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An exponential model of the curve of mean monthly hourly air temperature

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RESUMEN

Se presenta un modelo exponencial para la simulación de la marcha horaria media mensual de la temperatura del aire. Su bondad de ajuste es superior a la del modelo de De Wit *et al.* (1978), que a su vez resultó ser el más preciso en una revisión hecha por Reicosky *et al.* (1989).

ABSTRACT

An exponential model for the simulation of the monthly mean hourly shape of the air temperature curve is presented. Its accuracy was shown to be greater than the model of De Wit *et al.* (1978), which was shown to be one of the best methods according to a revision made by Reicosky *et al.* (1989).

1. Background

It is very important to know the hourly values of the air temperature by simulation or evaluation in biological process. For example, the atmospheric temperature in crop growth models or in thermal comfort calculations for humans or cattle is of particular relevance.

Most climatological stations do not register hourly temperatures. They mainly record the daily minima and maxima temperatures. As a result, several authors have attempted to modeled the shape of the diurnal temperature curve. Allen (1976), Hansen and Riscoll (1977), Johnson and Fitzpatrich (1977a, b), Parton and Logan (1981), Floyd and Braddock (1984), Wann *et al.* (1985) and Kimball and Bellamy (1986), proposed models based upon sine curves, and Carson (1963) suggested a model based upon Fourier analysis.

Reicosky *et al.* (1989) have determined the accuracy of five methods from the literature basing this selection on their use in existing growth models and their simplicity. Four year of hourly air temperature data collected during the growing season at 2 m over well-watered grass were used to test the various methods. Six days from each growing season were randomly selected for detailed analysis and an additional nine days were selected to cover a range of daily maximum temperatures and solar global radiation. The absolute mean error for a 24-hours period ranged from 0.5 to 9.3°C for the six randomly selected days for all four years of data.

2. The exponential model

The function

$$y = at^{b} exp(ct) \tag{1}$$

is capable of modeling a process of rapid increase and slow decrease (e.g. the daily course of the atmospheric temperature at 2 m over the ground). If T_H is the monthly mean hourly temperature, $T_{\max}$ and $T_{\min}$ the monthly mean maxima and minima temperatures respectively, and t is a lineal function of time:

$$T_H = T_{\min} + y(T_{\max} - T_{\min}). \tag{2}$$

where y should have the following characteristics:

i) $y = 0$ at $T_H = T_{\min}$

ii) $y = 1$ at $T_H = T_{\max}$,

iii) $y = \frac{1}{2}$ at $T_H = \frac{T_{\max}+T_{\min}}{2}$, the average temperature that occurs in the morning.

iv) $y = \frac{1}{2}$ at $T_H =$ average temperature in the afternoon.

As t (Eq. 1) cannot be negative, the condition (i) that define t as a function of local time (H) and local sunrise time (H_o), should be determined as follows:

$$T = H - H_o, \qquad \text{if } H \geq H_o$$

$$t = H + 24 - H_o, \qquad \text{if } H < H_o \tag{3}$$

The conditions ii, iii and iv were evaluated by statistical methods of ten cities in Mexico for the year 1979 (Table 1). Since it was assumed that the $T_{\min}$ occurs at sunrise, the values of (a, b, c) were functions of latitude and time of the year (Table 2).

Table 1. Cities used to evaluate the constant of Eq. 1.

Cities	Latitude (°N)	Longitude (°W)	Altitude (m)
Hermosillo	29°4'	110°58'	237
Torreón	25°32'	103°27'	1013
Monterrey	25°40'	100°18'	538
Tampico	22°13'	97°51'	12
Manzanillo	19°3'	104°17'	8
Jalapa	19°32'	96°55'	1427
Tacubaya	19°24'	99°12'	2308
Tapachula	14°55'	92°16'	182
S. Cristóbal de las Casas	16°44'	92°38'	2276
Mérida	20°59'	89°39'	9

Table 2. Values of a, b and c, as a function of latitude and time of year for Mexico.

Months	Latitude		Values	
		a	b	c
March to October	≥23.5°N	0.026	3.190	-0.375
November to February	≥23.5°N	0.023	3.436	-0.421
January to December	<23.5°N	0.096	2.422	-0.339

3. Comparative methods

The statistical measurements which were used by Reicosky *et al.* (1989) to evaluate the accuracy of the models, will be used here to compare the exponential model with the best model cited by Reicosky *et al.*

a) The absolute mean error (AME), defined as the sum of the absolute values of differences between the estimated (θ) and observed (o) temperatures:

$$AME - \frac{\sum\limits_{i=1}^{n} \mid T\theta i - Toi \mid}{n} \tag{4}$$

where n is the number of observations.

b) The root mean square error (RMSE) is obtained as follows:

$$RMSE = \left[\frac{\sum\limits_{i=1}^{n} (Toi - T\theta i)^2}{n}\right]^{1/2} \tag{5}$$

c) The sum of the residuals (RES) and the sum of the absolute residuals | RES |:

$$RES = \sum_{i=1}^{n}(Toi - T\theta i) \tag{6}$$

d) The determination coefficient or squared correlation coefficient (R^2):

$$R^2 = \frac{\sum\limits_{i=1}^{n} (T\theta i - \overline{To})^2}{\sum\limits_{i=1}^{n} (Toi - \overline{To})^2} \tag{7}$$

where $\overline{To}$ is the mean of Toi, $i = 1,\ 2, \ldots n$.

From the five models analysed by Reicosky (1989), the method of De Wit *et al.* (1978) was shown to be the most accurate. Its statistical evaluation and the comparisons with the *exponential* *model* is shown in Table 3. The best and the worst cases of exponential method simulations are shown in figures 1 and 2. The adjusting values in the *exponential model* for the other cases (Table 1), are shown in the last column of Table 3.

Table 3. Accuracy of the *exponential* and De Wit *et al.* models.

		De Wit *et al.*	Exponential model		
		Worst case	Best case	Worst case	Mean for all cases
AME	(°C)	1.0	0.3	1.4	0.9
RMSE	(°C)	1.5	0.5	2.0	1.3
RES	(°C)	-12.6	-1.3	-17.0	-9.2
| RES |	(°C)	24.0	3.7	16.3	10.0
R^2		0.95	0.97	0.84	0.91

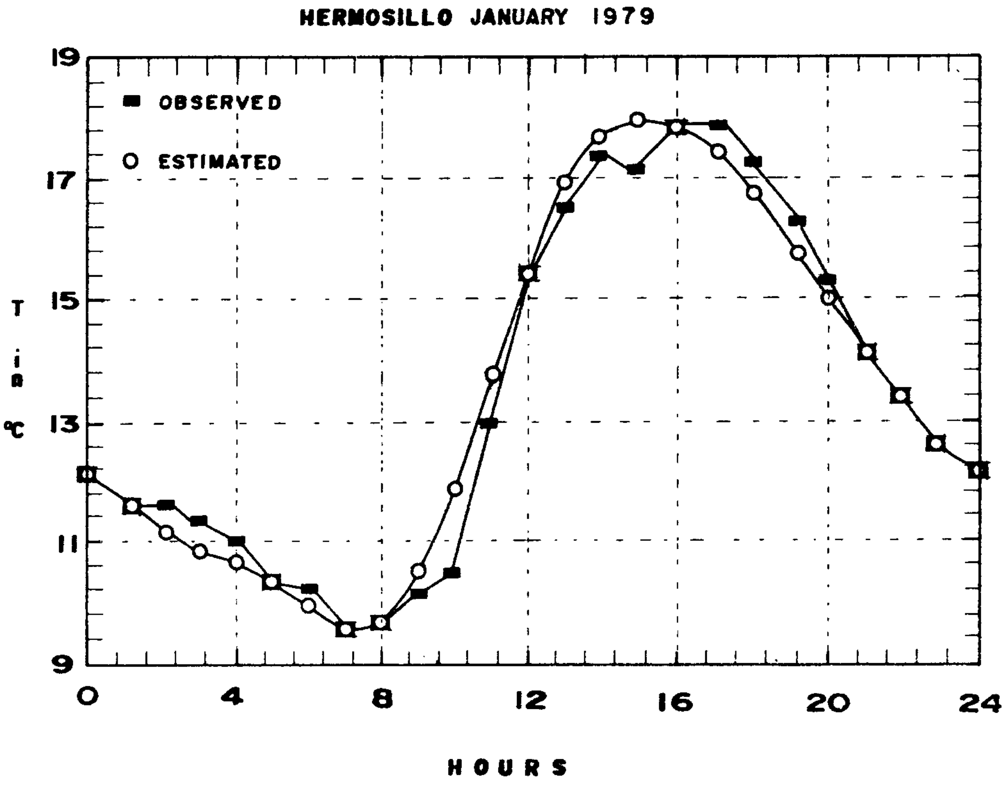

Fig. 1. Best case (Hermosillo, January 1979). Comparison of observed and estimated values of monthly mean hourly temperatures for the *exponential model.*

In the worst case (Tacubaya, Fig. 2), a "temperature delay" in the observed values, compared to the estimated ones, was probably due to the thermal effect of the high level of constructions and urbanization surrounding the climatological station.

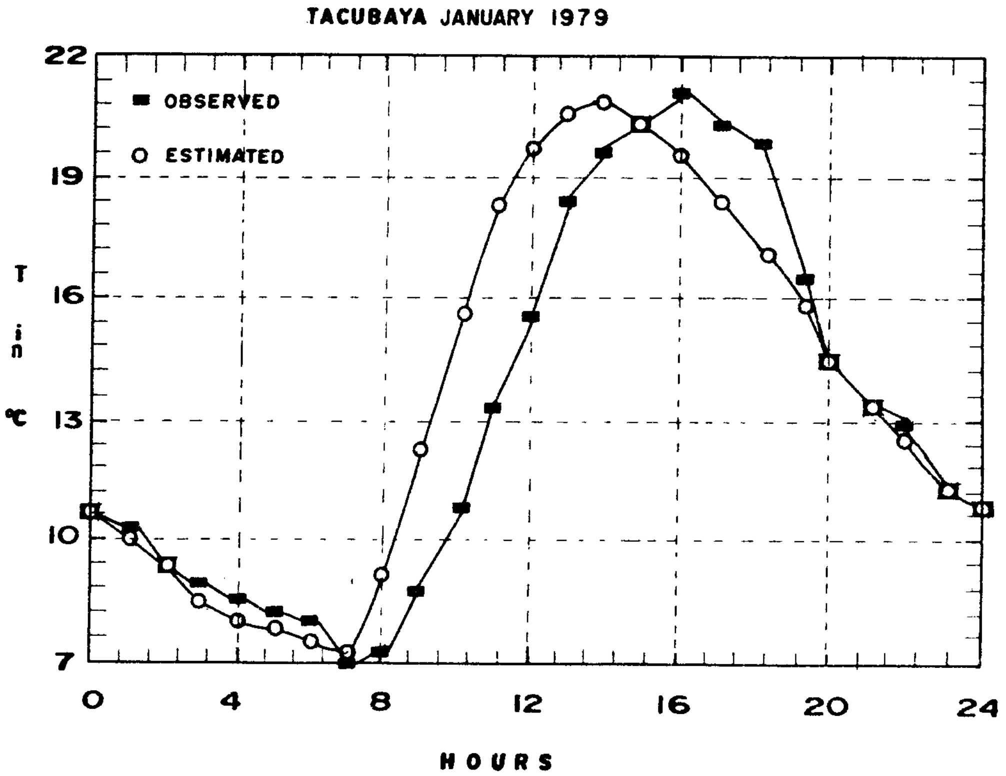

Fig. 2. Worst case (Tacubaya, January 1979). Comparison of observed and estimated values of monthly mean hourly temperatures for the *exponential model*.

The method of De Wit *et al.* (1978) assumes daily maxima temperature (T_M) to be at 14 h local time, and daily minima temperature (T_m) at sunrise. If H and Ho are as in Eqs. 3, the hourly temperatures are calculated as follows:

a) For $0 \leq H \leq Ho$ or 14 h $< H \leq 24$ h; with $H' = H + 10$ if $H < Ho$; $H' = 14$ if $H > 14$ h:

$$Th = \frac{Tm + T_M}{2} + \frac{T_M - Tm}{2} \cos[\frac{\pi H'}{10 + H_o}] \tag{8a}$$

b) For H_o, $H \leq 14$ h; with $H' = H + 10$ if $H < H_o$ $H' = 14$ if $H > 14$ h:

$$T_h = \frac{Tm + T_M}{2} - \frac{T_M - Tm}{2} \cos[\frac{\pi(H - H_o)}{14 - H_o}] \tag{8b}$$

4. Conclusions

It may be appreciated that the *exponential model* and the method of De Wit *et al.* have the same degree of difficulty for evaluating the hourly temperatures with the values of maxima and minima. The *exponential model* being for monthly mean hourly temperatures, and the De Wit's model for daily hourly temperatures.

The *exponential model*, here presented, has a better accuracy than De Wit's model (see Table 3). this is due to the fact that the *exponential model* has four adjustment points: the hour at minima, maxima, and the two hours of mean temperatures. Also, the monthly mean hourly temperature has a more regular shape than the daily hourly temperature, because the later may be affected by advection.

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