac{1}{r^2})\Psi_1 = \alpha A r.$$

The first condition in (8) leads to the solution

$$\Psi_1 = CJ_1(br) + \frac{\alpha A}{b^2}(br).$$

The arbitrary constant C is determined from the second condition in (8).

$$C = -\frac{\alpha A R}{b^2}/J_1(bR).$$

To find R (strictly speaking not R itself, but bR) we make use of the match condition (9) which after some manipulation provides us with the following equation:

$$\frac{d\Psi_1}{dr}\,|_{r=R} = J_2(bR) = 0. \tag{11}$$

The first two smallest roots are (Abramowitz and Stegun, 1964): $(bR)_1 \approx 5.14$; $(bR)_2 \approx 8.42$.

Thus, the solution of (10) corresponding to the root number n has the form

$$\Psi_1^{(n)} = \frac{\alpha A}{b^3}[\frac{r}{R_n} - \frac{J_1(br)}{J_1(bR_n)}]. \tag{12}$$

The modon for first root (bR) is precisely the Stern's (1975) modon, which he found for the barotropic vorticity equation, but in meridional plane *. Fig. 2 illustrates about the modon corresponding to this very root.

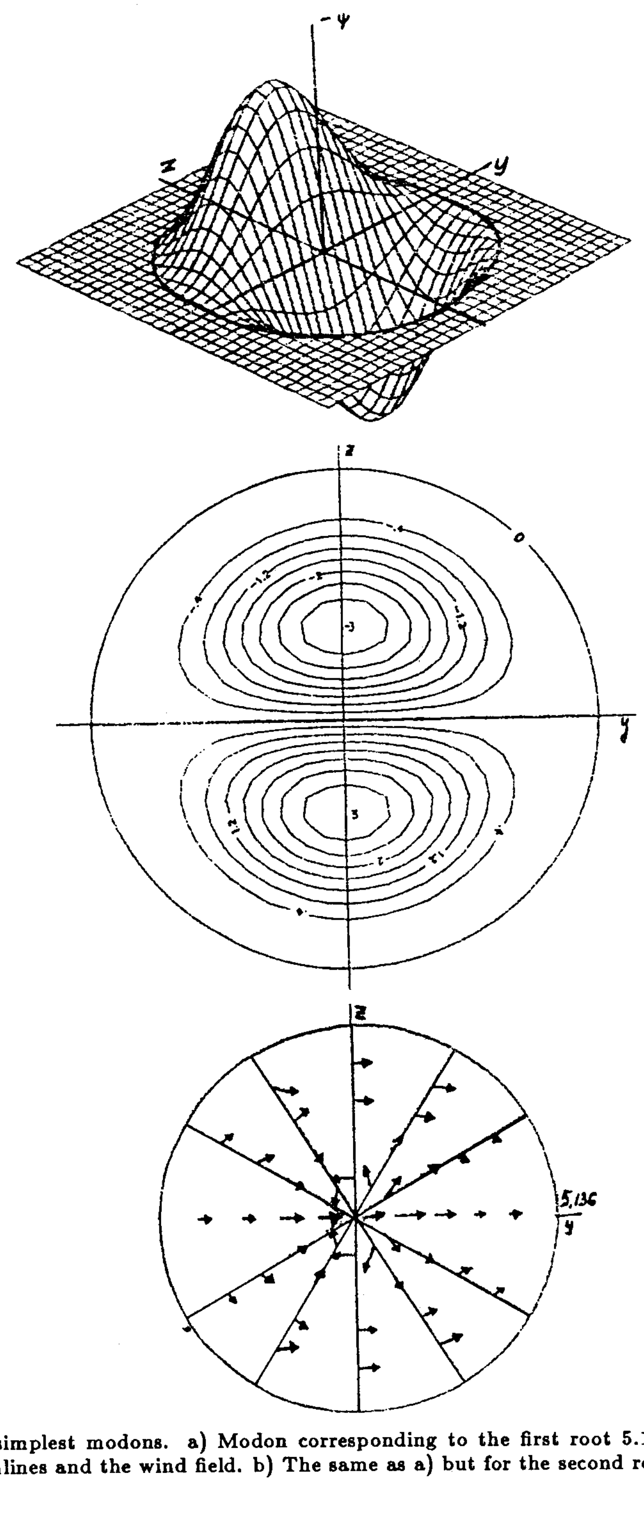

Fig. 2. The examples of the simplest modons. a) Modon corresponding to the first root 5.136 (Stern's modon). The three-dimensional view, the streamlines and the wind field. b) The same as a) but for the second root 8.417.

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* As far as the author knows,
Stern (1975) first introduced the title "modon" for the solitary disturbance which at finite radius smoothly conjugates with the surrounding flow.

b. General solution of equation (6')

Now it is necessary to search the solution in the form

$$\Psi = \Psi_o(r) + \Psi_1(r)\sin(\vartheta - \kappa_o) + \Psi_2(r)\cos 2\vartheta. \tag{13}$$

According to the physical terminology, Ψ_o is named as the monopole part of modon, Ψ_1 - the dipole part and Ψ_2 - the quadruple part. Each part satisfies the corresponding inhomogeneous Bessel equation. After using the conditions (8) Ψ_k may be written in the form

$$\Psi_o = \frac{\tilde{B}}{b^2}[r^2 - R^2\frac{1 - J_o(br)}{1 - J_o(bR)}]$$

$$\Psi_1 = \frac{\rho}{b^3}[r - R\frac{J_1(br)}{J_1(bR)}]$$

$$\Psi_2 = \frac{\tilde{B}}{b^2}[r^2 - R^2\frac{J_2(br)}{J_2(bR)}]. \tag{14}$$

The match condition (9) may be applied only to the quadruple part of the modon. This leads to the equation

$$4J_2(bR) = bRJ_1(bR).$$

The first two roots of this equation are: $(bR)_1 \approx 6.37$; $(bR)_2 \approx 9.77$. Fig. 3 shows the wind field in modon and the disturbance in homogeneous east flow.

It is important to keep in mind that in modon theory there is no possibility to find arbitrary parameters A, B, b (and consequently R). Choosing an appropriate value of these parameters it is possible to simulate a rather complex non-axisymmetric wind field in disturbances.

From the papers by Stern (1975) and Leith (1985) it is known that the Stern's modon in some sense is stable. As to the general solution (13) in form (14) there is no confidence in stability. As far as only quadruple part satisfies to the match condition, the monopole and dipole parts have the discontinuity, at $r = R$. The discontinuities are rather small. Nevertheless such discontinuities stipulate the instability. But the life time of small scale solitary disturbances in reality ranges consists of a few minutes up to several hours.

Sometimes the small disturbances in meridional plane move not only with general east flow but along the meridian or even along vertical axis. The equations (1) - (3) permit to simulate the movement of modon in a meridional plane too. The point is that system (1) - (3) has the invariant group transformation of translation type with constant velocity components along the y- and z-axes. Let us search the solution of equation (6) in the form

$$\Psi = \Psi^o - V(z - z_o) + W(y - y_o) + \tilde{\Psi}(y - y_o, z - z_o)$$

where V and W are constants.

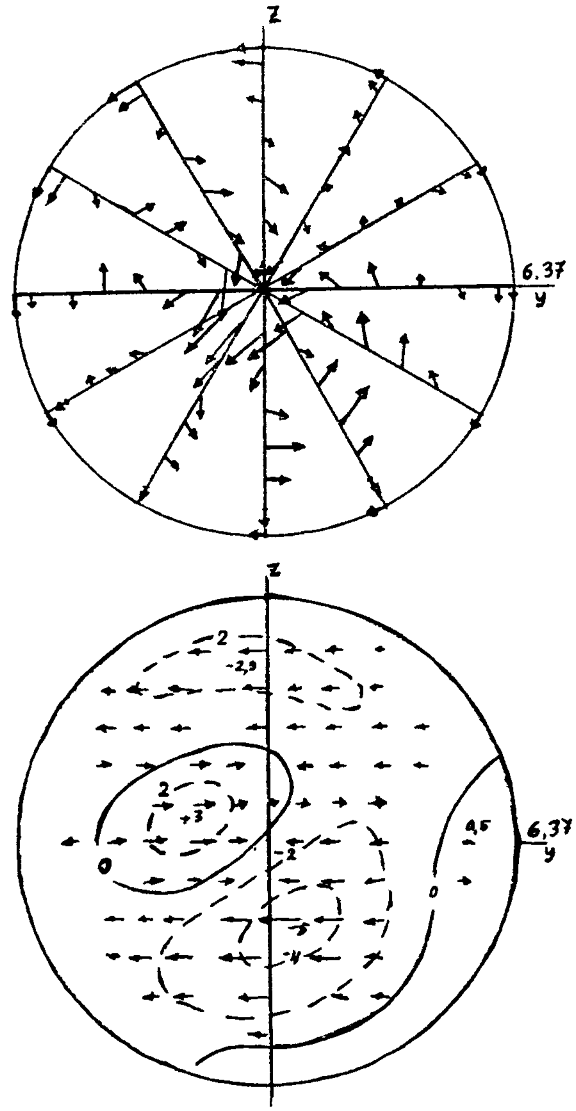

Fig. 3. Upper figure: The wind field in the complex modon with three parts. Lower figure: The field of disturbance in the homogeneous east flow in modon area.

After introducing the polar coordinates we get equation (6') with parameters $\tilde{\rho}$ and $\tilde{\kappa}_o$, instead of ρ and κ.

$$\tilde{\rho}^2 = (\alpha A + 2\omega B + b^2 V)^2 + (4\tilde{B} y_o^2 + b^2 W)^2$$

$$\tilde{\kappa}_o \doteq \tan^{-1} \frac{4\tilde{B} Y_o^2 + b^2 W}{\alpha A + 2\omega B + b^2 V}.$$

If the following relations exist

$$V = -(\alpha A + 2\omega B)/b^2; \qquad W = -4\tilde{B} y_o^2 / b^2,$$

then $\rho = o$ and the dipole part of modon vanishes. Such modon has the constant discontinuity of tangent wind along the circumference of radius R. The situation looks like the front but in form of circumference. The wind field resembles the field of two parallel vortices located one above the other (Fig. 4).

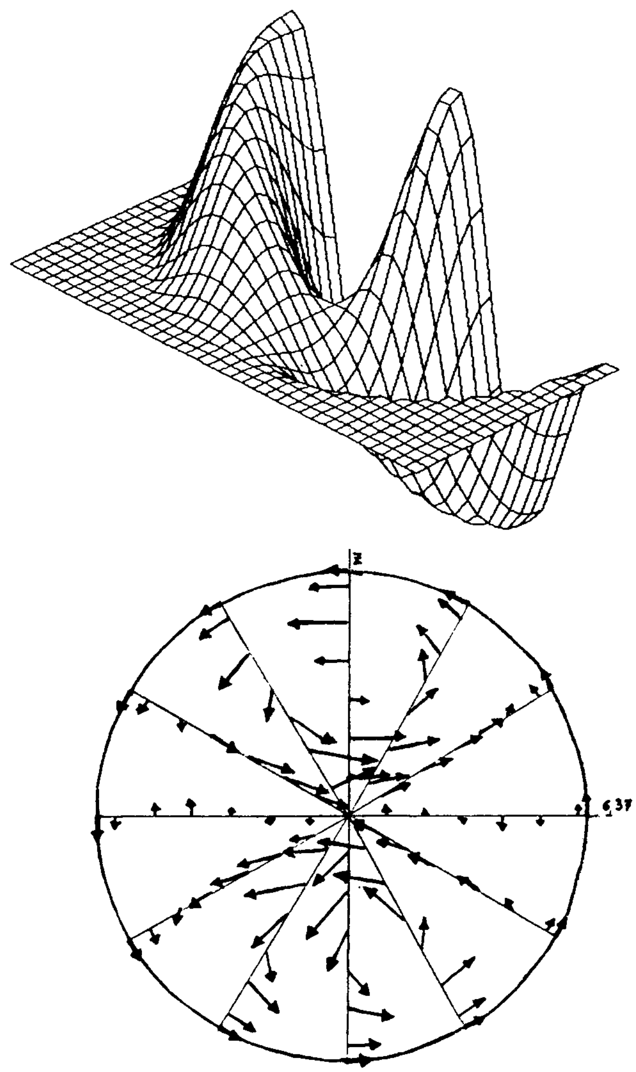

Fig. 4. Upper figure: The three-dimensional view of half modon containing two parts: monopole + quadrupole. Lower figure: The wind field in the modon. (The module of wind speed at the modon border does not depend upon coordinates).

II. The linear waves in tropical atmosphere

As it was already mentioned, to construct the wave theory valid for the whole tropical zone including the equatorial belt it is necessary to take into account all components of Coriolis acceleration. But a customary approach to tropical wave theory has to be used too. To simplify the investigation a method of "frozen" coefficients instead of dependence on the latitude is used (more detailed considerations are in Dobryshman, 1987). Let us assume that the mean state regime is the hydrostatic equilibrium with zero wind velocity. The governing equations are as follow:

$$\frac{\partial u}{\partial t} - 2\omega\varepsilon v + 2\omega w + \frac{\partial H}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + 2\omega\varepsilon u + \frac{\partial H}{\partial y} = 0$$

$$\frac{\partial w}{\partial t} - 2\omega u + \frac{\partial H}{\partial z} - \delta RT = 0 \tag{15}$$

$$\frac{1}{c^2}\frac{\partial H}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \varepsilon\eta v + \frac{\partial w}{\partial z} - \delta w = 0$$

$$\frac{\partial T}{\partial t} + \Gamma w = 0.$$

Here $\varepsilon \sim 0.1 - 0.2$ is the mean value of y/a, i.e., the first term of Taylor series of $\sin\varphi$; $\varepsilon\eta$ is the mean value of $\frac{1}{a}\tan\varphi \approx \frac{\varepsilon}{a}$. So in reality $\eta = \frac{1}{a}$. The symbol η is introduced to make clear the influence of small term $-\frac{v}{a}\tan\varphi$ in equation of discontinuity. (Almost in all tropical wave models the authors neglect this small term. However in some cases its role is not negligible). $\delta \sim 10^{-4} - 10^{-5}\mathrm{m}^{-1}$ is the parameter of atmospheric compressibility; R is the gas constant; $\Gamma \approx 3.10^{-3}\mathrm{Km}^{-1}$ is stability parameter, i.e., the difference between adiabatic lapse rate and mean lapse rate; T, H are deviations of temperature and geopotential from the equilibrium state. The other symbols are already known.

The searching of all dependent variables in form

$$\{u, v, w, H, T\} = \{u, iv, iw, H, T\}e^{i(mx-\sigma t)}e^{-ny-kz}$$

leads to the dispersion equation

$$\frac{\sigma^5}{c^2} + \sigma^3\{k(k+\delta) + n(n+\varepsilon\eta) - m^2 - \frac{1}{c^2}[\tilde{\delta} + 4\omega^2(1+\varepsilon^2)]\} +$$

$$\sigma^2 2\omega m(\delta - \varepsilon\eta) - \sigma\{4\omega^2(n+\varepsilon k)[n + \varepsilon(n+\delta+\eta)] + \tilde{\delta}[n(n+\varepsilon\eta) - m^2 - \frac{4\omega^2\varepsilon^2}{c^2}]\} + 2\omega\varepsilon^2 m\eta\delta = 0$$

$$\tilde{\delta} = \delta R\Gamma \approx 50 \times 10^{-6} s^{-2}. \tag{16}$$

The parameter $\tilde{\delta}^{1/2}$ plays the role of the Brunt-Vaisala frequency; its value corresponds to a period of about 15 minutes. The form of equation (16) permits to make several conclusions. The zero degree term in reality depends only upon the zonal wave number m, so far other multipliers are fixed. When this term is zero stationary waves exist. Thus the stationary waves appear in the following models: 1) $\tilde{\delta} = 0$, i.e., when the interaction between temperature and geopotential is not taken into account; 2) $m = 0$, i.e., in zonal models; 3) $\varepsilon = 0$, i.e., just at the equator; 4)$\eta = 0$, i.e., when the small term $-\frac{v}{a}\tan\varphi$ in the discontinuity equation is omitted. In other words, when we neglect the influence of the Earth's curvature. Parameter η affects mainly the coefficients generated by terms which depend upon y; 5) $\omega = 0$, i.e. on non-rotating Earth.

The replacement of n by $-n$ or/and k by $-k$ changes the coefficients. Therefore, the growing or decreasing waves along the corresponding axis have a different spectrum. If we replace n by in or/and k by ik then the coefficients in (16) become complex. In such cases the complex roots exist. (When the coefficients are real, a pair or two pairs of complex roots may exist too). When a complex root exists, the neutral waves do not arise in the model without fail.

To eliminate the fast waves it is sufficient to count $c^2 = \infty$. In such case (16) becomes a third order equation and the system (15) has the integral. If in addition we neglect the effect of compressibility ($\delta = 0$), then the changing sign before n or/and before k does not change the coefficients. Therefore in such models there is no difference between the growing and decreasing waves along the corresponding axis. Moreover such models permit easily to investigate the two or three dimensional waves, so far the replacing n by in or/and k by ik keeps the coefficients real.

Since $\tilde{\delta} >> 4\omega^2 = 2.10^{-8}s^{-2}$ and for real waves $| k |\sim O(\delta)$ and $| k |>>| n |$, it is convenient to simplify (16) to the form

$$\frac{\sigma^5}{c^2} + \sigma^3[2k^2 - m^2 - \frac{\tilde{\delta}}{c^2}] + \sigma^2 2\omega m\delta + \tilde{\delta}(m^2 - n^2) + 2\omega\varepsilon^2 m\eta\delta = 0. \quad (17)$$

For the two dimensional (in x, y plane) waves it is sufficient to change the sign before only one term, namely n^2.

For the high frequency part of spectrum of order $(\tilde{\delta})^{1/2}$ two roots are close to the expressions

$$\sigma_{1,2} \approx \pm(\tilde{\delta})^{1/2} \mp \frac{\omega c^2 \varepsilon\eta}{\tilde{\delta}} m.$$

On the spectrum of various parameters in low latitudes (zonal and vertical wind components, sea surface temperature, etc.) in the vicinity of 15 min period there is a more or less noticeable peak (GATE Monograph, 1982).

In the case when $n^2 \neq m^2$ the smallest of absolute value roots is determined approximately by the formula

$$\sigma_{\min} \approx \frac{2\omega\varepsilon^2\eta m}{n^2 - m^2}.$$

In fact $n^2 >> m^2$. Thus the one-dimensional wave corresponding to $\sigma_{\min}$ moves to the East. But the two-dimensional wave moves to the West, so far the $\sigma_{\min}$ is negative (we have to change the sign before n^2). Therefore the usually omitted small term in discontinuity equation which is created by the Earth's curvature is responsible for the generation of a west propagated very slow wave. This wave has a small amplitude. To detect such wave a very long series of reliable data are needed. For the first zonal model $m = 2\pi : 4.10^7 m$ the period of such wave $t_{\max} = 2\pi : \sigma_{\min}$ is of order 25 - 27 months. This period is comparable with the quasi-biennial cycle of zonal wind in the upper troposphere and lower stratosphere. However, the quasi-biennial cycle does not appear clearly in other wind components. It seems not quite reasonable to insist on the opinion that the only small term in continuity equation is capable of creating quasi-biennial oscillation in zonal wind. Another approach seems more realistic. Namely, the quasi-biennial oscillation is the manifestation of parametric resonance. Within framework of system (15) it is possible to construct rather a simple model of such parametric resonance.

From the data analysis it is known that the amplitude of biennial oscillation decreases along the meridian and grows with altitude. Fig. 5 corroborates this statement. So dependence of all functions in (15) upon y and z may be taken in form

$$exp[-ny + kz].$$

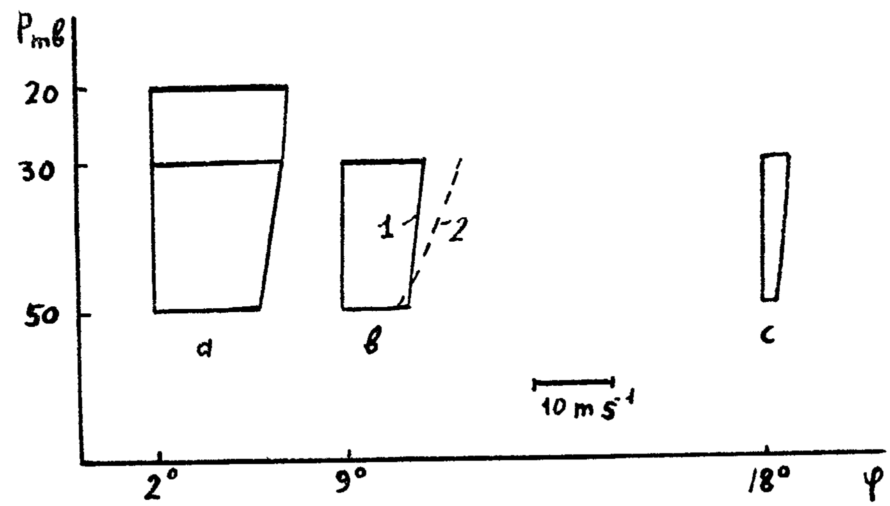

Fig. 5. The variability of amplitude of quasi-biennial oscillation with altitude and latitude. a) Canton Island (≈ 3°N; 1957-1967) and Gan Island (≈ 2° S; 1964-1974): b) 1 - Goward (≈ 9°N; 1950-1970) and Trivandrum (≈ 8°N 1964-1976); 2 - Ascension Island (≈ 8°S; 1964-1974); c) Kingstown (≈ 18°N; 1952-1964). and Nandy (≈ 18°S, 1952-1962).

In Fig. 6 the annual variations of heat flow from the Sun at two latitudes are presented: at the equator and at the tropics $\varphi = \pm 23^o 27'$. The amplitude of semiannual period at the equator is much greater than the annual one. Here the annual period is due only to the eccentricity of the Earth's

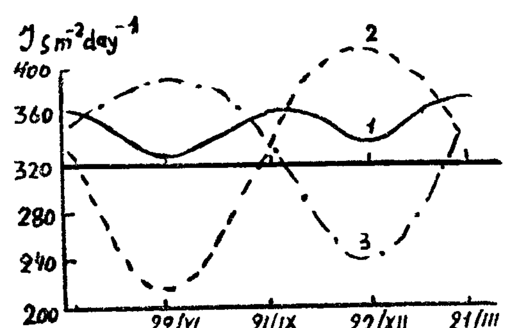

Fig. 6. The annual variation of mean day heat flow from the Sun: 1 - at equator; 2 - at latitude - 23° 27'; 3 - at latitude of 23°27'.

orbit. But at the tropics the amplitude of semiannual part practically vanishes. Following several authors, for instance Purgansky (1965), let us suppose that the semiannual change of heat flow is reflected in mean value of lapse rate.

$$\gamma = \gamma_o(1 + \chi \cos 2\Omega t),$$

where $\Omega = 2.02.10^{-7}s^{-1}$ is the angular frequency of the Earth's revolution around the Sun. $\chi \approx 0.01 - 0.05$ is a small parameter; γ_o is the mean annual value of lapse rate. For slow movements it is natural to neglect the fast waves i.e., to *take* $c^2 = \infty$. We omit also the small term $-\frac{v}{a}\tan\varphi$ in continuity equation. In present model its role is insignificant.

Under the mentioned assumptions, after cancellating the multiplier $exp[-ny + kz]$ the system (15) takes the form

$$\frac{du}{dt} - 2\omega\varepsilon v + 2\omega w = 0$$

$$\frac{dv}{dt} + 2\omega u - nH = 0$$

$$\frac{dw}{dt} - 2\omega u + kH - \delta RT = 0$$

$$\frac{dT}{dt} + [\gamma_a - \gamma_o(1 + \chi\cos 2\Omega t)]w = 0$$

$$-nv + (k - \delta)w = 0. \tag{18}$$

After defining v from the last equation and comparing the second equation with third one, two relations may be found

$$k = \delta - n\varepsilon; \qquad (\frac{h}{\varepsilon} + \delta - n\varepsilon)H = \delta RT.$$

Now the elimination of all functions but the zonal wind component leads to the Mathieu equation for $\frac{du}{dt}$:

$$\{\frac{d^2}{dt^2} + [4\omega^2(1+\varepsilon^2) + \frac{(\gamma_a - \gamma_o)n\delta R}{n(1-\varepsilon^2) + \delta\varepsilon} - \frac{\gamma_o n\delta R}{n(1-\varepsilon^2)+\delta\varepsilon}\chi\cos 2\Omega t]\}\frac{du}{dt} = 0 \tag{19}$$

The substitution of the dimensionless time $\tau = \Omega t$ transforms this equation to the final form

$$\frac{d^2x}{dt^2} + [A - 2q\cos 2\tau]x = 0, \tag{19'}$$

where

$$x = \frac{du}{dt}$$

$$A + [4\omega^2(1+\varepsilon^2) + \frac{(\gamma_a - \gamma_o)n\delta R}{n(1-\varepsilon^2)+\delta\varepsilon}]\frac{1}{4\Omega^2}; \;\; 2q = \frac{\gamma_o n\delta R}{n(1-\varepsilon^2)+\delta\varepsilon}\chi\frac{1}{4\Omega^2}.$$

Here either n or χ may be chosen as a free parameter to find necessary solution. The calculations are rather difficult due to big value of parameters A and q. (That is the very small parameter before the second derivative in (19')).

We wish to draw one's attention to the necessity to find such value of free parameters that gives the wave of *fourfold* period, not double period.*

The fourfold period means the exact biennial oscillation. But the empirical data show the mean period of 26-28 months. (See for instance GATE Monograph, 1982, Chapter V).

To get more realistic period probably non-linear interactions should be probably included. The book by Gledzer *et al.* (1981), contains another explanation of revealing 26 - 28 months oscillation. The explanation is based on the theory of hydrodynamic type systems. In such non-linear system a period break may occur. During the last three decades, three breaks may be revealed. These breaks strain the period in statistical treatment.

To terminate the investigation of equation (16), below the approximative formula for the last two roots is written. These roots are proportional to ω.

$$\sigma_{4,5} \approx \omega \frac{-\delta m \pm [\delta^2 m - 4(2k^2 - n^2 - m^2)n^2]}{2k^2 - n^2 - m^2}.$$

The corresponding two-dimensional waves (it is necessary to change the sign before n^2) are easterly waves in the tropics. The spectrum of such waves in k, m, n space is not large, so far

$$| k | >> | n | > m.$$

Thus the equation (16) comprises at least three parts of wave spectrum in the tropical atmosphere including the equatorial belt: 1) from ten minutes to several hours; 2) from one day to a few weeks and 3) biennial cycle.

It should be mentioned that if in equations (15) we replace ε by y/α, then the problem comes to eigenvalue problem for the meridional wind component v. The eigenvalues are found from the equation

$$\frac{d^2 v}{d\xi^2} - (\xi^2 + \mu^2) v = 0, \tag{20}$$

where ξ is the dimensionless coordinate in y direction; μ has rather unwieldy form and contains almost all parameters of system (15) including σ but n.

The solution of equation (20) is limited at $| \xi | \to \infty$ if and only if $\mu^2 = 2j + 1$; $j = 0, 1, 2, \ldots$ The eigenfuntions are expressed by Hermite Polynomials (Matzuno, 1966; Dobryshman, 1987).

Two models described here certainly cannot reflect the long list of peculiarities which occur in the tropical atmosphere. However, even the simple models are capable to outline not only the general view but sometimes give more or less appropriate value of quantitative parameters of waves and solitary disturbances.

*Mathieu equation contains the solution of any period $j\tau_{per}$ where j is integer $j = 1, 2, \ldots$ (see Abramowitz, 1964).

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АННОТАЦИЯ

Различного масштаба повторяющиеся и уединенные возмущения общего восточного потока являются характерными особенностями атмосферных процессов в низких широтах. Сравнительно простые системы уравнений гидротермодинамики позволяют с некоторым приближением моделировать такие особенности. Нелинейная система содержит решения типа уединенных возмущений в ограниченной области, так называемых модонов. Другая система - линейная - описывает практически все наблюдаемые в низких широтах, включая экваториальный пояс, волны. Особенно четко выявляются волны в интервалх частот, соответствующих периодам: а) десятков минут, б) суток - недель, в) квазидвухлетнему циклу.