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A model of the stable nocturnal boundary layer

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RESUMEN

Se construye un modelo unidimensional de dos capas de la capa límite nocturna que incluye varios procesos meteorológicos: turbulencia, radiación, interacción capa límite - suelo y capa límite - flujo sinóptico. El modelo predice el perfil termodinámico y dos alturas. Una es de la capa límite y otra de una capa superficial. La capa límite evoluciona como un sistema en contracción (en la noche) y en expansión (en el día), influenciada principalmente por el enfriamiento o calentamiento radiativo y el tipo de suelo. Los resultados preliminares de este trabajo son comparables a los obtenidos por otros autores.

ABSTRACT

A one-dimensional stable nocturnal boundary layer model of two-layers is constructed, which includes various meteorological processes: turbulence, radiation, interaction boundary layer - soil and boundary layer - synoptic flux. The model predicts the thermodynamic profiles and two heights. One is from the boundary layer, and the other from the turbulent surface sublayer. The boundary layer evolves as a system in contraction at night and expansion during the day, influenced mainly by strong radiative cooling or heating and soil type. Predictions agree with the profiles obtained by other authors.

Introduction

The geographical situation of Mexico permits masses of air of different origin to interact on the Mexican plateau, which provoke different processes. The phenomena of the atmospheric boundary layer may be the ones, which have the most important impact on our society. In the boundary layer one finds the swirl of sands, frost, inversions of temperature and fog (Pérez, 1986). For instance during summer fog and estratus are often formed west of the Californian coast (Schuber *et al.*, 1979) and during winter, all along the slopes of the Gulf of Mexico. They are also called the Northern Winds. In winter the inversions of temperature associated with air pollution have a big impact on the population of Mexico City. Therefore, forecast models of fog, minimum temperature and air pollution are necessary.

The atmospheric boundary layer is the layer, where the surface has the strongest effects over the atmosphere. They are: heating, cooling and surface friction. The layer can arise in any of 3 states: unstable, stable and neutral. The unstable or convective regime is usually under an upward sensible heat flux. In the day time the boundary layer normally coincides with the height, where the pollutants have a good mixing. Frequently the stable regime arrives during the night by the heat lost from the ground due to longwave radiation. In this state the turbulence generated by surface friction is suppressed by the downward sensible heat flux. The strong radiative cooling of the surface produces the cooling of the lowest part of the atmospheric boundary layer (surface layer). It is believed that due to this cooling the height of the surface layer decreases.

A large amount of research has been done on the evolution of the boundary layer in convective conditions (Lilly, 1968; Tennekes, 1973; Deardorff, 1979; Diendronk and Tennekes, 1984). Less research has been carried out on the development of the nocturnal boundary layer, particularly due to the lack of observations (Brost and Wyngaard, 1978; Zeman, 1979; Yamada, 1979; Nieuwstadt and Tennekes, 1982). Therefore, the mistery which covers the nocturnal boundary layer, still lacks a total clarification.

A few studies of the nocturnal boundary layer have considered the simultaneous interactions between soil-boundary layer, turbulence, radiative flux and subsidence of synoptic high-pressure systems. Gannon (1978) has shown the importance of the influence of the soil on the mesoscale modeling. André and Mahrt (1982), and Garrat and Brost (1981) considered the radiative cooling effect on the layer of the surface inversion. Zeman (1979), Nieuwstadt and Tennekes (1981) developed equations to predict the height of the nocturnal boundary layer. Generally these equations take the form:

$$\frac{dH}{dt} = T^{-1}(H_e - H)$$

where T is the time scale of relaxation, and H_e is an equilibrium height, which can be expressed by a diagnostic equation similar to the one proposed by Silitinkevich (1972):

$$H_e = 0.4(u_* L/f)^{1/2}$$

where u_* is the surface friction velocity, L the Monin - Obukhov length and f the Coriolis parameter. Observations carried out by Mahrt *et al.* (1982) on the depth of the boundary layer have shown that the diagnostic model perform reasonably well, when they are compared with observed heights, derived from the Richardson number.

Due to the well known importance of the nonstationary effects which occur in the atmospheric boundary layer, there is little doubt that prognostic models are required to parameterize the processes which occur there.

The purpose of this paper is to present a preliminary one-dimensional thermodynamic model for the nocturnal boundary layer which consists of two layers in stable condition. Using the approach of Nieuwstadt and Tennekes (1981) in subsections *b* and *c* an equation is derived, which describes the development of the boundary layer height and a small surface layer under stable conditions, respectively. In subsection *f* the interaction of the soil with the planetary boundary layer is considered, following McCumber and Pielke (1981), and Mahfouf (1986). The long and short wave radiative processes are incorporated following the parameterization of Mahrer and Pielke (1977). The coupling of the model with a large scale model is described in subsection *g*.

Basic equations of the model

a. Equations of the boundary layer under stable conditions

A plotted sounding on the $T, LogP$ with temperature inversion representing the typical temperature profiles of winter, frequently observed in Mexico City (Pérez, 1986) is shown in Fig. 1. The equations for potential temperature and humidity can be obtained according to the vertical profiles following the sounding of temperature and potential temperature. Considering the atmospheric layer $\Omega = (z_o, z_3) = \{z \mid z \in \cup_{k=1}^{3}(z_{k-1}, z_k), z_{k-1} < z_k\}$, subdivided into three layers, such that z_o represents the terrestrial surface height above mean sea level, z_2 the top of the atmospheric boundary layer, and z_3 a height slightly above the top of the boundary layer. It is assumed that z_3 is at just 50 mb.

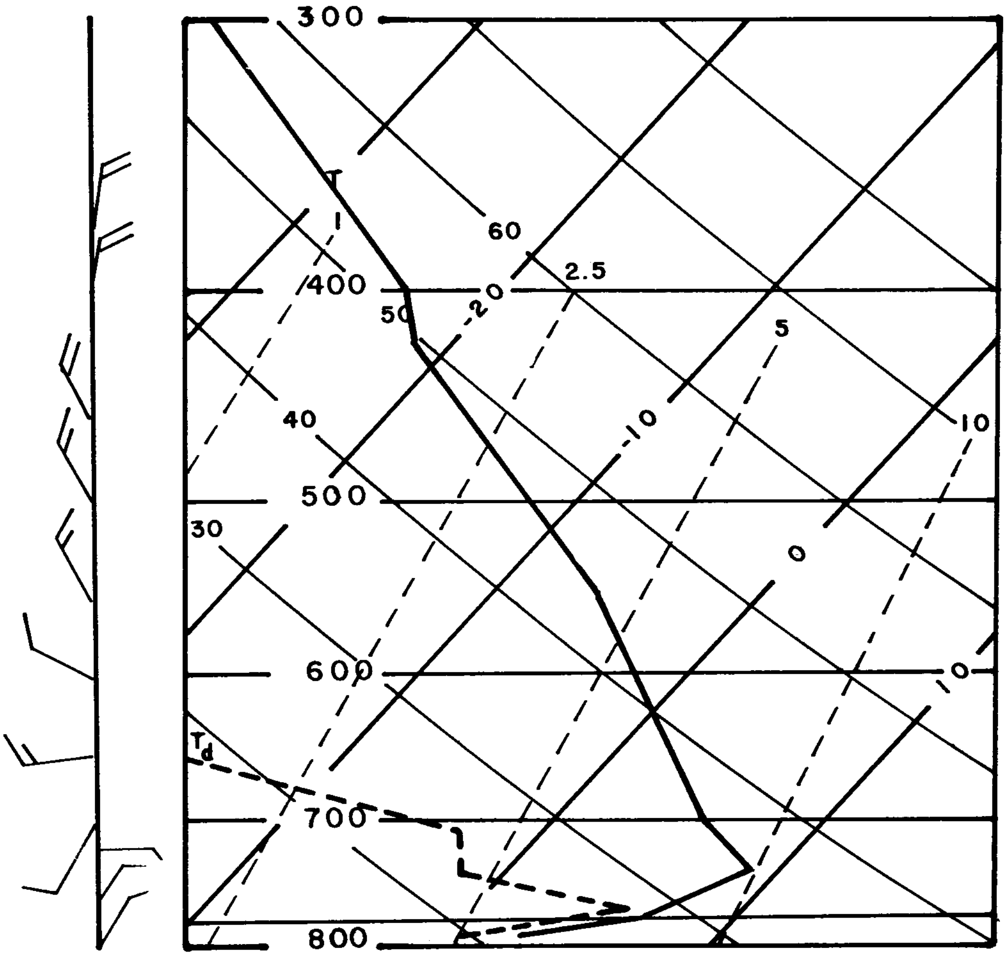

Figure 1. A plotted sounding on the thermodynamic chart, for 12 GTM on 3rd January, 1988, from Mexico City Airport, with temperature inversion.

For a more realistic description of the boundary layer it is necessary to have equations that represent in an average form the stable boundary layer (z_o, z_2). Therefore, it is split up into two sublayers. The thinest (z_o, z_1) here named one surface layer, and the other one remainder of the boundary layer (z_1, z_2).

On the assumption that the variation of the mean potential temperature $\overline{\theta}$ on (θ_{k-1}, θ_k) for $k = 1, 2$, follows the form

$$\overline{\theta}(z) = \theta_{k-1} + f_k(\vec{x}, t)(\theta_k - \theta_{k-1}) \tag{1}$$

where $\vec{x} = (x, y, z) = (0, 0, z), \theta_o, \theta_1$ and θ_2 are the potential temperatures on the surface, the top of the surface layer and the top of the boundary layer respectively, $f_k(\vec{x}, t)$ is a dimensionless shape factor such that $0 \leq f_k \leq 1$. In this paper it is assumed that $f_1(z) = \frac{z}{H_s}$ if $0 \leq z \leq H_s$ and $f_2(z) = \frac{z-H_s}{\Delta H}$ for $H_s \leq z \leq H$ where $H_s = z_1 - z_o, H = z_2 - z_o$ and $\Delta H = z_2 - z_1$.

The next step is to integrate both sides of the averaged thermodynamic and humidity equations, in the region $\Omega' = \cup_{k=1}^{2}(z_{k-1}, z_k)$ with boundary conditions $\overline{w'\theta'}(z) = 0 = \overline{w'q'}(z)$ for $z \geq z_2$. Then we find that the equations for the surface layer (z_o, z_1) are:

$$\frac{d\bar{\theta}_s}{dt} = \frac{\overline{w'\theta'_o} - \overline{w'\theta'_1} + (\frac{dH_s}{dt} - w_1)\frac{\delta\theta}{2}}{H_s} + \frac{1}{H_s}\int_{z_0}^{z_1}(-\frac{1}{\bar{\rho}c_p}\frac{\partial \overline{F}_r}{\partial z})dz, \tag{2}$$

$$\frac{d\bar{q}_s}{dt} = \frac{\overline{w'q'_o} - \overline{w'q'_1} + (\frac{dH_s}{dt} - w_1)\frac{\delta q}{2}}{H_s}, \tag{3}$$

where $\bar{\theta}_s = \frac{\theta_o+\theta_1}{2}$, $\bar{q}_s = \frac{q_o+q_1}{2}$, $\delta\theta = \theta_1 - \theta_o$, $\delta q = q_1 - q_o$, w_1 is the subsidence of synoptic scale in the top of the surface layer, z_o is the height at which the wind speed vanishes and H_s is the height of the surface layer. $\overline{F}_r(z) = \overline{F}_L(z) + \overline{F}_S(z)$ is the net radiative flux, where $\overline{F}_L(z)$ is the longwave radiative flux in level z and $\overline{F}_S(z)$ the solar flux. We assume that $q(z)$ varies similarly to (1). The thermodynamic and humidity equations for the remainder of the boundary layer (z_1, z_2) are:

$$\frac{d\bar{\theta}_B}{dt} = \frac{\overline{w'\theta'_1} + [(\frac{dH}{dt} - w_2) + (\frac{dH_s}{dt} - w_1)]\frac{\Delta\theta}{2}}{\Delta H} + \frac{1}{\Delta H}\int_{z_1}^{z_2}(-\frac{1}{\bar{\rho}c_p}\frac{\partial \overline{F}_r}{\partial z})dz, \tag{4}$$

$$\frac{d\bar{q}_B}{dt} = \frac{\overline{w'q'_1} + [(\frac{dH}{dt} - w_2) + (\frac{dH_s}{dt} - w_1)]\frac{\Delta q}{2}}{\Delta H}, \tag{5}$$

where $\bar{\theta}_B = \frac{\theta_1+\theta_2}{2}$, $\bar{q}_B = \frac{q_1+q_2}{2}$, $\Delta\theta = \theta_2 - \theta_1$, $\Delta q = q_2 - q_1$. H is the nocturnal boundary layer height. As a preliminary study and for simplicity $w_1 = 0$.

The equations for the potential atmospheric temperature, humidity and wind speed at the top of the boundary layer are similar to those of Zeman (1979):

$$\frac{d\theta_2}{dt} = (\frac{\partial\bar{\theta}}{\partial t})_{z=H} + (\frac{\partial\bar{\theta}}{\partial z})_{z=H}\frac{dH}{dt}, \tag{6}$$

$$\frac{dq_2}{dt} = (\frac{\partial\bar{q}}{\partial t})_{z=H} + (\frac{\partial\bar{q}}{\partial z})_{z=H}\frac{dH}{dt}, \tag{7}$$

$$\frac{d\overline{E}_H}{dt} = (\frac{\partial\overline{E}}{\partial t})_{z=H} + (\frac{\partial\overline{E}}{\partial z})_{z=H}\frac{dH}{dt}, \tag{8}$$

the local change at $z = H$ follows from Reynolds' average thermodynamic equation, humidity and velocity components $\bar{u}, \bar{v}$. Applying (9) and (10) at $z = H$ the turbulence terms vanish. Therefore we have $(\frac{\partial\bar{\theta}}{\partial t})_{z=H} = -\frac{1}{\bar{\rho}c_p}(\frac{\partial\overline{F}_r}{\partial z})_{z=H}$, $(\frac{\partial\bar{q}}{\partial t})_{z=H} = 0$ and $(\frac{\partial\overline{E}}{\partial t})_{z=H} = f(\bar{v}\,\bar{u}_g)_{z=H}$, where $\overline{E} = \frac{1}{2}(\bar{u}^2 + \bar{v}^2)$ is the mean kinetic energy.

Equations (2), (3), (4) and (5) are similar to those of the model of Reif *et al.* (1984) except that they do not consider the small surface layer and the radiative part.

b. The rate equation of nocturnal boundary layer

In all boundary layer models, the problem of closure of the equations exists. In the system of equations (2) to (8) there are more unknown ($\overline{\theta}_s$, $\overline{q}_s$, $\overline{\theta}_B$, $\overline{q}_B$, θ_2, q_2, $\overline{F}_r$, w_2, $\frac{dH_s}{dt}$, $\frac{dH}{dt}$) than equations, and the systems no longer closed. Following Nieuwstadt and Tennekes (1981) a growth rate equation for turbulent nocturnal boundary layer is derived by means of an integral method, which assumes a balance of the production and destruction of kinetic turbulent energy inside of the stable boundary layer. For that purpose a modified flux Richardson number is introduced which depends on the time, and is defined as the ratio of production and destruction average across the layer (z_1, z_2). The Richardson number is evaluated with the help of the self-similarity profiles of the boundary layer. (Caughey *et al.*, 1979; Nieuwstadt and Tennekes, 1981):

$$\overline{w'\theta'}(z) = \overline{w'\theta'_o}(1 - \frac{z}{H})^3, \tag{9}$$

$$\vec{\tau}\ (z) = \vec{\tau}_o\ (1 - \frac{z}{H})^{\frac{3}{2}}, \tag{10}$$

$$\vec{v}\ (z) = \vec{v}_H\ (\frac{z}{H}), \tag{11}$$

where $\overline{w'\theta'_o}$ is the turbulent temperature flux surface, and $\vec{v}_H$ the wind velocity at the top of the boundary layer. In the surface layer (z_o, z_1), the shear stress $\vec{\tau}$ is approximately equal to the surface value $\vec{\tau}_o = \overline{\rho} u_*^2(cos\alpha, sin\alpha)$. u_* is the friction velocity, α is the angle of $\vec{\tau}_o$ with the x axis, the x axis is taken along the geostrophic wind $\overline{u}_g$. Recently Nieuwstadt (1984) has indicated that the adoption of scaling (9), (10) and (11) is justified only, if the boundary layer height can be taken as the representative height scale of turbulence and that in stable conditions the turbulent eddies can not extend themselves to the whole layer. For this reason the use of H as a characteristic scale is not necessarily appropriated. Consequentely he adopted the hypothesis of the local scaling. Where the turbulent scale are z, and the local values of the fluxes $\overline{w'u'}$ and $\overline{w'\theta'}$. The adoption of the above mentioned hypothesis in this work will be the subject to a subsequent paper.

Using (9), (10) and (11):

$$\overline{R}_{iF} = \frac{-\frac{g}{T_r}\int_{H_s}^{H} \overline{w'\theta'}dz}{\int_{H_s}^{H} \frac{\vec{\tau}}{\rho} \cdot \frac{\partial \vec{v}}{\partial z} dz}, \tag{12}$$

we have

$$\overline{R}_{iF} = C'_1 \frac{H}{L}\frac{u_*}{U_H}, \tag{13}$$

where

$$C'_1 = \frac{(1 - \frac{H_s}{H})^4}{\frac{8}{5}k(1 - \frac{H_s}{H})^{\frac{5}{2}}} \approx \frac{(1 - 4\frac{H_s}{H})}{\frac{8}{5}k(1 - \frac{5}{2}\frac{H_s}{H})},$$

L is the Monin - Obukhov length, $k = 0.35$, is the von Kármán constant and U_H the mean velocity of the top of the boundary layer. In this paper the approximation $(1-y)^n = 1 - ny + ...$ has been used.

The integral of the buoyancy flux $\frac{g}{T_r}\overline{w'\theta'}$ (destruction of turbulence) is obtained from the thermodynamic equation integrated from H_s to H, neglecting the term of advection, using (1) i.e. $\overline{\theta}_H - \overline{\theta} = (1 - \frac{z-H_s}{\Delta H})(\overline{\theta}_H - \overline{\theta}_{H_s})$ similar to Zeman (1979) and (6):

$$-\frac{g}{T_r}\int_{H_s}^{H}\overline{w'\theta'}dz = 4\overline{E}_H R_{iB} C_3'' \frac{dH}{dt} - C_3 \frac{g}{T_r} H(H + H_s + \frac{H_s^2}{H})\frac{d\overline{\theta}_{H_s}}{dt} + 2R_{iB}\overline{E}_H C_4'' \frac{dH_s}{dt}$$

$$+\frac{2R_{iB}\overline{E}_H}{(\overline{\theta}_H - \theta_o)}(\frac{H_s}{H})(1+\frac{H_s}{H})(1-\frac{H_s}{H})^2\overline{w'\theta_o'} - \frac{2R_{iB}\overline{E}_H}{(\overline{\theta}_H - \theta_o)H}[\frac{1}{\overline{\rho}c_p}(H\overline{F}_{rH} - H_s\overline{F}_{rH_s}) - \frac{H_s^2}{2}(-\frac{1}{\overline{\rho}c_p}\frac{\partial \overline{F}_r}{\partial z})_{z=H_s}$$

$$+A_H(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=H} + \int_{H_s}^{H}(-\frac{1}{\overline{\rho}c_p}\overline{F}_r)dz], \tag{14}$$

where $\overline{E}_H = \frac{1}{2}U_H^2$, $A_H = C_2H^2 - C_3H_s(H_s + H)$, $C_2 = \frac{1}{3}$ and $C_3 = \frac{1}{6}$. The derivation of (14) is discussed in the appendix, where we have used the Leibnitz's rule and the bulk Richardson number R_{i_B} defined as

$$R_{i_B} = \frac{g}{T_r}\frac{(\overline{\theta}_H - \theta_o)}{2\overline{E}_H}H,$$

$$C_3'' = C_3(1+\frac{H_s}{2H})\frac{\Delta\theta}{(\overline{\theta}_H - \theta_o)} - C_3[1 - \frac{H_s}{2H}(1+\frac{H_s}{H})][\frac{H}{(\overline{\theta}_H - \theta_o)}\frac{\partial\overline{\theta}}{\partial z}]_{z=H},$$

$$C_4'' = C_3\frac{\Delta\theta}{(\overline{\theta}_H - \theta_o)}(1+\frac{2H_s}{H}) + \frac{H_s}{2H}[\frac{H_s}{(\overline{\theta}_H - \theta_o)}\frac{\partial\overline{\theta}}{\partial z}]_{z=H_s}.$$

Using the equations of movement of the boundary layer, (9a) and (9b) from Nieuwstadt Tennekes (1981), the term of mechanic production yields

$$\int_{H_s}^{H}(\frac{\vec{\tau}}{\overline{\rho}}\cdot\frac{\partial\vec{v}}{\partial z})dz = 2\overline{E}_H C_2''\frac{dH}{dt} + \int_{H_s}^{H} f\overline{v}\,\overline{u}_g dz - (1 - \frac{H_s^3}{H^3})C_2 H f(\overline{v}\,\overline{u}_g)_{z=H} - (\frac{\vec{\tau}}{\overline{\rho}}\cdot\vec{v})_{z=H_s}, \tag{15}$$

where f is the Coriolis parameter, $\vec{v}$ and $\overline{\theta}$ with overbar refer to mean values,

$$C_2'' = C_2(1 - \frac{H_s^3}{H^3})[1 - \frac{H}{2\overline{E}_H}(\frac{\partial\overline{E}}{\partial z})_{z=H}].$$

Substitution of (14) and (15) in (12), and arranging terms, we have the rate equations for H

$$\frac{8}{3}\left(1-\frac{C_2''}{2C_3''}\frac{\overline{R}_{iF}}{R_{iB}}\right)\frac{dH}{dt}=-\frac{4C_3\frac{d\overline{\theta}_{H_s}}{dt}}{3C_3''(\overline{\theta}_H-\theta_o)}(\mathbf{H}_e''-\mathbf{H}'')-4C_2\frac{C_4''}{C_3''}\frac{dH_s}{dt}$$

$$-\frac{8C_3H_s}{C_3''(\overline{\theta}_H-\theta_o)H}\left(1+\frac{H_s}{2H}\right)\left(1-\frac{H_s}{H}\right)^2\overline{w'\theta_o'}+\frac{8C_3}{C_3''(\overline{\theta}_H-\theta_o)H}\Big[\frac{1}{\overline{\rho}c_p}(H\overline{F}_{r_H}-H_s\overline{F}_{r_{H_s}})$$

$$-\frac{H_s^2}{2}\left(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z}\right)_{Z=H_s}+A_H\left(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z}\right)_{z=H}+\int_{H_s}^{H}\left(-\frac{1}{\overline{\rho}c_p}\overline{F}_r\right)dz\Big], \tag{16}$$

where

$$\mathbf{H}_e''=-\frac{\overline{R}_{iF}\mathbf{P}''}{C_3\frac{g}{T_r}\frac{d\overline{\theta}_{H_s}}{dt}},$$

$$\mathbf{P}''=\frac{1}{H}\int_{H_s}^{H}f\overline{v}\,\overline{u}_g dz-C_2\left(1-\frac{H_s^3}{H^3}\right)f(\overline{v}\,\overline{u}_g)_{z=H}-\frac{1}{H}\left(\frac{\vec{\tau}}{\rho}\cdot\vec{v}\right)_{z=H_s},$$

$$\mathbf{H}''=H+H_s+\frac{H_s^2}{H}.$$

When $H_s \rightarrow O$, equation (16) converge to the general equation (31) of Nieuwstadt and Tennekes (1981). Equation (16) containing the direct contribution of radiation by means of the last term. The difference that exists between equations (16) and (31) of Nieuwstadt and Tennekes are evident. Apart from the modifications of the forcings of the right side of the equation (16) i.e. the effect of the turbulence, the heat ratio of radiative cooling of the surface layer plus the upper boundary layer. At the same time the production term contained explicity in $\mathbf{H}_e''$ has been modified. For example the mechanical production term is $\frac{\vec{\tau}}{\rho}\cdot\vec{v}|_{z=H_s}$ close to the surface.

On the other hand the evaluation of C_2'' necessarily require a nonstationary description of the wind's velocity at the top of the boundary layer. For this reason similar to Nieuwstadt and Tennekes a steady election of the value of C_2'' in this paper gets to be somewhat arbitrary. Therefore, in this model the dynamic nonstationary effect must be incorporated. The temperature change of the top surface layer $\frac{d\overline{\theta}_{H_s}}{dt}$ is computed from (2) and (4). However, the parameterization of the temperature change at the surface $\frac{d\theta_o}{dt}$ and $\frac{dH_s}{dt}$ is necessary.

c. The equations for the surface layer

The surface layer is the part of the planetary boundary layer immediately above the surface, where the vertical variations of the heat fluxes, momentum and humidity, can be ignored. Inside this internal layer the similarity laws of Monin - Obukhov are assumed to be valid. As the depth of the mixed layer varies enormously between day and night, the same characteristic can be said about the thickness of the surface layer. Panofsky and Dutton (1984) have noticed that during clear nights with slight winds the thickness of this layer is very thin (less than 10 m). They adopt the definition that the surface layer occupies less than 10% of the mixed layer depth. In this paper it is assumed that H_s varies according to this definition, and that it is one "surface layer". If H_s were the height of the surface layer, this assumption would be false, because the surface layer is considered in equilibrium with its current boundary conditions.

Substituting $\frac{dH}{dt}$ from (16) into (4), solving the resulting equation for $\frac{1}{2}\frac{d\theta_1}{dt}$, and using (2), we obtain:

$$(C_5''\frac{\delta\theta}{H_s} - \frac{\Delta\theta}{\Delta H} + \frac{8C_3C_4''}{A_oC_3''}\Gamma_\theta)\frac{dH_s}{dt} = C_5''\frac{d\theta_o}{dt} - \frac{\Delta\theta}{\Delta H}w_H$$

$$+\{\frac{2}{\Delta H}(1-\frac{3H_s}{H}) - \frac{6}{H} + \frac{8C_3\Gamma_\theta}{A_oC_3''(\overline{\theta}_H-\theta_o)H}[\mathbf{H}'' - H_s(1+\frac{H_s}{2H})(1-\frac{H_s}{H})^2]\}\overline{w'\theta_o'}$$

$$+\frac{2C_2H\Gamma_\theta}{A_oC_3''\overline{E}_H}(\frac{\overline{R}_{iF}}{R_{iB}})\mathbf{P}'' - [1 - \frac{8C_3A_H\Gamma_\theta}{A_oC_3''(\overline{\theta}_H-\theta_o)H}](1-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=H}$$

$$+[1-\frac{2C_3H\Gamma_\theta}{A_oC_3''(\overline{\theta}_H-\theta_o)}]2(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=H_s} - 2C_5''(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=0}, \qquad (17)$$

where (6) and the following approximation

$$\int_{H_s}^{H}(-\frac{1}{\overline{\rho}c_p}\overline{F}_r)dz \approx -\frac{1}{\overline{\rho}c_p}(\frac{\overline{F}_{rH}+\overline{F}_{rH_s}}{2})\Delta H,$$

$$(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=H_s} \approx -\frac{1}{\overline{\rho}c_p}(\frac{\overline{F}_{rH}-\overline{F}_{rH_s}}{\Delta H}),$$

$$(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=0} \approx -\frac{1}{\overline{\rho}c_p}(\frac{\overline{F}_{rH_s}-\overline{F}_{ro}}{H_s}),$$

have been utilized, with

$$C_5'' = 1 - \frac{4C_3\mathbf{H}''\Gamma_\theta}{3A_oC_3''(\overline{\theta}_H-\theta_o)},$$

$$A_o = \frac{8}{3}(1-\frac{C_2''}{2C_3''}\frac{\overline{R}_{if}}{R_{iB}}),$$

$$\Gamma_\theta = \frac{\Delta\theta}{\Delta H} - (\frac{\partial\overline{\theta}}{\partial z})_{z=H}.$$

The value of $\frac{dH}{dt}$ from (16) is obtained using (17) together with formulas (18) to (26) given below. The change of the height of the surface layer is forced by the mechanical production $\mathbf{P}''$, the temperature change in the surface $\frac{d\theta_o}{dt}$, the thermal turbulence $\overline{w'\theta_o'}$, and the difference of the cooling ratio of the radiative heating at the 2 layers of the boundary layer.

d. Turbulence in the surface boundary layer

The method of profile is used to compute the momentum turbulence fluxes, the sensible heat and the evaporation. Temperature, humidity and wind speed at two levels are necessary in order to derive fluxes from the profile method. The wind speed in the z'_o roughness lenght is $\overline{U}(z'_o) = 0$ and at H_s is $\overline{U}_{H_s}$. The formulas used are the following:

$$\overline{\rho}\,\overline{u'w'_o} = -\overline{\rho}u_*^2 = -\overline{\rho}C_D\,|\vec{V}_{H_s}|\,\overline{U}_{H_s},$$

$$\overline{\rho}c_p\overline{w'\theta'_o} = -\overline{\rho}c_p u_*\theta_* = -\overline{\rho}c_p C'_D\,|\vec{V}_{H_s}|\,(\overline{\theta}_{H_s} - \theta(z'_o)),$$

$$\overline{\rho}L_v\overline{w'q'_o} = -\overline{\rho}L_v u_* q_* = -\overline{\rho}L_v C'_D\,|\vec{V}_{H_s}|\,(\overline{q}_{H_s} - q(z'_o)), \tag{18}$$

where

$$C_D = k^2[Ln(\frac{H_s}{z'_o}) - \psi_m(R_{ib}, \frac{H_s}{z'_o})]^{-2},$$

$$C'_D = k^2\{[Ln(\frac{H_s}{z'_o}) - \psi_h(R_{ib}, \frac{H_s}{z'_o})][Ln(\frac{H_s}{z'_o}] - \psi_m(R_{ib}, \frac{H_s}{z'_o})]\}^{-1}, \tag{19}$$

C_D and C'_D depend on the atmospheric stability in the vecinity of the soil, represented by the parameter $\frac{z}{L}$ or $R_{ib} = \frac{g}{T_r}\frac{H_s(\theta_1-\theta_o)}{\overline{U}_{H_s}}$. The functions ψ_m and ψ_h can be obtained in terms of $\frac{z}{L}$ (see Paulson, 1970; Barker and Baxter, 1975; or Nickerson and Smiley, (1975)).

In this paper I have followed a similar procedure as Louis (1979), where the relationships (19) have been approximated by analytical formulae. H_s is the height of the first level in the model and its change in time is forecasted from equation (17). The expressions $\theta(z'_o)$ and $q(z'_o)$ are computed from Silitinkevich (1970):

$$\theta_o = \theta(z'_o) = \theta_g + 0.0962(\theta'_*/k)(u_* z_o/\nu)^{0.45},$$

$$q_o = q(z'_o) = q_g + 0.0962(q'_*/k)(u_* z_o/\nu)^{0.45},$$

where the kinematic viscosity of air is $\nu \approx 1.5x10^{-5}m^2s^{-1}$,

$$\theta_g = T_g(\frac{1000}{p_g})^{R/c_p},$$

$$q_g = R_h q_s(T_g),$$

$$\theta'_* = (\theta_1 - \theta_g)[Ln(\frac{H_s}{z'_o}) - \psi_h]^{-1},$$

$$q'_*(q_1 - q_g)[Ln(\frac{H_s}{z'_o}) - \psi_h]^{-1},$$

R_h is the relative moisture in the surface, T_g ground surface temperature, and P_g the ground surface pressure.

e. Radiative flux equations

We have used the longwave flux equations in the emitance form of wide band (Sasamori, 1968; Stephens, 1984). The upward and downward longwave fluxes at altitude z are computed from

$$F_L^{\uparrow}(z) = \sigma T_g^4(1 - \epsilon(z,0)) + \int_0^z \sigma T^4(z')\frac{d\epsilon}{dz'}(z,z')dz' \quad (20)$$

$$F_L^{\downarrow}(z) = \sigma T_{top}^4(1 - \epsilon(z,top)) + \int_z^\infty \sigma T^4(z')\frac{d\epsilon}{dz'}(z,z')dz', \quad (21)$$

Here $\epsilon(z,z')$ is the emissivity for the correct mass of the absorptivity that corresponds to a vertical path from the height z to z', and σ being the Stephan-Boltzman constant.

The equations (20) and (21) are solved numerically following the method of Mahrer and Pielke (1977), in condition of clear sky, where the emissivities of the wide band are calculated starting from the empirical relations of Kuhn (1963). The heating rate, due to solar flux, takes into consideration the absorption and dispersion and is determined from the parameterization, proposed by Mahrer and Pielke (1977) and Mahfouf (1986):

$$\overline{F}_S(z) = F_o(1 - A_g)cosZ\tau_s,$$

where F_o is the radiative solar intensity at the top of the atmosphere, τ_s is a term which considers the transmissitivity and the absorptivity of the atmosphere, and **Z** is the zenith angle. A_g is the total surface albedo, which is the sum of a term that takes into account the effect of the zenith angle and another that considers the degree of wetness of the soil (Idso *et al.*, 1975).

f. The heat equation at the atmosphere - soil interface

In the present model the ground surface temperature is required. It is obtained by the surface energy balance equation in the interface soil - atmosphere:

$$-G(0) = \epsilon_g \sigma T_g^4 - (1 - A_g)F_S^{\downarrow} - \epsilon_g F_L^{\downarrow}(0) - \overline{\rho} L u_* q_* - \overline{\rho} c_p u_* \theta_*, \quad (22)$$

where ϵ_g is the emissivity, and A_g is the albedo of the underlying surface and depends on the content of the soil humidity η, $G(0)$ is the soil heat flux at the surface which is obtained from

$$G(0) = -\lambda\frac{\partial T_s}{\partial z}\mid_{z=0},$$

where T_s is the soil temperature, $\lambda \approx 1.05JK^{-1}m^{-1}s^{-1}$ is the thermal conductivity of the soil. In this paper we use the λ which depends on the soil moisture content and the soil texture (Nakshabandi and Kohne, 1965; Clapp and Hornberger, 1978). As the heat fluxes change, T_s also changes and is given by means of the soil's heat conduction equation

$$\rho c \frac{\partial T_s}{\partial t} = \frac{\partial}{\partial z}(\lambda \frac{\partial T_s}{\partial z}), \tag{23}$$

where ρc is the volumetric heat capacity. McCumber and Pielke (1981) have associated ρc with certain characteristics of the soil according to the formula

$$\rho c = (1 - \eta_s)\rho_i c_i + \eta \rho_w c_w$$

where η_s is the volumetric saturation moisture content, $\rho_i c_i \approx 1.26 \times 10^6 WK^{-1}m^{-3}$ is the calorific capacity for a unit of solid matter, which varies slightly with different soil types and with the water content, and $\rho_w C_w \approx 4.2 \times 10^6 WK^{-1}m^{-3}$.

Deardorff (1978) reanalyzed various methods to solve equation (22). He compared the results of these methods with those which integrate the multi-layer model of the soil's heat conduction. He finds that the surface heating rate $\frac{\partial T_g}{\partial t}$ is computed by

$$\frac{\partial T_g}{\partial t} = \frac{2\pi^{1/2}}{\rho c\sqrt{k_s \tau_1}}[-G(0)] - \frac{2\pi}{\tau_1}(T_g - T_s), \tag{24}$$

where $k_s = \frac{\lambda}{\rho c} \approx 5 \times 10^{-7} m^2 s^{-1}$ is the thermal diffusivity of the soil, $-G(0)$ is the forcing function, and $\tau_1 = 86400$ sec which is the period of one day.

The local rate of change of the volumetric moisture content is described by the equation

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial z}(D_\eta \frac{\partial \eta}{\partial z} + K_\eta), \tag{25}$$

where D_η is the soil's diffusivity and K_η the hydraulic conductivity. Values of D_η, η_s, K_η for different types of soil are given in descriptions by McCumber and Pielke (1982). Following McCumber and pielke (1982) and Mahfouf (1986) the continuity of the mass in the interface soil -air is secured by an iterative method of the expression $\bar{\rho} u_* q_* = W_s \mid_{z=0}$ where $W_s \mid_{z=o} = D_\eta \frac{\partial \eta}{\partial z} + K_\eta$ is the soil's humidity flux in the surface.

g. Coupling of the model with synoptic scale flux

The effect of the synoptic scale circulation over the development of the boundary layer is achieved through the rate penetration:

$$\frac{dH}{dt} - w_H = w_e,$$

where $w_H = w_2$ is the vertical velocity on synoptic scale in the top of the planetary boundary layer. This term represents the direct interaction of the boundary layer with the free atmosphere above. For this calculation of w_H we have followed a method proposed by Fleagle and Nuss (1985), which depends on measurements of vorticity, and uses the following formula:

$$w_H = \frac{\overline{\rho} C_D}{\rho_H f}(\overline{V}_H - \overline{V}_{H_s})\xi + \frac{\overline{\rho}}{\rho_H f}\nabla \cdot$$

$$[C_D(\vec{V}_H - \vec{V}_{H_s}) \times (\vec{V}_H - \vec{V}_{H_s}) \cdot \hat{k}] + \frac{\beta H}{f}\frac{\overline{\rho}\overline{V}_H}{\rho_H} + \ldots, \tag{26}$$

where ξ is the vorticity, f the Coriolis parameter, $\vec{V}_H = (u_H, v_H)$ the horizontal velocity, and ρ_H the density at the top of the boundary layer; C_D is the computed by the first equation from (18), $\overline{V}_{H_s} = |\vec{V}_{H_s}|$ is the surface velocity, and $\beta = \frac{\partial f}{\partial y}$. In this paper, as a preliminary step, it is assumed that the vorticity and the horizontal velocity in (26) are geostrophic ($\xi_g = \frac{g}{f}\nabla^2 z$, $\vec{V}_H = \vec{V}_g - (z - z_2)(\frac{\partial \vec{V}_g}{\partial z})_{700}$) and are forecasted with a simple barotropic model for the 700 mb level (Pérez *et al.*, 1986):

$$\nabla^2 \frac{\partial z}{\partial t} = -J(z, \frac{g}{f}\nabla^2 z + f),$$

$$\vec{V}_g = f^{-1}\hat{k} \times \nabla g z.$$

The integration is carried out in the region shown in Fig. 2 with time steps of half hour using $(\frac{\partial \vec{V}_g}{\partial z})_{700} = \frac{g}{f}\hat{k} \times \nabla T\,|_{700}$ (the thermal wind). In this way w_H and $\vec{V}_H$ are computed each half hour and these values are incorporated at the same time step in the integration of the equation for the model of nocturnal boundary layer.

Numerical experiments

In this paper we follow a method similar to that of Mahfouf (1986), in which (22) is integrated by means of a bisection method, and by solving numerically (23) and (25) using the multiple soil layer model. The depth of the soil is 1 m, subdivided into 13 sublayers, by employing one vertical co-ordinate transformation with a very high resolution near the ground. Therefore, a system of 13 equations is obtained and solved for $T^{t+\Delta t}$ and $\eta^{t+\Delta t}$, with $\Delta t = 30$ sec.

To solve the prognostic equations (2), (4), (5), (6), (8), (16) and (17), a predictor-corrector method with time steps of $\Delta t = 30$ sec is used.

At present we have no data on the turbulent atmospheric boundary layer. With the preliminary intention of testing the scheme, the model is tested with the slightly idealized sounding of Fig. 3. This sounding shows the mean height, where the surface cooling is extended. The inversion depth H_i is then higher than the height H, where the turbulence is eliminated. Nieuwstadt (1984) indicates that in stable conditions the turbulent eddies cannot extend themselves to the whole boundary layer. For a data set, Driedonks *et al.* (1985), find a correlation between H_i and H: $H_i = 2.8/H$. Therefore we may assume that $H = H_i/2.8$. The depth H_s of the surface layer is initialed with a value equal to 15.3 m.

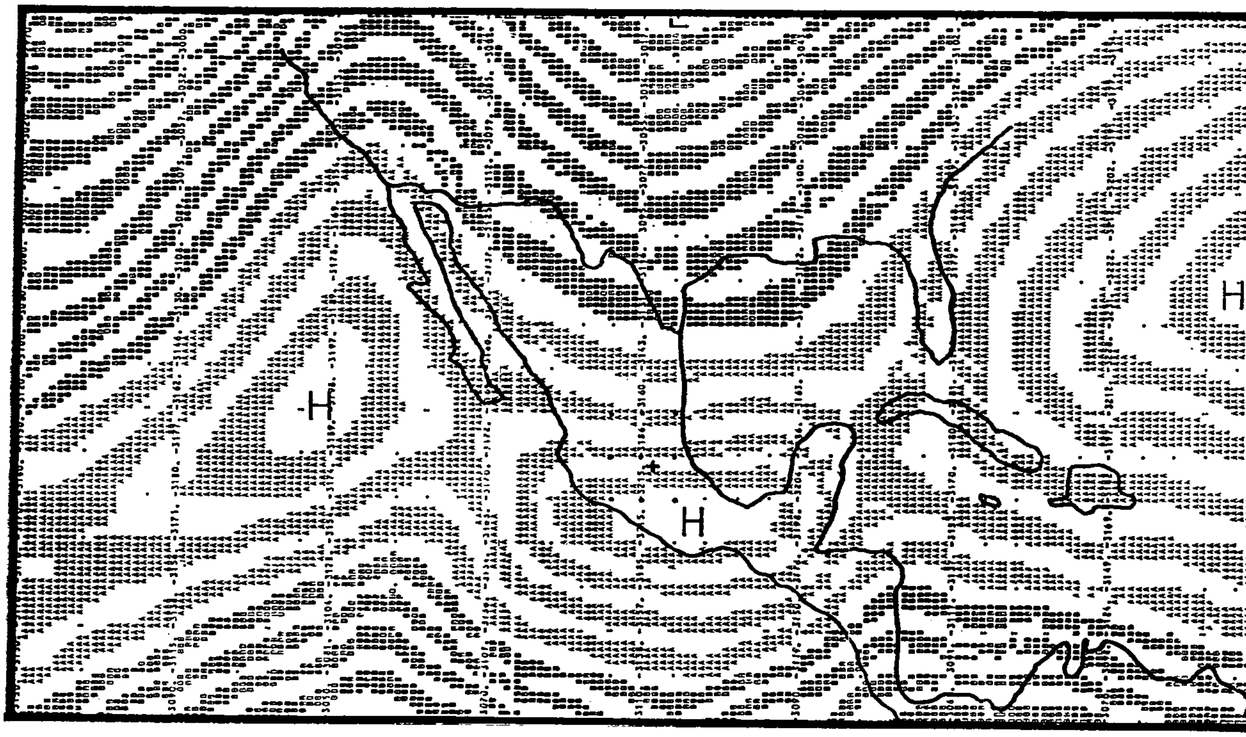

Figure 2. Integration region of the vorticity equation and 700 mb synoptic weather maps for 12 GMT on 3rd January, 1988. The vorticity in the point "+" (the Mexico City location is indicated with this sign) is determined with nine points Lagrangian interpolation.

The synoptic height for 700 mb level is shown in figure 2, with the intention of starting the integration at night and observing the behaviour of the model in the transition period of the rising Sun. Figs. 2 and 3 provide the dynamic and thermodynamic conditions which exist 0.49 hours before sunrise. The barotropic model is run and the vertical velocity is computed every half hour from equation (26). Furthermore, the various surface types found in the cities, such as pavement, highways and buildings are the most characteristic. This surface consists of armoured surfaces composed of rock fragments, concrete, sand and other materials. Therefore, the sandy soil is assumed to have a roughness lenght $z_o' = 0.04$, $\rho_i c_i = 1.463 \times 10^6 W K^{-1} m^{-3}$, $\rho c_w = 4.18 \times 10^6 W K^{-1} m^{-3}$ and the humidity of the soil equal to 0.4. The volumetric saturation moisture content is taken as $\eta_s = 0.395$. Initially the soil temperature and the humidity are equal to the surface values.

The experimental problem that we have in this model, is the adequate evaluation of C_2'' and C_3'', which require a detail description of the wind and the temperature at the top of the boundary layer. Zeman (1979) pointed out the distinction between the gradients of $\overline{u}$ and $\overline{\theta}$ above and below $H(H^+, H^-)$, where for a growing nocturnal boundary layer ($\frac{dH}{dt} > 0$) the value of $\frac{\partial \overline{\theta}}{\partial z}\mid_{z=H}$ ($or \frac{\partial \overline{u}}{\partial z}\mid_{z=H}$) would be replaced by $\frac{\partial \theta^+}{\partial z}$, which corresponds to the nocturnal field flow above the nocturnal boundary layer. For a descending boundary layer $\frac{\partial\overline{\theta}}{\partial z}(H^-) \approx A\frac{\Delta\theta}{\Delta H}$, A is a free constant. However, in this paper the form of the profile in H changes, leading to varying values of C_2'' and C_3''.

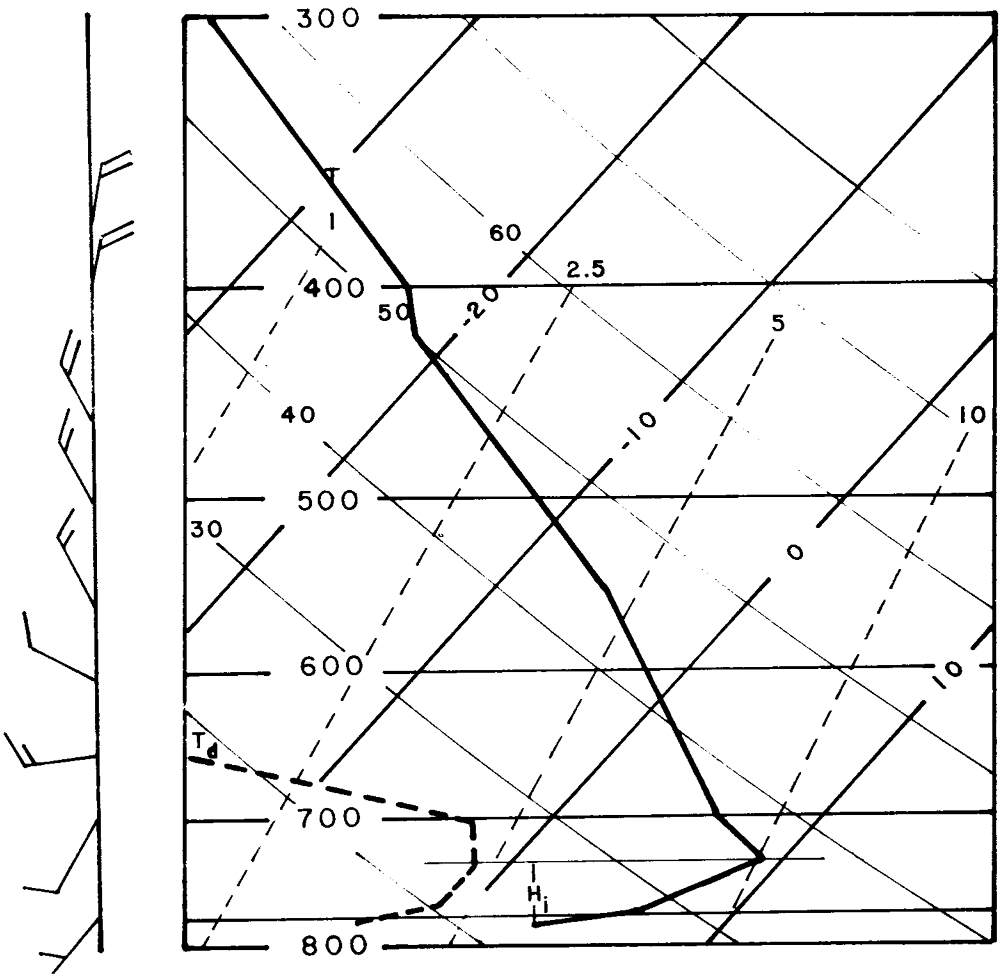

Figure 3. Initial vertical profiles for wind temperature and dew-point temperature. The conditions are slightly modified with respect to those of Figure 1.

With the above mentioned initial conditions and considering the problem of expressing the gradients at $z = H$, the following experiments are carried out:

i. Specifying the C_2'' and C_3'' in the form

$$C_2'' = C_2\left(1 - \frac{H_s^3}{H^3}\right)\left[1 - \frac{H}{2\overline{E}_H}\left(U_H\frac{U_H - \overline{U}_{H_S}}{\Delta H}\right)\right]$$

$$C_3'' = C_3\left(1 + \frac{H_s}{2H}\right)\frac{\Delta\theta}{(\overline{\theta}_H - \theta_0)} - C_3\left[1 - \frac{H_s}{2H}\left(1 + \frac{H_s}{H}\right)\right]\left[\frac{H}{(\overline{\theta}_H - \theta_0)}A\frac{\Delta\theta}{\Delta H}\right]$$

with $A = -2$. Initially $H = 248.8m, H_s = 15.33m, \overline{\theta}_H = 303.47^oK, \overline{\theta}_{H_s} = 296.5^oK$. For $A = 2$ we find a growing boundary layer.

ii. A fixed choice for $C_3'' = \frac{1}{3}$ and C_2'' similar to i with $H = H_i/2.8$.

iii. Similar to 1 and the model's sensitivity to radiative fluxes.

Results of experiment i

They are shown in Figs. 4 and 5 respectively. In Fig. 4 the initial profile of the temperature is plotted (with continuous line) and the forecastings every 15 minutes for the boundary layer and soil. The decreasing of the soil's temperature close to the surface is much noticed. This is mainly to the longwave radiation emission of the soil.

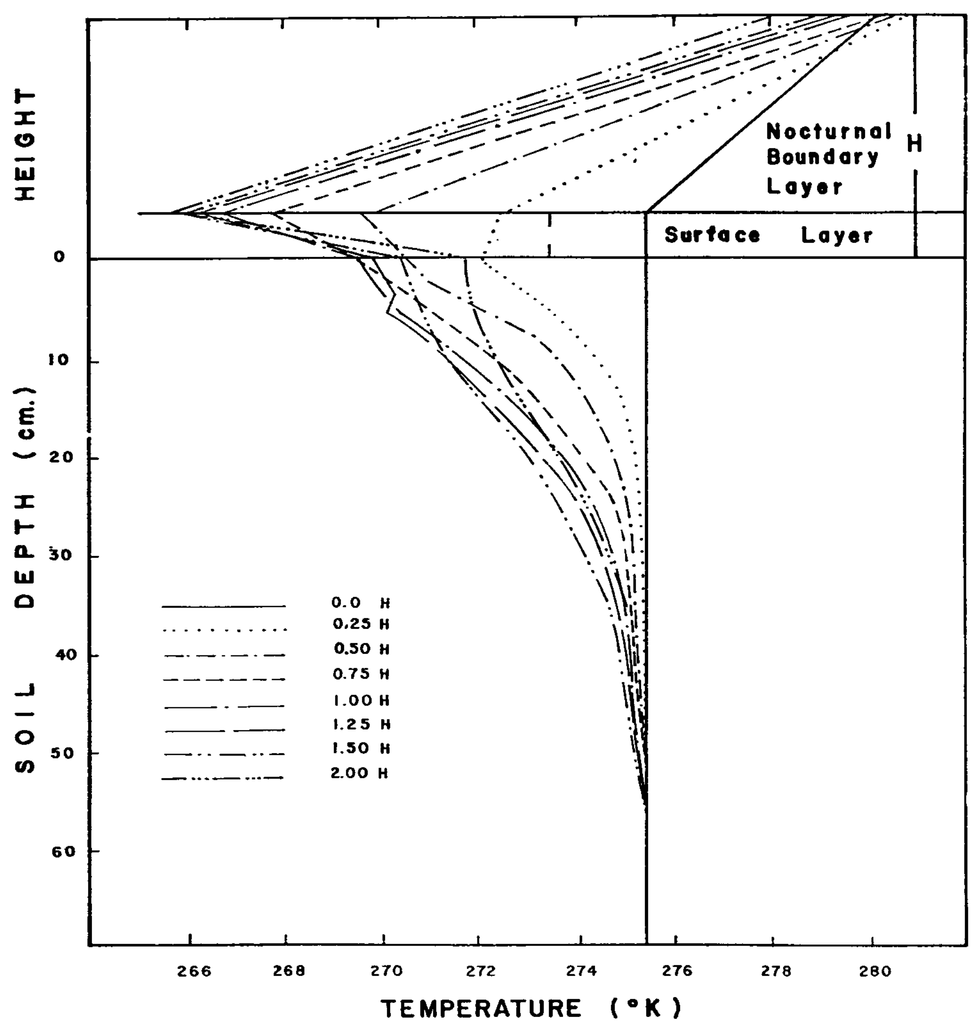

Figure 4. Initial (solid line) profile for the temperature of the boundary layer and the soil, and computed time evolution (from 0 to 2 hours) for experiment 1.

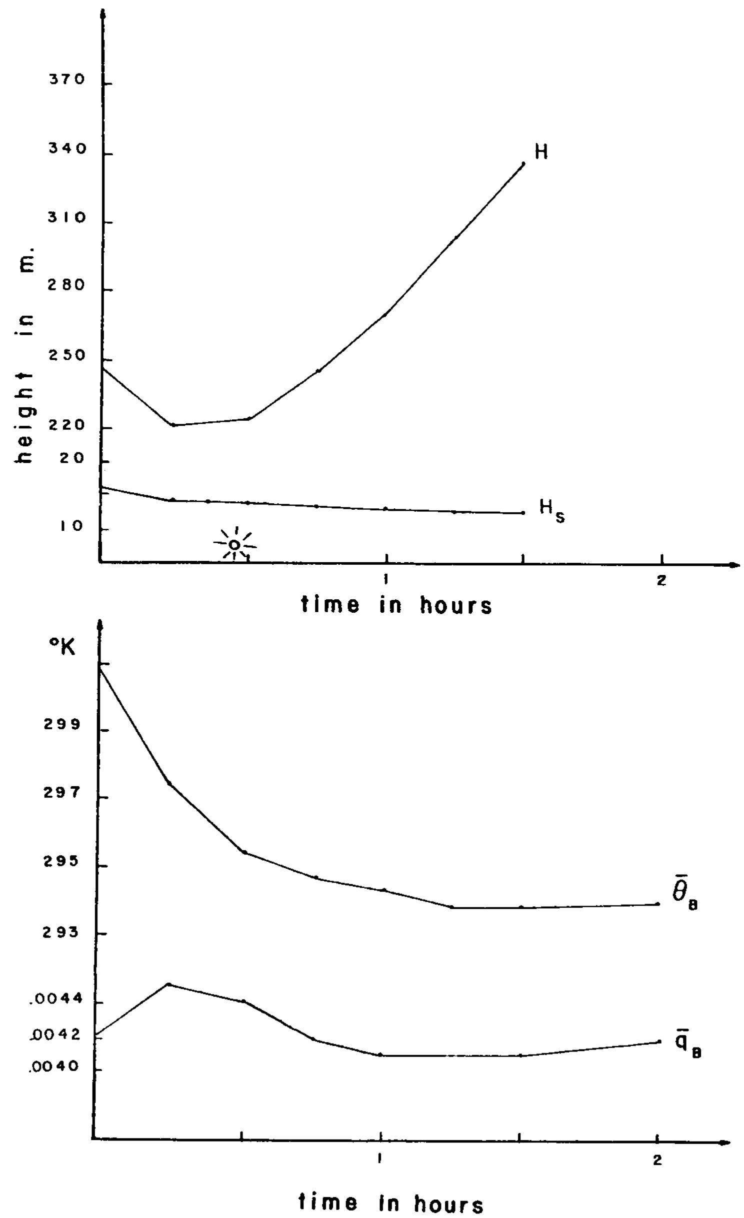

Figure 5. Top: calculated time evolution of the boundary layer height for experiment 1. Bottom: corresponding potential temperature and water-vapor mixing ratio profiles for the remainder boundary layer. The sun shows the time of sunrise.

Initially the surface temperature is 274.75 ^oK. At the moment of sunrise (see Fig. 5) the surface temperature is 270.82 ^oK and three minutes after sunrise it is 270.55 ^oK. The ground cooled by radiation tends to cool the air above it and affects the model's first level. Thus the temperature in the first level has been reduced to 265.97 ^oK. After sunrise the radiation reaches the ground, and the portion no reflected is absorbed to heat the surface. This heat raises the temperature of a very thin layer of soil. Therefore, the tendency of the surface temperature is now reversed, retarding the decrease of the temperature in H_s. The rate of heating $\frac{d\theta_0}{dt}$ varies between $.28 \times 10^{-4}$ and 2.4×10^{-5} $^oK sec^{-1}$.

Fig. 5 shows the evolution of the boundary layer, the surface layer and the thermodynamic profile. The two layers have a tendency to estratificate, and the boundary layer after sunrise increases. This behaviour is in accordance with the changes of temperature in the soil.

Results of experiment ii

They are presented in Figs. 6 and 7. Fig. 6 illustrates the evolution of the temperature profile of the boundary layer and temperature profile of the soil each 15 minutes. Initially, the temperatures of the top surface layer and of the thin layer of soil, increase. Afterwards they decrease. 7 minutes before sunrise the temperature of the top surface layer is lower that of the surface,. leading, to an unrealistic positive surface heat flux, which delays the fast fall of H (see Fig. 7). After sunrise the depth of the boundary layer increases. However, the temperature of the top surface layer continues decreasing, making the surface layer very stable and shallow.

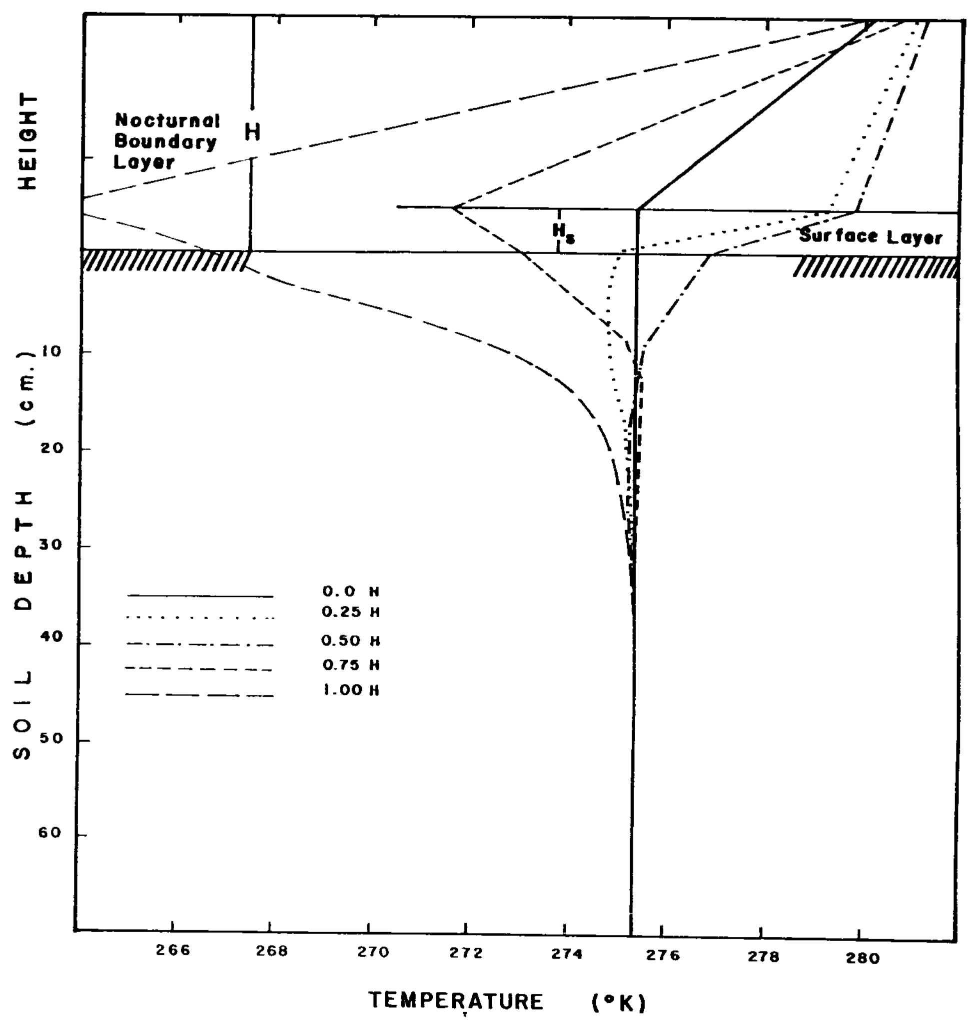

Figure 6. As in Figure 4 except for $C_3'' = \frac{1}{3}$.

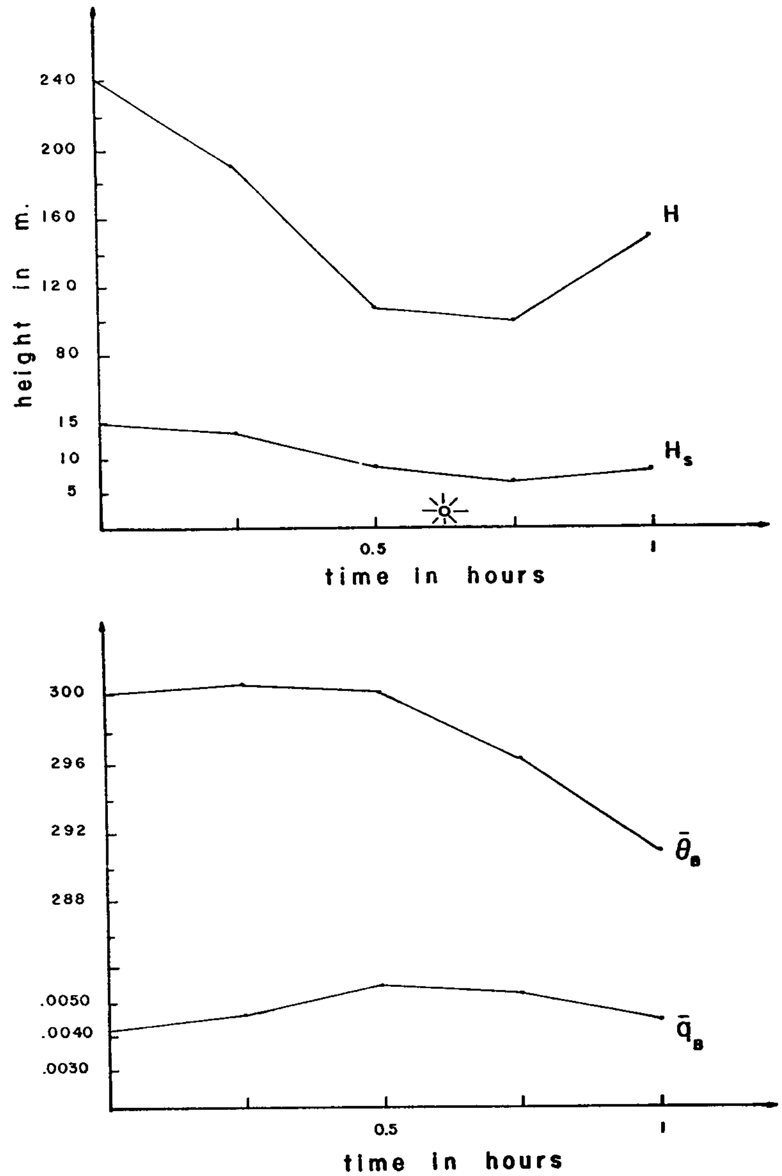

Figure 7. As in Figure 5 except for the case, when a fixed choice for $C_3'' = \frac{1}{3}$ is used.

Results of experiment iii

Fig. 8 shows the evolution of the boundary layer, the surface layer and the profiles, when all the terms of radiation in equations (16) and (17) are incorporated (with plus sign) and for the case, when the radiation is neglected (with continuous line). Fig. 8 shows that when radiation is included the layers H and H_s decrease, making the boundary layer and the surface layer stable and shallow, and showing the important influence of radiation over the boundary layer. In this case $\overline{\theta}_B$ also decreases. The moisture $\overline{q}_B$ behaves similarly in both cases.

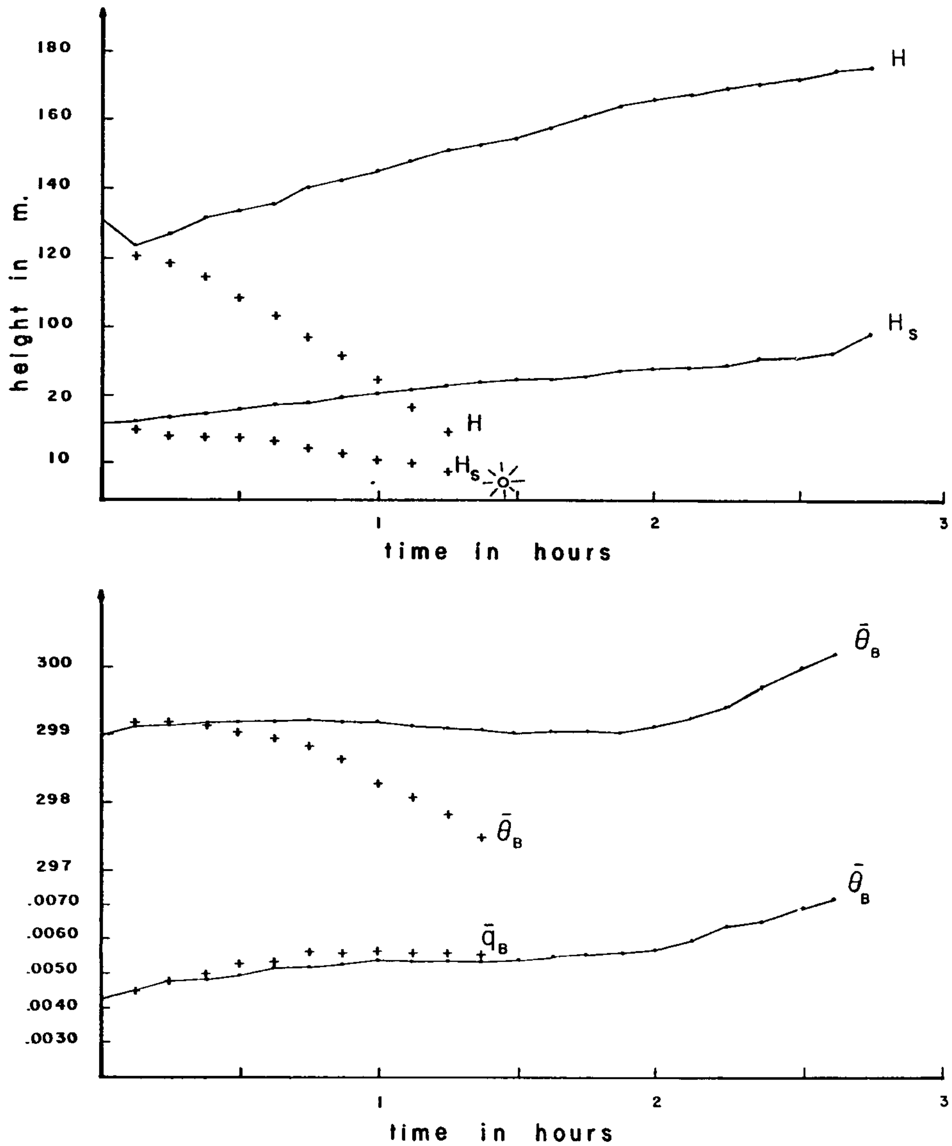

Figure 8. As in Figure 7 except for the experiment 3.

General remarks and conclusions

In this paper a preliminary thermodynamic model for the system of nocturnal boundary layer-soil has been presented. The model includes various meteorological processes: turbulence, radiation, interaction boundary layer-soil and boundary layer synoptic flux. The model forecasts the thermodynamic profiles in stable conditions. An equation to predict the change of height of the planetary boundary layer was also developed, as well as one equation for the surface layer. The presence of this thin layer is necessary for computing the drag coefficient that appears in the computation of the sensible and latent heat fluxes and in the study of the influence of the micrometeorological factors of the nocturnal surface layer. For example, the vegetation cover. The present model is of the contraction expansion type, therefore it differs from the one described by Reiff *et al.* (1984) and Nieuwstadt and Tennekes (1981) in the following aspects: radiative heating rate of the boundary layer has been incorporated, the interaction of the soil with the planetary boundary layer has been considered and is achieved through the surface energy budget. The coupling of the model with synoptic scale flux is obtained by means of a simple barotropic model. Furthermore, the nonstationary effects of the horizontal velocity at the top of the boundary layer is achieved through a coupling with the synoptic flux scale.

The results of the experiments show that turbulent sensible heat flux and strong radiative cooling or heating of the soil contribute to development of the boundary layer.

When the surface cooling or heating rate is computed using the surface heating rate from Deardorff (1978) the boundary layer decrease is large, making it very stable and shallow.

The variables of the boundary layer H_s, H, $\overline{\theta}_{H_s}$ and $\overline{\theta}_B$, are the most sensitive ones of the changing values of $\frac{d\overline{\theta}_o}{dt}$ and C_3''. The determination of the optimum value of C_3'' will be the subject of future work.

A comparison of the computed variations of surface temperature for different types of soil shows that they are influenced by η and $\frac{d\overline{\theta}_o}{dt}$. Furthermore, an increase of the moisture content reduces the variations of temperature. Therefore, the soil behaviour was predominantly determined by the moisture content. The behaviour surface of the layer is more stationary than the upper part of the boundary layer.

Research is being carried out to improve this model and possibly apply it to predict the evolution of the nocturnal boundary layer of Mexico City. An attempt will be made to introduce a Richardson number radiation and a parameterization during unstable conditions.

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APPENDIX

The buoyancy destruction averaged across the upper part of the boundary layer

In this Appendix we derive the numerator of $\overline{R}_{iF}$, which is defined as $-\int_{H_s}^{H} \frac{g}{T_r}\overline{w'\theta'}dz$. We use the horizontally homogeneous equation for the potential temperature, which is the following:

$$\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial\overline{w'\theta'}}{\partial z} + \left(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z}\right) \tag{27}$$

Multiplying the equation (27) by z and integrating the resulting expression across the upper part of the boundary layer (H_s, H), we obtain:

$$\int_{H_s}^{H} z\frac{\partial\overline{\theta}}{\partial t}dz = H_s\overline{w'\theta'}_{H_s} + \int_{H_s}^{H}\overline{w'\theta'}dz - \frac{1}{\rho c_p}(HF_{r_H} - H_sF_{r_H}) + \int_{H_J}^{H}\frac{F_r}{\overline{\rho}c_p}dz, \tag{28}$$

where we have used the product derivated rule and the boundary condition. Next we derive an explicit expression for the first member of (28), by integrating by parts and with the aid of Leibnitz's rule:

$$\int_{H_s}^{H} z\frac{\partial\overline{\theta}}{\partial t}dz = \frac{H^2}{2}[(\frac{\partial\overline{\theta}}{\partial t})_{z=H} + (\frac{\partial\overline{\theta}}{\partial z})_{z=H}\frac{dH}{dt}] - \frac{H_s^2}{2}[(\frac{\partial\overline{\theta}}{\partial t})_{z=H_s} +$$

$$(\frac{\partial\overline{\theta}}{\partial z})_{z=H_s}\frac{dH_s}{dt}] - \frac{d.}{dt}[\frac{H_s^2}{2}(\theta_H - \theta_{H_s}) + \int_{H_s}^{H}(\overline{\theta}_H - \overline{\theta})zdz]. \tag{29}$$

Consistent with (1), the profile of $\overline{\theta}$ in the layer (H_s, H) is of the form

$$\overline{\theta}_H - \overline{\theta} = (\frac{H-z}{\Delta H})(\overline{\theta}_H - \overline{\theta}_{H_s}). \tag{30}$$

Using (30) and (9), formula (29) becomes:

$$\int_{H_s}^{H} z\frac{\partial\overline{\theta}}{\partial t}dz = -\{C_3(2H + H_s)\Delta\theta - [C_2H^2 - C_3H_s(H_s + H)](\frac{\partial\overline{\theta}}{\partial z})_{z=H}\}$$

$$\frac{dH}{dt} - [C_3(2H_s + H)\Delta\theta + \frac{H_s^2}{2}(\frac{\partial\overline{\theta}}{\partial z})_{z=H_s}]\frac{dH_s}{dt} + C_3(H^2 + H_sH + H_s^2)\frac{d\overline{\theta}_{H_s}}{dt}$$

$$-\frac{3}{2}\frac{H_s^2}{H}(1 - \frac{H_s}{H})^2\overline{w'\theta'_o} + [C_2H^2 - C_3H_s(H_s + H)](-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=H} - \frac{H_s^2}{2}(-\frac{1}{\overline{\rho}c_p}\frac{\partial\overline{F}_r}{\partial z})_{z=H_s}, \tag{31}$$

substituting (31) in (28) and multiplying by $-\frac{g}{T_r}$, we obtain formula (14).

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