Atmósfera (1990), **3**, pp. 239-253

The estimation of the upper ocean mixed layer depth at ocean weather station Papa using the variational approach

YU. D. RESNYANSKY

Hydrometeorological Center of the USSR, Moscow, USSR

(Manuscript received Sept. 4, 1989; accepted Feb. 23, 1990)

RESUMEN

Las variaciones de las características térmicas de las capas oceánicas cercanas a la superficie ejercen un impacto significativo en el flujo de calor cedido a la atmósfera y, a través de esto, en la circulación atmosférica. Para examinar las fuentes de calor oceánicas se necesitan las estimaciones del estado actual de los campos térmicos oceánicos, obtenidos de mediciones operacionales de temperatura. La distribución de estas mediciones respecto a la profundidad, es altamente irregular. Información relativamente detallada existe sólo para la temperatura superficial (SST) medida mediante barcos y boyas, así como mediante percepción remota con satélites. Además, se necesitan mediciones de perfiles verticales de temperatura, para obtener la profundidad h de la capa oceánica mezclada, que junto con la temperatura superficial, es uno de los principales parámetros que caracterizan el almacenamiento de calor en la capa superior del océano. El número de mediciones XBT generalmente contenidas en las bases de datos operacionales, es altamente restringido ($\sim$ 150 mediciones XBT diarias en el Hemisferio Norte) y no es suficiente para obtener las profundidades de la capa mezclada en las espaciosas regiones oceánicas. Por lo tanto, se necesitan métodos indirectos para estimar h. Uno de los métodos, basado en un enfoque variacional, ha sido propuesto por Resnyansky (1985). Las estimaciones de h se obtuvieron de las sucesiones temporales de SST en una localidad oceánica. Los valores de h correspondientes a la diferencia mínima en las temperaturas modeladas y las observadas en un intervalo dado de tiempo, se toman como una estimación resultante. El método ha sido comprobado usando datos simulados. El presente artículo tiene por objeto comprobar el método, usando las observaciones de 1972 a 1974, obtenidas en la estación Papa, situada en la región noreste del océano Pacífico. En los cálculos se considera para las incertidumbres estimadas, su dependencia de los parámetros del modelo, así como del tipo de estado básico establecido para estabilizar las estimaciones.

ABSTRACT

The variations of the thermal characteristics of the near-surface ocean layers exert a significant impact on the heat influx to the atmosphere, and through this on the atmosphere circulation. To examine the ocean heat sources, the estimates of the current state of the oceanic thermal field drawn from operational temperature measurements are needed. The distribution of these measurements over the depth is highly irregular. Relatively detailed information is available only on the sea surface temperature (SST) measured both *in situ* from opportunity ships and drifting buoyes and by means of remote sensing from satellites. To obtain immediately the upper ocean mixed layer depth h, which in conjunction with the SST represents one of the main parameters characterizing oceanic near-surface heat storage, the measurements of vertical temperature profiles are needed. The number of XBT measurements commonly contained in operational data bases is highly restricted ($\sim$ 150 daily XBT measurements in the Northern Hemisphere) and is not enough for immediate obtaining of the mixed layer depths over the spacious ocean domains.

In this connection indirect methods of estimating h are needed. One of the methods based on variational approach has been proposed by Resnyansky (1985). The estimates of h were obtained from the temporal sequences of SST at an ocean location. The values of h corresponding to minimal differences between modelled and observed SSTs within a given time interval are taken as a resultant estimate. The method has been tested using simulated data. The present paper is devoted to the further testing of the method using the actual observations at ocean weather station Papa in the North Eastern Pacific over the period from 1972 to 1974. The dependence of the estimation uncertainties on the model parameters as well as on the type of the basic state set to stabilize the estimates is considered.

1. Introduction

A heat storage of the upper ocean is one of the energy sources of the large-scale atmosphere motions. The most important indexes of this storage are the temperature of the upper mixed layer (UML) and its depth h. To monitor the current state of the UML, operational observational data are needed. The operational data base includes marine meteorological observations from opportunity ships and drifting buoyes, XBT measurements of the vertical temperature profile and remotely sensed sea surface temperatures (SSTs) from satellites. Relatively detailed information, though not sufficient for tracing all the space scales of variability interesting for practical purposes, is available only for SSTs. Approximately, 1500 SST *in situ* measurements are added daily to the Northern Hemisphere data base (Clancy and Pollak, 1983). Applying one of objective analysis procedures (e.g., Bagrov and Kozhevnikova, 1981; Zelenko and Nesterov, 1986) to these data, supplemented by climatological information, the SST fields with the horizontal resolution of few hundreds of kilometers may be constructed.

Operational data of XBT measurements suitable for direct determination of the UML depth are essentially more limited in number amounting only approximately 150 measurements per day in the Northern Hemisphere (Clancy and Pollak, 1983). It is a highly complicated task to restore the distribution of the UML depth from this quite limited data set. That is why there appears a task of deriving indirect estimates using available SST measurements and marine meteorological observations.

The deriving of informative state estimates from incomplete data sets is commonly performed by means of data assimilation procedures; or, as they are often called in meteorological applications, methods of four-dimensional objective analysis (e.g., Bengtsson, 1981). The lack of the actual data for current time is filled with data from the previous times taking into account possible temporal variations determined by means of a suitable model. The existence of this model which relates mathematically different physical variables to each other allows us to derive a judgement on the behaviour of directly nonobservable variables provided that there are available observations of other variables.

A method of deriving an indirect estimate of h from temporal sequences of SSTs, heat and momentum fluxes has been proposed by Resnyansky (1985). The method is based on a variational approach widely used in meteorological applications (Penenko, 1981; Penenko *et al.*, 1983; Ryasin and Svidritsky, 1982; Le Dimet and Talagrand, 1986). The values of h which produce the minimal differences between modelled and observed SSTs within a selected time interval are taken as the resultant estimates. To stabilize the results which are highly sensitive to uncertainties in SST data, an additional a priori information on the expected UML depths is taken into account. The method has been tested with the usage of simulated data which imitate the UML evolution under the seasonal atmospheric forcing.

The present paper is aimed at the further verification of the above method using the actual observations at ocean weather station Papa in the North Eastern Pacific (50°N, 145°W). The sensitivity of the derived results to the variations of the parameter values is being examined. Then an estimation accuracy which can be attained for the available observational data is being determined. In Section 2 basic model equations used to evaluate the evolution of UML during deriving indirect estimates of h are presented. The method of indirect estimation of h is described in Section 3, while the regularization method used to stabilize the derived estimates is considered in Section 4. A description of a technique for searching a minimum of the function quantifying the deviations of calculated water temperatures from the observed ones is briefed in Section 5.

Section 6 contains a description of input data used in the simulations. The results of numerical experiments performed with various parameter values and with various types of the basic state are discussed in Section 7. Section 8 contains the concluding remarks .

2. Basic equations of the upper ocean mixed layer model

The main features of the evolution of the upper ocean except its specific regions (boundary currents, upwelling areas, near-equatorial band) can be simulated with the help of simple one-dimensional models (e.g., Niiler and Kraus, 1977; Zilitinkevich *et al.*, 1977, for reviews). One of these models developed by Resnyansky (1983) is used in the present paper. The basic equations of a one-dimensional version of the model are:

$$dT/dt = (2/h)[q/(\rho_r c_p) - P/(\beta h)], \tag{1}$$

$$dT_A/dt = q/(\rho_r c_p H), \tag{2}$$

$$T_A = T_H + (1 - c_T + c_T h/H)(T - T_H), \tag{3}$$

$$P = \Lambda c_1 (u_* - c_2 f h) u_*^2 + \frac{\beta h(q - \mid q \mid)}{4\rho_r c_p}[1 - (1 - \frac{h}{H})^m], \tag{4}$$

where t is time; T is UML temperature assumed coincident with SST; h is UML depth; T_A is vertically averaged temperature within an active layer; T_H is temperature at the lower boundary of the active layer; H is depth of the active layer; ρ_r and c_p are reference values of sea water density and specific heat capacity correspondingly; $\beta = g\alpha$ (g is gravity acceleration, α is thermal expansion factor) is buoyancy parameter; f is Coriolis parameter; q is net heat flux at the ocean surface; $u_* = (\tau/\rho_r)^{1/2}$ is friction velocity in water (τ is tangential wind stress); Λ is Heaviside unit function equalled to one (zero) if $(u_* - c_2 f h)$ is less (greater) than zero; $c_1 = 10.$, $c_2 = 4.$, $c_T = 0.85$, $m = 3$ are empirical coefficients.

The model equations are derived through the vertical integration of the thermal energy equation and the equation of the turbulent kinetic energy budget assuming the vertical homogeneity of temperature within the UML and the certain self-similarity of the temperature profile within the seasonal thermocline. The details of the derivation and the discussion of other assumptions used in models like that can be found, for example, in Zilitinkevich *et al.* (1979). Eqs. (1) and (2) in conjunction with formulae (3) and (4) constitute a set for calculating temporal variations of the UML temperature T and vertical mean active layer temperature T_A. The active layer is determined as that part of a water column from the surface to the depth $z = H$ within which seasonal variations of temperature are discernible. It includes the upper mixed layer and the seasonal thermocline. The temperature T_H at the lower boundary of the thermocline, $z = H$, is supposed to be fixed. The UML depth h is obtained from (3) using T and T_A evaluated at each time t from (1) and (2). The heat flux q and the friction velocity u_* (the tangential wind stress τ) are assumed to be known as deduced from meteorological observations in the atmosphere surface layer.

The above set of equations may be rewritten in equivalent from with time derivative dT_A/dt replaced by dh/dt. To obtain this form, T_A should be eliminated through the substitution of (3) into (2) taking into account (1). However, the accepted form is more compact and convenient for numerical integration.

3. Method for obtaining indirect estimate of h

Let $t = t_i$ be a current time, for which an estimate of the UML depth is to be obtained. Furthermore, let there be $M + 1$ measurements of the UML temperature $\tilde{T}_j = \tilde{T}(t_j)$, $j = i - M$, $i - M + 1$, ..., i, distributed over some time interval $t_{i-M} \leq t_j \leq t_i$ prior to the current time t_i. Here and hereafter the symbol " ˜ " marks the measured values which differ from the actual ones owing to various types of uncertainties, and the symbol " ˆ " will designate that the corresponding variables are obtained as a result of applying the estimation procedure.

Let us introduce a function

$$I_i^l = I_i^l(h_v) = \sum_{j=i-M}^{i} [T_j(h_v) - \tilde{T}_j]^2, \tag{5}$$

the magnitude of which is a measure of the "distance" between the UML temperatures $T_j = T_j(h_v)$ deduced from model (1)-(4) and the observed temperatures $\tilde{T}_j$ at the same times t_j.

An argument of function I_i^l is h_v representing the initial value of the UML depth for the integration of Eqs. (1)-(4) over the time interval $t_{i-M} \leq t \leq t_i$:

$$h(t)\ |_{t=t_{i-M}} = h_v. \tag{6}$$

One more initial condition is the observed value of the UML temperature which is assumed coincident with the SST:

$$T(t)\ |_{t=t_{i-M}} = \tilde{T}_{i-M}. \tag{7}$$

It seems natural to accept as the resultant estimate $\hat{h}_{i-M}$ for time $t = t_{i-M}$ such value of UML depth, for which the corresponding departures of T_j from $\tilde{T}_j$ turn out to be minimal in the sense of measure (5). That is,

$$I_i^l(\hat{h}_{i-M}) = \inf_{h_v \epsilon D} I_i^l(h_v), \tag{8}$$

where $D = [h_\varepsilon, H]$ is the interval of permissible values for h_v; h_ε is a lower limit set to the UML depth during the integration of Eqs. (1)-(4) to prevent the declination of the calculated model state outside the boundaries of the model applicability (the model becomes invalid as $h \to 0$).

Then it is easy to estimate the depth $\hat{h}_i$ related to the current time $t = t_i$. This estimate yields from time integration of the model over the interval $t_{i-M} \leq t \leq t_i$ using the following initial conditions:

$$h(t)\mid_{t=t_{i-M}} = \hat{h}_{i-M}, \quad T(t)\mid_{t=t_{i-M}} = \tilde{T}_{i-M}, \tag{9}$$

where $\hat{h}_{i-M}$ is obtained from (8).

4. Regularization of the estimates

The application of the above procedure to inaccurately specified values of $\tilde{T}_j$ showed, however, that the resultant estimates $\hat{h}_i$ appeared to be unstable. Small perturbations of $\tilde{T}_j$ yielded large variations of $\hat{h}_i$ (Resnyansky, 1985). This is a common property of many inverse problems (Tikhonov and Arsenin, 1974), the problem of indirect estimation of h from temperature measurements with the use of the UML model being one of them.

To stabilize the estimates, function I_i' is supplemented with one more term representing the regularizational part of the function to be minimized. The introduction of this term is based on the use of *a priori* information on the parameters being estimated.

The succession of steps for obtaining the regularized estimate $\hat{h}_i$ remains the same as above, except that I_i' is replaced by

$$I_i = I_i(h_v) = \sum_{j=i-M}^{i} [T_j(h_v) - \tilde{T}_j]^2 + rM[h_i - \overline{h}_i]^2, \tag{10}$$

where $h_i = h_i(h_v)$ is the UML depth at time $t = t_i$ in a model run corresponding to initial conditions (6) and (7); $\overline{h}_i$ is the UML depth corresponding to the basic state; $r = \delta_T^2/\delta_h^2$; δ_h^2 is the variance of the departures of model derived values h_i from the basic state $\overline{h}_i$ relatively to which the regularization is performed; δ_T^2 is the variance characterizing the errors in specified temperatures $\tilde{T}_j$. It should be noted that an immediate determination of δ_h^2 and δ_T^2 turns out to be complicated both due to the scantness of the data base and the fact that the word "error" means in this case not a simple instrumental inaccuracy but a more general measure of uncertainties including also deviations of the actual water temperatures from those values, the evolution of which can be reproduced by the model. Since δ_h^2 and δ_T^2 enter into (10) only as a relation, so taking into account the above reasons the relation r may be considered as an adjustment parameter chosen during trial calculations of estimates $\hat{h}_i$.

There are various possibilities for setting up the basic state $\overline{h}_i$. The simplest one is the use of climatological data. If a succession of cyclically intermittent analyses and forecasts is available (a typical case for operational forecasting systems of the type described by Clancy and Pollak, 1983), the forecast for the current time t_i can be taken as a basic state. While estimating the atmosphere current state Ryasin and Svidritsky (1982) took the stationary solution of model equations as a basic state.

The stationary solution of Eqs. (1)-(4) exists only when $q = 0$. However, during the periods of surface heating ($q > 0$), in spite of the increase of the UML temperature, the UML depth tends to approach a limiting value (during the period of the order of a day; e.g. Resnyansky, 1975) provided that q and u_* change sufficiently slowly. For model (1)-(4) the limiting value is determined as

$$h_+ = \frac{2c_1u_*^3}{(1+b)\beta q/(\rho_r c_p) + 2c_1c_2fu_*^2}, \tag{11}$$

where $b = (1-c_T)(H-h_+)/[h_+ + (1-c_T)(H-h_+)]$.

Formula (11) is derived by assuming $dh/dt = 0$ in an equation yielded from (1)-(4) under fixed q and u_*.

Obviously, the actual atmospheric forcing, expressed through q and u_*, never remains strictly invariable. Yet a typical time scale of its changes (the period related to synoptic maximum in the atmosphere motions spectrum may be assumed for this scale) usually exceeds the time necessary to stabilize the UML depth. So the assumption on the proximity of the actual UML depths to the limiting ones, expressed by formula (11), seems quite acceptable.

During the periods of surface cooling ($q < 0$), which is usually accompanied with buoyancy convection, the situation is essentially different. The assumption on the quasi-stationary nature of the UML depth changes is already invalid and some other ***a priori*** information is needed. The additional information may be extracted, for instance, from the fact that the periods of convection are characterized by a monotonic increase of h. To introduce this information into the estimation procedure, we assume that during cooling $\overline{h}_i$ is equal to

$$h_- = [2\int_{t_{i-M}}^{t_i} q/(\rho_r c_p \gamma)dt + h_v^2]^{1/2}, \tag{12}$$

where γ is a mean temperature gradient below the mixed layer. Formula (12) is obtained from the equation which describes the nonpenetrative convective deepening (Zilitinkevich *et al.,* 1979), and, thus, is a formal expression of the tendency to the monotonic increase of h.

Substituting one of the expressions for $\overline{h}_i$ (for instance, $h_i = h_+$ when $q \geq 0$, $\overline{h}_i = h_-$ when $q < 0$, or climatological data $\overline{h}_i = h_{\mathrm{clim}}$) into (10), one will obtain various versions of cost-function I_i the minimization of which forms the base for deriving regularized estimates $\hat{h}_i$.

5. Procedure for searching minimum of I_i

A minimum of function I_i in the below experiments was searched by the golden section method (Himmelblau, 1972). According to the searching algorithm, at first the interval Δ_o which contained an extremum of I_i along h_v was established; and then Δ_o was squeezed to the prescribed level by means of successive division of the interval into the two parts so that the relation of the original interval length to its greater part was equal to the relation of the greater part to the lesser one. The initial segment Δ_o including an extremum of I_i was obtained by evaluating I_i in a number of points. The points were placed along h_v from a chosen initial guess h_{vo} with fixed increments towards decreasing I_i values.

Preliminary calculations have shown that function may be not unimodal (Resnyansky, 1985). Therefore, a triple search of the minimum of I_i was performed to raise the probability of attaining the point of the global minimum within the interval of permissible values of the UML depth $D = [h_\epsilon, H]$. The searchers differed from each other in the values of the initial guess h_{vo} (10, 40 and 200 m) and in the corresponding increments Δ_o (1, 1 - 10 m).

6. The data

The initial verification of the above algorithm for estimating $\hat{h}_i$ was performed using simulated data $\tilde{T}_i$ and $\tilde{h}_i$ which had been obtained as daily samples from the UML temporal evolution simulated by Eqs. (1)-(4) under the given seasonally evolving q and u_*. The results of this verification have been discussed formerly (Resnyansky, 1985). Highly sensitive nature of the estimates derived from the function similar to (5) was demonstrated as well as an increase of their stability as a result of replacement of (5) with (10). Below we discuss the results of applying the above procedure to actual observational data from ocean weather station Papa in the North Eastern Pacific ocean (50°N, 145°W). The data were taken from the Monthly bulletin (1972-1974) and from the oceanographic observations at ocean station P (1972-1974). The data selected from these sources (for the period from January 1972 to December 1974) were ordered into the sequences of daily mean values (averaged over the two observations at 0000 and 1200 GMT) of meteorological variables (wind velocity, air temperature and air humidity in the surface layer, cloudiness) as well as daily measured values of SST. Then daily mean net surface heat fluxes q were calculated from these sequences using the empirical formulae of Sheremetevskaya (1972). The net heat flux q was the sum of short-wave solar radiation, residual infrared radiation, sensible and latent heat fluxes. The friction velocity in water u_* was determined using the data on wind velocity U measured at conventional height,

$$u_* = (c_D \rho_a / \rho_r)^{1/2} U, \tag{13}$$

where ρ_a is air density, and c_D is friction coefficient ($= 1.5 \times 10^{-3}$).

The temporal sequences of U, q and T for one of the seasonal cycles are shown in Fig. 1. To make the graphical representation more visual, the sequences were subjected to 30 day running smoothing. The original daily mean values are shown in the inserts for one of the months as well. The variations of daily mean values are obviously comparable in size with the seasonal changes, especially the variations of the heat flux.

For the first two years of the period considered the resultant heat influx appeared to be negative, though not very much different from zero, while during 1974 it came out positive. The possibility is not to be excluded that the obtained magnitudes of q are somewhat underestimated, since the results of Tabata (1965), who used a different method of evaluating q, indicate the preference of annual mean heating over cooling at the site. However, a bias in the heat flux, even if it really exists, is apparently not so large to influence sufficiently the resultant estimates except when q approaches zero. Zero crossing of q results in changing the mixing regime in the upper ocean, and in those cases even the small inaccuracies in specifying q result in appreciable fluctuations of the estimates $\hat{h}_i$. This is apparent even from the fact that different formulae (11) and (12) are used for specifying $\bar{h}_i$ in I_i during heating and cooling periods.

In order to verify the above algorithm, the derived estimates $\hat{h}_i$ were compared with the control UML depths $\tilde{h}_i$ which had been determined by a visual inspection of the vertical temperature profiles borrowed from the oceangraphic observations at ocean station P (1972-1974). The depiction of these profiles is based on the CTD measurements amounted in number to an order of 100 observations for each of the three years under consideration. The visual evaluation of $\tilde{h}$ does not always lead to a definite result. The greatest uncertainties appear at the end of a cooling season and early in spring at the beginning of ocean heating (from February to April) when a

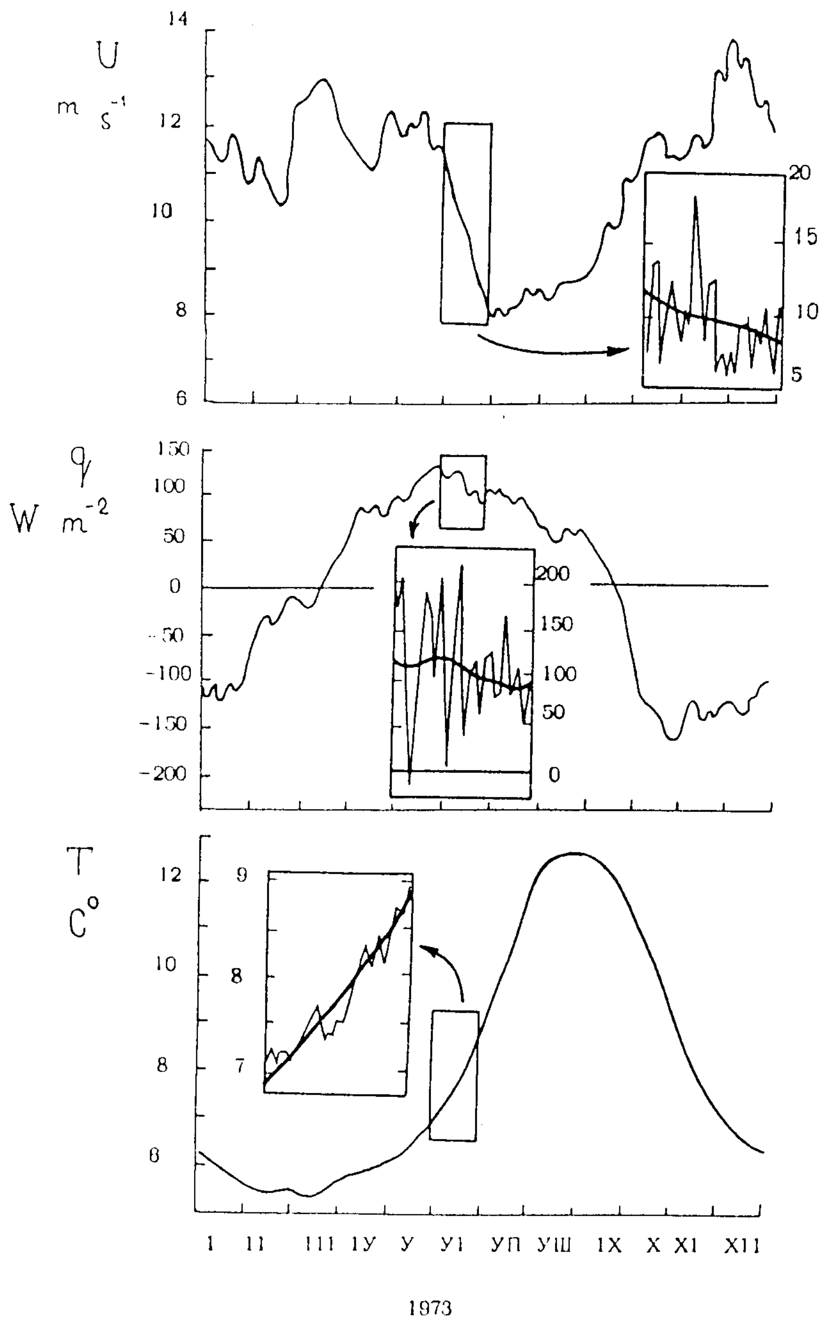

Fig. 1. Seasonal changes of the wind velocity U (m s^{-1}), the net surface heat flux q (W m^{-2}), and the sea surface temperature T (°C) at station Papa in 1973. The curves are plotted from the moving 30 day averages of daily mean values. The initial daily mean values for one of the months are shown in the inserts.

secondary, often being very feebly marked thermocline appears periodically against a background of the deep UML which has been formed in winter. Later this thermocline may disappear due to wind enhancing and/or heat flux sign changing, and then under the favourable conditions it may restore again. In these situations the determination of $\tilde{h}$ is obviously characterized by a large degree of uncertainties. We chose the least value of $\tilde{h}$ from all the discernable ones in the temperature profile. When the thermocline is strongly pronounced, the accuracy of the UML depth determinations seems to be about 5 m; when a feebly marked secondary thermocline is present, it may be much lower.

7. Results of testing the indirect method of UML depth determination

The values of T_j entering function I_i were evaluated through the numerical integration of Eqs. (1)-(4) according to the Matsuno scheme. The time step Δt was 2 hours. At each current time t the values $q = q(t)$ and $u_* = u_*(t)$ were determined using a linear interpolation between the neighbouring daily mean values related to the middle of a day. During heating periods the basic state h_+ was evaluated using formula (11) with q and u_* averaged over the two days (the current day with number i and the preceding day with number $i-1$). The integral in formula (12) (for the cooling basic state h_-) was evaluated using a trapezoidal scheme. A version of I_i was also considered where $\overline{h}_i$ was specified by climatology during the whole time period.

The estimates $\hat{h}_i$, $i = 1, 2, \ldots, N$, were evaluated for each day i provided that the information on the actual UML depths $\tilde{h}_i$ ($\sim$ 100 per year) for this day existed. A root-mean-squared deviation

$$S = [N^{-1} \sum_{i=1}^{N} (\Delta \hat{h}_i)^2]^{1/2} \tag{14}$$

(where $\Delta \hat{h}_i = \hat{h}_i - \tilde{h}_i$, N is the number of the UML depths $\tilde{h}_i$ coincident with the number of available temperature profiles over the considered time period) was taken as a measure of the estimation accuracy.

A performance of the estimation procedure depends on many conditions, such as the structure of input data, the number M of temperature measurements over the time interval $t_{i-M} \leq t \leq t_i$, the type of the basic state $\overline{h}_i$ in function I_i, the value of the relation $r = \delta_T^2/\delta_h^2$ which stands for the regularization parameter, the value of the parameter γ entering formula (12) for h_-, the numerical scheme for integration Eqs. (1)-(4), and, possibly, the value of the time step Δt.

The task is to determine conditions ensuring the best possible estimates $\hat{h}_i$.

The determination of these conditions was perfomed on the base of numerical simulations with varying M, r, γ, with the use of various methods to present $\overline{h}_i$ and with some preliminary transformations of input data $\tilde{T}$. The simulations were performed separately for heating ($q \geq 0$) and cooling ($q < 0$) cases taking into account that in those two cases the mixed layer evolution is controlled by sufficiently different physical processes, namely, by wind mixing during heating and by thermal convection during cooling. The visual appearance of these differences is the observed peculiarities of mixed layer evolution: monotonic deepening in autumn and winter, and cyclic, atributed mainly to variations of wind velocity, fluctuations in summer. The use of different formulae to evaluate $\overline{h}_i = h_+$ during heating and $\overline{h}_i = h_-$ during cooling is a formal expression of the peculiarities characteristic for each of the mixing regimes. Apparently, optimal parameter values used to estimate $\hat{h}_i$ may differ for these two cases, as well.

Table I shows the RMS deviations S of the estimates obtained with various M for each of the three years from 1972 to 1974. Other conditions for this series of simulations were fixed: $H = 200$ m, $h_\epsilon = 10$ m, $T_H = 4°$C, $f = 1.1 \times 10^{-4}$ s^{-1}; the sequences $\tilde{T}$, q and u_*, used as input data, were taken in the original form without any transformations; the regularization was performed relatively to $\overline{h}_i = h_+$ during heating ($r = 0.01°\text{C}^2\ \text{m}^{-2}$ in this case) and $\overline{h}_i = h_-$ during cooling ($r = 0.5°\text{C}^2\ \text{m}^{-2}$).

The figures in Table I suggest that the RMS deviations S for the cases with $q < 0$ generally decrease as M (i.e., the number of measured temperatures $\tilde{T}_j$ used to obtain a single estimate $\hat{h}_i$) increases. The most pronounced decrease of S appears for M varying from 1 to 3. There are relatively small variations in S for $M > 3$. It should be remembered that the computational cost of obtaining a single estimate $\hat{h}_i$ increases in proportion to M. The general decrease of S may be explained by the increase of the ratio of the actual temperature variations over the time interval $t_{i-M} \leq t \leq t_i$ to the uncertainty level in $\tilde{T}_j$ simultaneously with M, i.e. essentially the signal to noise ratio. Besides, the inclusion of additional values of $\tilde{T}_j$ is likely to act as a filter suppressing the influence of errors to some degree. There is no discernable trend in S changes for the cases with $q \geq 0$. The RMS deviations S remain roughly the same for all of M values considered.

Table I. The root-mean-square deviations S of the UML depth estimates $\hat{h}_i$ from the control values $\tilde{h}_i$ for various sample lenghts M^*.

Sample length	Root-mean-square deviations S (meters)					
	1972		1973		1974	
M	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$
1	19.3	78.9	32.8	44.2	19.5	67.0
2	19.4	64.6	32.7	27.9	19.7	31.2
3	19.6	21.1	32.9	18.4	19.5	29.0
4	20.0	36.7	32.8	17.9	19.7	43.8
5	20.4	29.3	30.9	20.0	20.0	27.8
6	20.7	30.6	31.1	16.0	21.7	26.8
7	20.6	31.3	29.9	15.1	21.5	25.5
8	20.8	31.3	29.4	15.7	21.9	23.2
9	20.3	29.9	29.4	15.0	22.0	20.0
10	20.3	29.8	29.2	15.3	22.2	22.1
15	19.8	27.4	29.3	14.8	20.8	18.3
30	20.4	30.4	24.6	15.5	17.6	12.0
45	19.8	30.5	23.5	15.1	16.8	10.4

* The regularization of the estimates was performed relative to $\overline{h}_i = h_+$ during heating ($q \geq 0$) and relative to $\overline{h}_i = h_-$ during cooling ($q < 0$). Parameter values are: $r = 0.01°\ \text{C}^2\ \text{m}^{-2} (q \geq 0)$, $r = 0.5°\text{C}^2\ \text{m}^{-2}$ ($q < 0$), $\Delta t = 2$ h, $H = 200$ m, $h_\epsilon = 10$ m, $T_H = 4$ °C, $f = 1.1 \times 10^{-4}$ s^{-1}, $\gamma = -7.5 \times 10^{-2}$ °C m^{-1}.

A similar behaviour is observed for the dependance of S on r (Tables II and III). In the heating cases the magnitude of S shows, in fact, no dependancy on r over the broad range of its variations from 10^2 to 10^{-5} °C^2 m^{-2} (Table II), while in the cooling cases there is an apparent increase of S at $r < 0.5$°C^2 m^{-2} (Table III).

Table II. The root-mean-square deviations S of the UML depth estimates $\hat{h}_i$ from the control values $\tilde{h}_i$ for various values of $r = \delta_T^2/\delta_h^2$ *.

r	Root-mean-square deviations S (meters)					
°C^2 m^{-2}	1972		1973		1974	
	$M = 3$	$M = 5$	$M = 3$	$M = 5$	$M = 3$	$M = 5$
10^2	19.5	20.0	32.8	31.6	19.8	19.8
10^1	19.5	20.0	32.8	31.6	19.8	19.8
1	19.5	20.0	32.8	31.6	19.8	19.9
10^{-1}	19.6	20.1	32.8	31.5	19.8	19.9
10^{-2}	19.6	20.4	32.9	30.9	19.5	20.0
10^{-3}	19.7	20.8	32.2	31.2	19.7	20.2
10^{-4}	21.6	21.0	31.6	30.5	20.9	20.1
10^{-5}	23.8	21.8	35.0	29.7	23.3	20.9

* The data are for the periods of the surface heating only, $q \geq 0$. The estimates are obtained for two sample lengths, $M = 3$ and $M = 5$. the values of other parameters are the same as in a footnote to Table I.

Table III. The root-mean-square deviations S of the UML depth estimates $\hat{h}_i$ from the control values $\tilde{h}_i$ for various values of $r = \delta_T^2/\delta_h^2$ *.

r	Root-mean-square deviations S (meters)					
°C^2 m^{-2}	1972		1973		1974	
	$M = 3$	$M = 5$	$M = 3$	$M = 5$	$M = 3$	$M = 5$
10^2	20.8	29.3	18.6	17.5	29.4	27.9
10^1	20.8	29.3	18.6	20.1	29.3	27.9
1	20.8	29.3	18.5	20.3	29.2	27.9
0.5	21.1	29.3	18.4	20.0	29.0	27.8
10^{-1}	37.1	42.4	29.3	30.1	49.9	42.2
10^{-2}	33.3	60.7	62.7	54.8	68.5	64.1

* The data are for the periods of the surface cooling only, $q < 0$. The estimates are obtained for two sample lengths, $M = 3$ and $M = 5$. The values of other parameters are the same as in a footnote to Table I.

During cooling quite noticeable influence on S is exerted by variations of the underlying UML temperature vertical gradient γ, which enters formula (12) for the basic state (Table IV). The optimal values appear to be $\gamma = (-5.0/7.5)10^{-2}$ °C m^{-1}. The use of both lower and higher γ values leads to the worse performance of the estimation procedure.

Table IV. The root-mean-square deviation S of the UML depth estimates $\hat{h}_i$ from the control values $\tilde{h}_i$ for various values of γ, mean vertical temperature gradient below the UML *.

γ °C m^{-1}	Root-mean-square deviations S (meters) 1972		1973		1974	
	$M = 3$	$M = 5$	$M = 3$	$M = 5$	$M = 3$	$M = 5$
-2.5×10^{-2}	45.0	38.7	60.7	46.0	37.8	29.3
-5.0×10^{-2}	32.8	27.8	35.6	25.2	28.0	27.2
-7.5×10^{-2}	26.0	29.3	18.6	17.5	29.4	29.4
-10.0×10^{-2}	30.0	33.4	18.9	19.1	32.1	32.8
-12.5×10^{-2}	32.0	36.8	24.0	25.1	37.0	38.1

* The data are for the periods of the surface cooling only, $q < 0$. The estimates are obtained for two sample lengths, $M = 3$ and $M = 5$, $r = 10^6$ °C^2 m^{-2}. The values of other parameters are the same as in a footnote to Table I.

A preliminary smoothing of $\tilde{T}$ series, which had been believed to reduce the accidental error level, on the contrary, did not lead to any noticeable reduction of the resultant deviations S (Table V). The smoothing was performed as a moving averaging with varying period p. Only in the cooling cases, when $q < 0$, with M = 1-2 the increase of p resulted in some reduction of S to the level, which had been, however, also obtained when the unsmoothed data with $M > 2$ were used. Apparently, the extension of time interval $t_{i-M} \leq t \leq t_i$ (i.e. the increase of M) has some filtering properties by itself, without explicit smoothing of data sequences, as can be judged by the revealed reduction of S with growing M (Tables I and V). The applying of time smoothing to q and u_* sequences in conjuction with $\tilde{T}$ smoothing not only improves the results, but, on the contrary, even involves some increase of S.

Table V. The root-mean-square deviations S of the UML depth estimates $\hat{h}_i$ from the control values $\tilde{h}_i$ in 1973 for various periods p (days) of moving averaging, applied to data on water temperature, and for various M *.

	Root-mean-square deviations S (meters) $q \geq 0$				$q < 0$			
M	$p = 1$	$p = 5$	$p = 10$	$p = 15$	$p = 1$	$p = 5$	$p = 10$	$p = 15$
1	32.8	32.8	32.8	32.8	44.2	20.2	30.7	30.8
2	32.7	32.7	32.6	32.6	27.9	18.4	18.9	18.9
3	32.9	32.4	32.5	32.4	18.4	20.9	21.1	21.1
4	32.8	33.0	32.8	32.8	17.9	20.6	20.6	20.7
5	30.9	31.2	31.1	31.3	20.0	17.2	19.9	20.0
6	31.1	31.5	31.3	31.3	16.0	15.6	15.7	15.7
7	29.9	29.9	29.8	30.1	15.1	15.1	15.2	15.2
8	29.4	29.5	29.4	29.4	15.7	15.4	15.4	15.4
9	29.4	29.8	29.8	29.5	15.0	15.1	15.2	15.2
10	29.2	29.3	29.4	29.3	15.3	15.0	15.0	15.0

* The values of other parameters are the same as in a footnote to Table I.

Quite appreciable influence on the procedure performance, though not a simple one, is exerted by the replacement of the basic state in the regularization part of function I_i (Table VI). During some periods, for instance in 1973 or during winter of 1974, the replacement of $\bar{h}_i$ evaluated from (11) and (12) by climatological data h_{clim} resulted in more successful estimates, whereas during other periods the deviations S turned out the same or somewhat higher. In the course of calculations, the climatology h_{clim} was given as monthly mean values related to the middle of a month, and for a specific day t_i it was linearly interpolated between the adjacent monthly values.

Table VI. The root-mean-square deviations S of the UML depth estimates $\hat{h}_i$ from the control values $\tilde{h}_i$ for various types of basic state $\bar{h}_i$ in function $I_i(h_v)$*.

Type of basic state	Root-mean-square deviations S (meters)											
	1972				1973				1974			
	$M = 3$		$M = 5$		$M = 3$		$M = 5$		$M = 3$		$M = 5$	
	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$	$q \geq 0$	$q < 0$
from formulae (11),(12)	19.6 (14.0)	21.1 (13.1)	20.4 (15.0)	29.3 (13.3)	32.9 (19.0)	18.4 (16.9)	30.9 (15.6)	20.0 (15.3)	19.5 (15.2)	29.0 (14.1)	20.0 (16.1)	27.8 (12.8)
climatology	21.6 (14.5)	24.6 (13.7)	22.0 (15.2)	24.7 (13.8)	18.7 (8.4)	10.6 (8.5)	19.1 (9.2)	10.6 (8.6)	26.5 (14.1)	16.2 (11.9)	26.5 (14.2)	16.3 (12.0)

*The estimates are obtained for two sample lenghts, $M = 3$ and $M = 5$. The values of other parameters are the same as in a footnote to Table I. The figures in parenthesis are the values of S obtained when data for the period from February to April (from day 31 to day 120) are excluded.

Now let us examine how the deviations $\Delta\hat{h}_i = \hat{h}_i - \tilde{h}_i$ are distributed over the seasonal cycle. This distribution is illustrated in Fig. 2. The deviations $\Delta\hat{h}_i$ are shown here as vertical segments. Their marked by circles endpoints correspond to the control values $\tilde{h}_i$, while the opposite endpoints to the estimates $\hat{h}_i$. The depicted estimates were evaluated using the above shceme under the following conditions: $H = 200$ m, $h_\epsilon = 10$ m, $T_H = 4$ °C, $f = 1.1 \times 10^{-4}$ s^{-1}, $M = 10$, $\bar{h}_i = h_+$ when $q > 0$ ($r = 0.01$ °C^2 m^{-2}) and $\bar{h}_i = h_-$ when $q < 0$ ($r = 0.5$ °C^2 m^{-2}). The irregularity in the horizontal distribution of the segments is a consequence of gaps present in data series. In particular, the relatively broad gaps were encountered during the periods from day 52 to day 92 and from day 176 to day 216. (The days are numbered from January 1, 1973). The rest of the year was covered with the observed $\tilde{h}_i$ in sufficient details.

First of all, Fig. 2 illustrates the typical for middle latitudes picture of the seasonal evolution of the UML depth, that is the monotonic deepening from about 20 m in September to near 110 m at the end of a cold season, the periodical appearance of a new mixed layer 20-40 m in depth in the initial phase of the heating season (from February to April), and the relatively stable positioning of the UML lower boundary during summer (from June to August). The fluctuations of h are extremely large during the initial phase of the heating period, reaching 60 m for day to day changes. However, it's worthwhile bearing in mind that these fluctuations are produced not by the actual motion of the same mixed layer boundary but as a result of forming a new boundary and its consequent disappearing. In situations like that the missed layer temperature differs only slightly from the underlying water temperatures, and, as already mentioned, the itself identification of the mixed layer boundary involves substantial uncertainties. Therefore, in these cases the estimate deviations $\Delta\hat{h}_i$ turn out quite expectatively large; the period, when $\Delta\hat{h}_i$ are large, coincides with the period when the fluctuations of $\tilde{h}_i$ itself are large too. It should be noted that during this period the large uncertainties $\Delta\hat{h}_i$ are not fatal for the estimation of the temperature profile since the UML temperature differs only slightly from the underlying water temperature.

If now, taking into account the above peculiarities, we exclude the deviations corresponding to

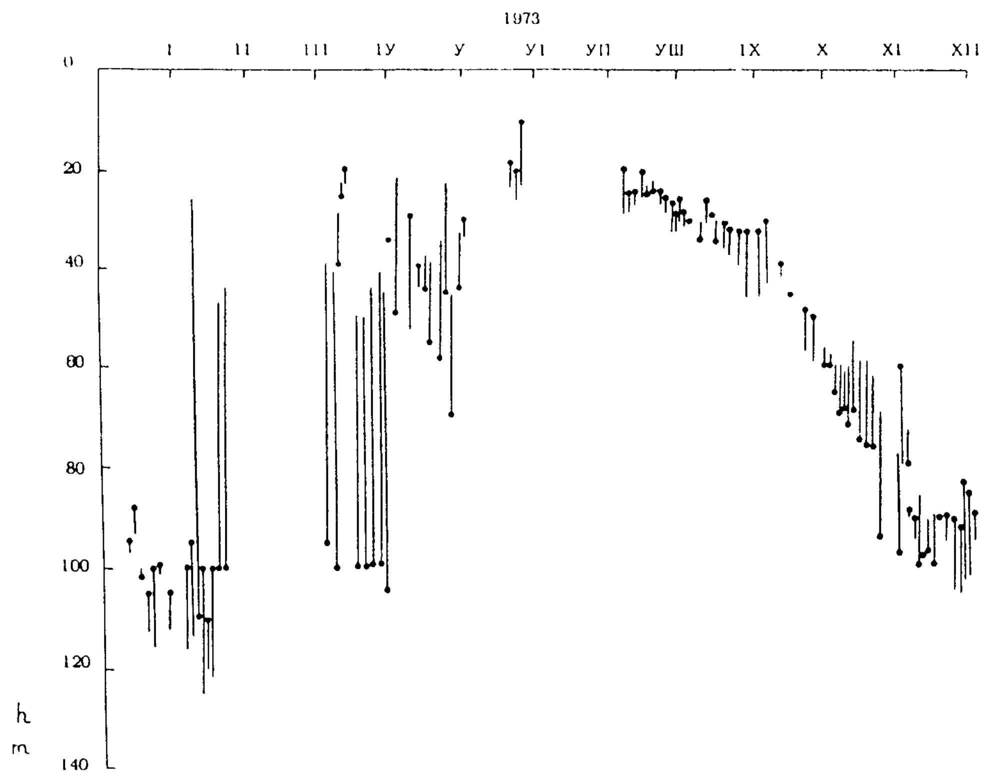

Fig. 2. Comparison of the UML depth estimates $\hat{h}_i$ with the control values $\tilde{h}_i$ at ocean station Papa in 1973. The estimates are obtained through the minimization of function $I_i(h_v)$ with $M = 10$, $H = 200$m, $h_\epsilon = 10$m, $\gamma = -7.5 \times 10^{-2}$ °C m^{-1}, $f = 1.1\ 10^{-4}$ s^{-1}. The basic state was set as $\bar{h}_i = h_+$ during heating ($r = 0.01$ °C^2 m^{-2}) and $\bar{h}_i = h_-$ during cooling ($r = 0.5$ ° C^2 m^{-2}. The vertical segment endpoints marked with circles correspond to the control values $\tilde{h}_i$ while the opposite endpoints to the estimates $\hat{h}_i$.

the period from February to April, the resulting RMS values S will be contained within 13-19 m, when the basic state $\bar{h}_i$ is evaluated from (11), (12), and within 8-15 m when $\bar{h}_i$ is set according to climatological data (see Table VI, figures in parentheses). These RMS deviations S seem to be rather representative for the attainable level of uncertainties in UML depths, which are indirectly estimated from the data of marine meteorological observations at an ocean weather station.

8. Concluding remarks

The above figures evidence that the indirectly estimated UML depths may be considered only marginally informative. Indeed, during the cold season the UML depth may reach 100 m and even more due to convective mixing, and the RMS deviations of 10-15 m seem quite acceptable. However, during the warm season, when the wind mixed layer is only 20-30 m in depth, the estimation uncertainties turn out to be comparable in magnitude with the UML depth itself. The direct determinations, obtained from the vertical profiles of water temperature, appear more accurate. If the profiles are available, the UML depth can be determined to an accuracy of the order of $\pm$ 5 m. The residual uncertainty is conditioned by the vagueness of the temperature jump, separating the UML from the underlying seasonal thermocline, and by the accidental displacements of the UML base in individual measurements due to the influence of gravitational wave field. However, as noted above, direct measurements of vertical profiles in the ocean are relatively rare, and in these cases even approximate indirect estimates of the UML depth may be useful. Thus obtained information on the directly nonobserved elements of ocean thermal structure could help to fill up the missing data which are necessary to determine the heat storage of the upper ocean and the set initial conditions for integrations of UML models.

REFERENCE

Bagrov, A. N. and N. N. Kozhevnikova, 1981. Objective analysis of the Northern Hemisphere sea-surface temperature. *Meteorol. Gydrol.*, **12**, 69-76.

Bengtsson, L. 1981. Current problems in four-dimensional data assimilation. In: *Seminar 1980.* Data assimilation methods. ECMWF, 195-217.

Clancy, R. M. and K. D. Pollak, 1983. A real-time synoptic ocean thermal analysis/forecast system. *Progr. Oceanogr.*, **12**, 383-424.

Himmelblau, D. M., 1972. Applied nonlinear programming. McGraw-Hill Book Comp. Austin (Tex.), 498 pp.

Le Dimet, F. -X. and O. Talagrand, 1986. Variational algorithms for analysis and assimilation of meteorological observations: Theoretical aspects. *Tellus*, **A38**, 97-110.

Niiler, P. P. and E. B. Kraus, 1977. One-dimensional models of the upper ocean. In: *Modelling and prediction of the upper layers of the ocean* (Ed. by E. B. Kraus). Pergamon Press, 143-177.

Monthly bulletin, 1972-1974. Canadian upper air data. Department of Transport. Meteorological Branch. Toronto (Ont.), I-XII.

Oceanographic observations at ocean station P (50°N, 145°W), 1972-1974. Environment Canada. Water Management Service. Marine Sciences Directorate. Pacific Region. Pacific Marine Science Report. Victoria (B. C.), vol. 52-59.

Penenko, V. V., 1981. Methods of numerical simulation of atmospheric processes. Gidrometeoizdat, Leningrad, 352 pp.

Penenko, V. V., A. V. Protasov and V. F. Raputa, 1983. An assimilation of hydrometeorological information on the basis of numerical models of atmosphere and ocean dynamics. In: *Proc. Third Soviet-France Symp. on Oceanography.* Novosibirsk, June 9-11, 1982. Part I (Ed. by V. P. Kochergin). Novosibirsk, 16-29.

Resnyansky, Yu. D., 1975. On the parameterization of the integral turbulent energy dissipation in the upper ocean quasi-homogeneous layer. *Izv. Akad. Nauk USSR. Atmos. Ocean. Physics*, **11**, 726-733.

Resnyansky, Yu. D., 1983. An influence of ocean currents on the evolution of the oceanic active layer. *Trans. Hydrometeorol. Center USSR*, **255**, 23-33.

Resnyansky, Yu. D., 1985. On an indirect estimation of the mixed layer thickness from data on the ocean surface temperature. *Meteorol. Gydrol.,* **11**, 46-56.

Ryasin, V. A. and V. P. Svidritsky, 1982. A comparison of two methods of Bayesian estimation of the atmosphere state based on the barotropic model. *Trans. Hydrometeorol. Center USSR*, **248**, 91-97.

Sheremetevskaya, O. I., 1972. A graphical method for the calculation of air-sea heat exchange. Hydrometeorological Center USSR. Metodicheskoe posobie. Moscow, 8 pp.

Tabata, S., 1965. Variability of oceanographic conditions at ocean station "P" in the Northeast Pacific Ocean. *Trans. Roy. Soc. Canada*. Ser. 4, vol. 3, sec. 3, 367-418.

Tikhonov, A. N. and V. Ya. Arsenin, 1974. Methods for solving the incorrect problems. Nauka, Moscow, 224 pp.

Zelenko, A. A. and E. S. Nesterov, 1986. Objective analysis of the upper ocean layer temperature in the North Atlantic. *Trans. Hydrometeorol. Center USSR,* **281** 76-83.

Zilitinkevich, S. S., D. V. Chalikov and Yu. D. Resnyansky, 1979. Modelling the oceanic upper layer. *Oceanologica Acta*, **2**, 219-240.