Atmósfera (1991), **4**, *pp.* *257-264*

An equation relating temporal changes of relative humidity and temperature: case of variable intervals of time

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(Manuscript received July 4, 1990; accepted in final form February 1, 1991)

RESUMEN

Este trabajo está dirigido a establecer si la relación lineal $h_2 - h_1 = -0,03 - 3,586(T_2 - T_1)$, que se supone válida en cualquier punto de la Tierra y en cualquier época del año, y cuyos coeficientes se obtienen haciendo uso de series síncronas de datos trihorarios de humedad relativa y de temperatura, medidos en el Observatorio de Matacán (6.794 datos), se cumple cuando se consideran intervalos de tiempo variables para establecer las variaciones de humedad relativa y de temperatura.

Para ello se recalculan, con la misma metodología, los coeficientes de la relación, usando datos de las variables obtenidos en el mismo observatorio durante el año 1978 (268 datos).

Los resultados se comparan con los valores correspondientes de la relación estudiada, tratándose de justificar las diferencias. En función de estos resultados se puede llegar a la conclusión de que la ecuación de estudio se cumple, también, cuando los intervalos de tiempo en los que se deteminan las variaciones de humedad relativa y temperatura del aire son variables.

ABSTRACT

This work is directed to establish how the linear relation $h_2 - h_1 = -0,03 - 3,586(T_2 - T_1)$, which is suppose to be valid at any point on the Earth and time of the year, and whose coefficients are obtained using synchronous series of three hourly data of relative humidity and temperature obtained from observations made in the Meteorological Observatory of Matacán (6, 794 data points), is verified if variable time intervals to establish the relative humidity and temperature variations are considered.

To do this, the equation coefficients were recalculated, with the same methodology, using values of the variables which were obtained in the same observatory during the whole year of 1978 (268 data points).

The results are compared with the corresponding values of the studied relation and the differences are justified. In function of this results it is possible to infer that the study equation is also valid when the time intervals used to determine the variations of relative humidity and temperature of the air are variable.

1. Introduction

One of the most interesting problems posed in the study of atmospheric phenomenology is undoubtedly that of establishing the relation between humidity and air temperature. It has not been possible to establish such relation on a basis of formal laws, one of the reasons being that the function linking both variables in an exact form is unknown.

In the other hand, present empirical relations between air temperature and humidity parameters (such as the formula of Magnus) give results that, although good, are subject of improvement.

The limited number or references consulted in the literature suggests that the studies carried out in this field have approached the problem only marginally (Fisher and Heimann, 1981; Kvetak, 1985) or from a basically statistical point of view (Yang and He, 1978; Andreas, 1978).

Labajo *et al.* (1989a) analized different empirical or semi-empirical exponential relations between humidity and air temperature. In a latter study (Labajo *et al.*, 1989b) a linear approach to the real relation function between air temperature and humidity, shown as follows,

$$h_2 - h_1 = a + b(T_2 - T_1) \tag{1}$$

where h is in % T in K, the coefficient a in % and b in $\%K^{-1}$, was found to improve the results of the above empirical, non-linear approaches. The results of this linear approach suggest that it is the best for the temperature range found in the atmosphere.

In the study made to obtain and justify this linear relation, three-hourly variations series of temperature and relative humidity were used for whole year 1981, 1982 and 1983 (6,794 data points), i.e. a constant interval of time was considered.

The results, which were obtained for the whole year and for each month, made it possible to deduce a single linear relation - which was shown to be independent of space and time - as follows:

$$h_2 - h_1 = -.03 - 3.586(T_2 - T_1). \tag{2}$$

As the utility of this relation function may be affected by the use of variations of the magnitudes in constant intervals of time, in the present study the linear relation function between variations in relative humidity and air temperature will be defined in time intervals of different values, comparing the results with those obtained in previous studies.

2. Preliminary considerations

As in the above-mentioned studies, in order to approach the problem it is necessary to assume the hypothesis that the vapour pressure, "e", measured at the observation point, remains almost constant. This requires the following conditions:

a) There should be no change in air mass over the observation point during the interval of time considered.

b) There should not be vapour advection either from the ground or from surrounding masses.

If a change in mass occurs over the observation point, the hypothesis of the constancy of vapour pressure will only be fulfilled in cases in which the humidity of the substituted mass and that of the mass which substitutes it are almost equal.

In view of this, the data that do not comply with the proposed hypothesis should be discarded. This entails filtering the series to be used.

Such a filter can be achieved by using a statistical criterion determined in previous studies. This consists in admitting as valid only those cases in which the values of temperature variation and those corresponding to humidity variation are included within certain intervals.

These intervals are calculated by taking a class of temperature variation to an extent of 1 degree, establishing the series of humidity variations corresponding to each class, calculating the average value and the standard deviation for each of the series and taking as the extremes of the intervals the average value plus-minus the standard deviation. Table 1 shows the values thus obtained.

In absolute terms this statistical criterion does not mean that, in the cases selected with it, the vapour pressure should remain constant, but it does make it possible to determine those in which the variation is large enough for such a hypothesis not to be considered a valid approach. Consequently, it will enable one to exclude all those cases in which such a working hypothesis is not met.

Table 1. Criterion for determining the intervals of variation of relative humidity corresponding to each class of temperature variation [interval = mean value of $(h_2 - h_1) \pm \sigma$].

$(T_2 - T_1)$ (K)	Mean value of $(h_2 - h_1)$ (%)	σ	interval of $(h_2 - h_1)$ (%)
< -8,0	27,2	12,2	30 to 15
-7,9 to -7,0	22,4	10,4	33 to 12
-6,9 to -6,0	22,2	8,3	31 to 14
-5,9 to -5,0	19,9	8,1	28 to 12
-4,9 to -4,0	17,7	7,5	25 to 10
-3,9 to -3,0	13,4	7,5	21 to 6
-2,0 to -2,0	8,9	6,9	16 to 2
-1,9 to -1,0	4,6	6,1	11 to -2
-0,9 to 0,0	1,8	5,4	7 to -4
0,1 to 1,0	-1,5	5,5	4 to -7
1,1 to 2,0	-6,0	6,9	1 to -13
2,1 to 3,0	-9,9	7,2	-3 to -17
3,1 to 4,0	-13,9	8,0	-6 to -22
4,1 to 5,0	-18,9	8,0	-11 to -27
5,1 to 6,0	-22,5	8,5	-14 to -31
6,1 to 7,0	-24,1	7,6	-17 to -32
7,1 to 8,0	-27,5	9,0	-19 to -37
8,1 to 9,0	-28,2	8,4	-21 to -38
9,1 to 10,0	-32,1	7,8	-24 to -40
10,1 to 11,0	-37,0	8,1	-29 to -46
11,1 to 12,0	-40,8	7,6	-33 to -52
12,1 to 13,0	-45,1	9,2	-38 to -55
13,1 to 14,0	-52,2	9,1	-43 to -62
14,1 to 15,0	-57,0	9,3	-48 to -68
15,1 to 16,0	-59,1	10,2	-50 to -69
16,1 to 17,0	-62,4	10,4	-52 to -72
17,1 to 18,0	-65,0	11,0	-55 to -75
18,1 to 19,0	-69,6	12,3	-58 to -79
> 19,0	-72,2	13,1	-60 to -81

However, since the criterion is not absolute, the series of cases to be studied will include, besides those in which there is no change in mass or in vapour advection those in which the mass changes but the vapour pression(e) is approximately constant.

These requirements are positive because, among other things, the results are not limited by strict conditions, and their validity could be extended to less restricted situations.

3. Data analysis and results

In order to establish a coherent limitation to temporal variability it seems logical to fix one of the extremes of the time interval and to take the other as that corresponding to the peak values of the magnitudes to be considered. 0700 **UTC** was taken as one fixed extreme of the interval. The other extreme was the moment at which the minimum daily humidity value was recorded.

The choise of these extremes is arbitrary and others could be used.

However, the interval of time that they encompass is sufficiently short for the changes of mass occurring within it to be produced with relative infrequence, but for the variability of the amplitude of the interval to be sufficient.

The humidity and temperature data series used were synchronous series obtained from daily observations at ground level at 0700 **UTC**, and from mimimun humidity values and the corresponding temperature values at the Matacán Observatory (Salamanca), during the whole year of 1978 (I. N. M., 1978). The minimum values of relative humidity are usually observed shortly after midday.

The complete series of values considered was used to calculate the coefficients "a" and "b", of the following expression:

$$h_2 - h_1 = a + b(T_2 - T_1), \qquad (3)$$

- where h_2 is the minimum value of relative humidity (h_m), h_1 is the value of relative humidity at 0700 **UTC** (h_7), T_2 is the temperature value at the time of the minimum relative humidity (T_{hm}) occurrence and T_1 is the temperature value at 0700 **UTC** (T_7) - for each month and for the whole year. The results are shown in Table II.

Table II. Regression coefficients and statistical indices of the expression $(h_m - h_7) = a + b(T_{hm} - T_7)$, calculated with the complete data series (90% confidence level).

Month	Regression coefficents a	Regression coefficents b	Correlation coefficents r	Explanation variation r^2(%)	Standard error	F-test	Degree freedom (n_1;n_2)
Ja	-4,00	-1,57	0,480	23,0	7,28	8,67	1; 29
Fe	-8,47	-2,35	0,700	49,0	8,35	24,95	1; 26
Mr	-11,61	-2,79	0,841	70,7	8,42	69,93	1; 29
Ap	-13,08	-2,66	0,749	56,1	9,25	35,77	1; 28
My	-13,04	-2,29	0,692	47,9	9,37	26,64	1; 29
Jn	-13,14	-2,54	0,674	45,5	9,66	23,33	1; 28
Jl	-22,75	-1,78	0,501	25,1	9,64	9,72	1; 29
Au	-40,94	-0,72	0,149	3,8	8,70	1,13	1; 29
Se	-20,86	-1,83	0,660	43,5	7,94	21,59	1; 28
Oc	-22,99	-1,59	0,731	53,4	6,40	33,20	1; 29
No	-0,43	-3,12	0,884	78,2	8,36	100,49	1; 28
De	-4,71	-2,27	0,616	37,9	7,92	17,70	1; 29
Year	**-7,19**	**-2,70**	**0,850**	**72,2**	**9,50**	**943,00**	**1;363**

The series were then filtered according to the above statistical criterion, recalculating the coefficients "a" and "b" of the linear approach, also for each month of the year and for the whole year. In this case the number of data of the working series is:

Month	Ja	Fe	Mr	Ap	My	Jn	Jl	Au	Se	Oc	No	De	year
No. dat.	22	22	22	22	21	22	25	18	19	26	22	27	268

This means that the average percentage of data discarded is 27%, as opposed to the 22% reported in previous studies. This difference is logical if one takes into account the fact that the probability of changes in mass is greater because the interval of time is larger than the one considered in previous studies. However, as pointed out before, because of the adequacy of the extent of the interval, the changes in mass do not occur with a high frequency.

Table III. Regression coefficients and statistical indices of the expression $(h_m - h_7) = a + b(T_{hm} - T_7)$, calculated with the filtered data series (90% confidence level).

Month	Regression coefficents		Correlation coefficents	Explanation variation	Standard error	F-test	Degree freedom
	a	b	r	r^2(%)			$(n_1;n_2)$
Ja	-0,74	-2,99	0,749	56,0	5,92	25,50	1; 20
Fe	-5,28	-2,62	0,872	76,0	5,31	63,47	1; 20
Mr	-5,18	-3,20	0,905	81,8	7,35	90,18	1; 20
Ap	-6,53	-3,16	0,891	79,4	6,04	77,16	1; 20
My	-6,62	-4,02	0,905	82,0	5,40	86,36	1; 19
Jn	-5,49	-3,13	0,837	70,0	7,14	46,70	1; 20
Jl	-7,38	-2,69	0,725	52,6	8,02	25,53	1; 23
Au	-34,53	-1,40	0,616	37,9	4,91	9,76	1; 16
Se	-4,84	-2,82	0,770	59,2	8,34	24,69	1; 17
Oc	-17,30	-1,99	0,810	65,6	5,84	45,83	1; 24
No	-0,74	-3,29	0,943	88,9	5,72	160,83	1; 20
De	-3,71	-2,16	0,703	49,5	6,24	24,47	1; 25
Year	**-3,87**	**-3,01**	**0,921**	**84,9**	**7,10**	**1504,01**	**1;266**

The results obtained from the filtered data series are shown in Table III. Figures 1 and 2 show the regression straight lines obtained from the annual series. Figure 1 shows the results of the non-filtered annual series, and Figure 2 the results obtained from the annual series of data after filtering.

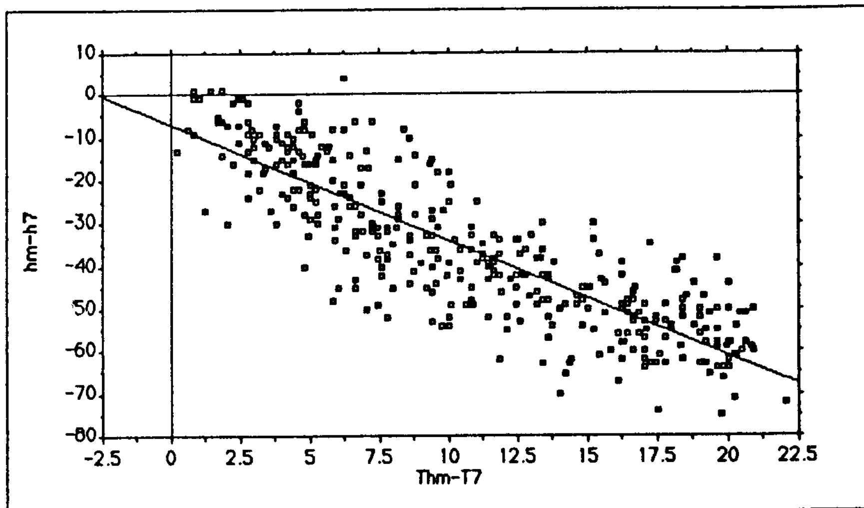

Fig. 1. Regression straight line obtained from the non-filtered annual series ($h_m - h_7 = -2,7(T_{hm} - T_7) - 7,19$).

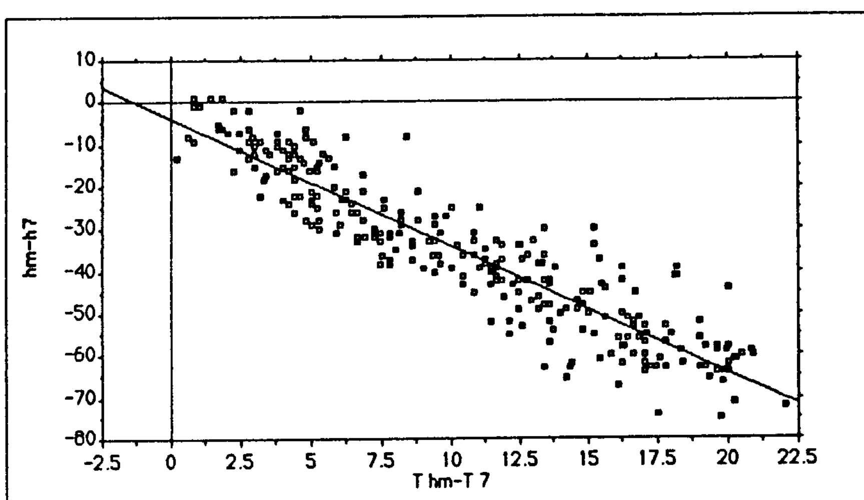

Fig. 2. Regression straight line obtained from the annual series of data after filtering ($h_m - h_7 = -3,01(T_{hm} - T_7) - 3,87$).

4. Discussion

In the tables of results it can be seen that when the data series are used without filtering, the value of coefficient "a" of the regression straight line displays great variability throughout the months of the year, the lowest values being in the summer months and the highest in winter. Coefficient "b" shows little variability, no regularity being noted in the distribution of values. The values of the correlation coefficients are quite disparate, but more or less significant, except for August.

The variability in the values of coefficient "a" and "b", and the different degrees of significance of the correlation coefficient of each of the months can be explained, among other causes, by the greater or smaller number of times that the constancy (or quasi-constancy) of the vapour pressure of the air mass in the interval considered is not fulfilled in the data series, or by the greater or smaller number of times that there are changes in mass, or in vapour advection, in the time intervals considered, which amounts to the same.

On consideration of the annual period the results obtained improve on the monthly periods, except for November. Two possible causes are:

- Due to the greater number of data in the series, these are more stable.
- The different distribution over the months of the cases that do not comply with the working hypothesis.

If the results of Table III are now considered, it is seen that the variability of coefficient "a", although still high, has decreased and the correlation coefficients have increased notably, as was to be expected since those cases in which the working hypothesis was clearly not fulfilled were not taken into account.

However, as the filtering criterion is not absolutely strict, only clearly anomalous data are excluded by its application. This justifies the persistence of variability in the values of "a" after filtering.

In the previous study by Labajo *et al.* (1989b) a single expression was obtained and justified for the linear relation between the variations of relative humidity and air temperature as given by equation (2).

In the present study, the annual expression of linear relation between these variations, obtained by applying the working hypothesis, is as follows:

$$h_m - h_7 = -3,87 - 3,01(T_{hm} - T_7) \tag{4}$$

On comparing both expressions it is observed that the values for coefficent "b" are almost equal (-3,01 $\%K^{-1}$ and 3,586$\%K^{-1}$). The values for coefficient "a" show a somewhat more pronounced difference, although the sign coincides.

The difference in the value of coefficient "a" in the expressions compared may be explained as being a consequence of the fact that since the time interval considered to obtain expression (4) was generally greater than that considered to obtain expression (2), there is a greater possibility of the vapour pressure not remaining constant in the corresponding time interval at the observation zone.

As in the study mentioned before, a theoretical justification is made of the linear approximation under study. From this it could be deduced that coefficient "a" is an index of the variation in the vapour pressure over the time interval considered. The value of said interval for expression (4) must be greater than the corresponding to expression (2).

Additionally another possible cause of the differences observed in both expressions is that the number of data of the series used to obtain expression (4) was much smaller (268 data points) than that used to obtain expression (2) (6,794 data points).

It therefore seems plausible to assume that if variations in humidity and temperature are considered over variable time intervals this does not affect the expression of the linear relation between both variations. Consequently there should be no objection to considering expression (2) as a valid expression, without restrictions, of the linear approximation between relative humidity and air temperature.

5. Conclusions

In the light of the foregoing, the following conclusions may be deduced:

On comparing the values of coefficients "*a*" and "*b*" of the linear relation between the variations of relative humidity and air temperature over a variable time interval - values obtained from complete data series and from those filtered as function of the working hypothesis - it was observed that in both cases the values of coefficient "*a*" show important variability throughout the year, and that the values of coefficient "*b*" are much less variable, and almost stable in the second case.

The consideration of a variable time interval does not seem to affect the expression of the linear relation between the variations of relative humidity and air temperature. The small differences that appear between the values of coefficients "*a*" and "*b*" in the relation for fixed intervals of time (3 hours) and for variable intervals of time may be caused by the fact that in the latter case the working hypothesis (constant vapour pressure) was less rigorously observed.

Therefore it seems logical to consider the expression

$$h_2 - h_1 = -.03 - 3.586(T_2 - T_1)$$

as a valid expression of the relation between relative humidity and air temperature for both fixed and variable intervals of time.

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