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Symmetric instability of monsoon zonal flows

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RESUMEN

Hemos examinado la posibilidad de inestabilidad simétrica usando un conjunto de ecuaciones zonalmente simétricas linealmente perturbadas, en la región del monzón cercana al ecuador que incorpora convección húmeda. Tanto en el modelo seco como en el húmedo encontramos que se dan modos inestables y neutros. Los modos inestables tienen periodos de alrededor de 1 ó 2 días y con tiempos de duplicación del orden de 1 ó 2 días. Las estructuras de las amplitudes de dichos modos inestables están principalmente confinadas a las regiones ecuatoriales. Así que hemos demostrado que los flujos zonales en las regiones del monzón exhiben inestabilidad simétrica.

ABSTRACT

We have examined the possibility of symmetric instability using a set of zonally symmetric linear perturbation equations in the near equatorial monsoon region incorporating moist convection. In the dry model as well as moist model both unstable and neutral modes were obtained. The unstable modes have period of around 1 to 2 days and with doubling times of around 1 to 2 days. The amplitude structure of such unstable modes are mainly confined to the equatorial regions. Thus we have shown that zonal flows in the monsoon region exhibit symmetric instability.

1. Introduction

There have been a number of studies on the inertial instability of the zonal flows with both vertical and horizontal shear in the equatorial region. Inertial instability in rotating fluids arises from an imbalance of pressure gradient and centrifugal (Coriolis) forces when the absolute value of angular momentum decreases with radius. A detailed review with a historical prespective of inertial instability also called symmetric instability has been given by Emanuel (1979). Charney (1973) showed that a basic state of the type $u = u(y, z)$ on a β-plane is unstable to zonally symmetric disturbances if $f\overline{p} < 0$ somewhere, where f is the Coriolis parameter and

$$\overline{p} = \frac{1}{\rho_o}\left\{\frac{\partial\overline{\theta}}{\partial z}\left(f - \frac{\partial\overline{u}}{\partial y}\right) + \frac{\partial\overline{\theta}}{\partial y}\frac{\partial\overline{u}}{\partial z}\right\} \tag{1}$$

is the Ertel potential vorticity of the basic state. Here ρ_o is the mean density, $\overline{\theta}$ is the basic state potential temperature $\frac{\partial\overline{u}}{\partial y}$ is the horizontal wind shear and $\frac{\partial\overline{u}}{\partial z}$ is the vertical wind shear.

Using the thermal wind equation, the equation (1) becomes:

$$1 - Ri^{-1} - \frac{1}{f}\frac{\partial\overline{u}}{\partial y} < 0 \tag{2}$$

where Ri is the Richardson number which is given by $Ri \equiv \frac{N^2}{(\frac{\partial \bar{u}}{\partial z})^2}$ where N is the buoyancy (Brunt-Väisällä) frequency, i.e., $N^2 = g\frac{\partial ln\bar{\theta}}{\partial z}$.

The importance of this instability and its various applications in explaining many atmospheric phenomena have been recognized in the recent studies by Emanuel (1979), Dunkerton (1981), Boyd and Christidis (1982). Schubert and Hack (1982), Stevens (1983), Stevens and Ciesielski (1986) and Ciesielski *et al.* (1989). Emanuel (1979) studied the possible connection between inertial instability of the flow and the middle latitude mesoscale circulation. He showed that the horizontal length scale of the most unstable normal mode is determined dominantly by the depth of the unstable domain and the slope of isentropic surfaces and has only a weak dependence on diffusive processes. Dunkerton (1981) studied the inertial instability of the equatorial middle atmosphere and he showed the inertially unstable modes have only a weak dependence on the magnitude of the assumed eddy viscosity. Schubert and Hack (1982) pointed out the important role of inertial stability in the rapid growth and development of cyclones. Boyd and Christidis (1982) have shown that a linear shear flow which is always barotropically stable according to Rayleigh-Kuo criterion at low latitude can become unstable on the equatorial β-plane. They found two distinct types of instability the "mixed Kelvin-inertial mode" and the "mixed gravity-Kelvin mode". Stevens (1983) investigated the possibility of the equatorial symmetric instability of the mean flow in the troposphere with horizontal shear in the presence of Rayleigh friction and Newtonian cooling. He considered basically two types of shears - quadratic and linear shear. He found that the instability is confined to the region where the vertical component of absolute vorticity is of opposite sign to the local Coriolis parameter i.e., where the square of the inertial frequency is negative. The most unstable mode in his calculations has the structure of a single cell in the horizontal with meridional overturning in the $y-p$ plane. In a follow up study, Stevens and Ciesielski (1986) invesigated both symmetric and asymmetric instabilities for horizontally sheared flow away from the equator. Ciesielski *et al.* (1989) applied the theory of asymmetric inertial instability to explain a mesoscale phenomenon which appeared as a group of propagating cloud wavelets.

Observation taken and compiled during MONEX over the Indian Ocean region as shown by Young (1981) indicate the possibility of existence of inertial instability between equator to around 10°N. He has computed the absolute vorticity and the inertial instability growth rate from 40°S to 20°N in latitude and from 3°E to 90°E in longitude. His computations show that the possibility of inertial instability exists between the zero-line of absolute vorticity (around 10°N) and the equator. In this paper, we will address the question whether the zonal monsoon flow of July is symmetrically unstable. This aspect has not been given much attention earlier.

In Section 2, we derive an equation in the streamfunction using a set of zonally symmetric perturbation equations. The method of solution is given in Section 3. Section 4 gives the results of the stability analysis. Section 5 presents the concluding remarks.

2. Governing equations

The governing equations are the two momentum equations, the continuity equation and the thermodynamic energy equation in (x, y, p) co-ordinates. We consider the basic flow $U(y, p)$ in the Indian summer monsoon region during July. Zonally symmetric perturbations are considered. The linearized set of equations are given by

$$\frac{\partial u'}{\partial t} + \left(\frac{\partial U}{\partial y} - f\right) v' + \omega' \frac{\partial U}{\partial p} = 0 \tag{3}$$

$$\frac{\partial v'}{\partial t} + fu' + \frac{\partial \phi'}{\partial y} = 0 \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial p}\right) - v' f \frac{\partial U}{\partial p} + \sigma\omega' = -\frac{R}{C_p p}\dot{Q}' \tag{5}$$

$$\frac{\partial v'}{\partial y} + \frac{\partial \omega'}{\partial p} = 0 \tag{6}$$

where equations (3), (4) are the x and y component of the momentum equations, equation (5) is the thermodynamic energy equation and equation (6) is the continuity equation. In equations (3) to (6) u', v', ω' are the perturbations of the eastward, northward and vertical velocities respectively, f is the Coriolis parameter, ϕ' is the perturbation geopotential, R and C_p are the specific gas constant and the specific heat at constant pressure, $\dot{Q}'$ is the rate of nonadiabatic heating and σ is the static stability.

We include here a simple parameterization of cumulus heating as proposed by Lau and Peng (1987). The rate of cumulus heating here is given by

$$\frac{\dot{Q}'}{C_p} = -mrLq_{sat}(T_L)\eta(p)\frac{\Delta p}{C_p p_o}\left(\frac{\partial v'_L}{\partial y}\right) \tag{7}$$

where m is the moisture availability factor (0.8); r the relative humidity (0.75); L the latent heat of condensation; $q_{sat}(T_L)$ the saturation specific humidity at temperature T, the suffix $_L$, meaning the values defined at the lowest model layer; p_o surface pressure. Here $\frac{\partial v'_L}{\partial y}$ is the divergence at the model level (referred by the suffix $_L$). We have considered only the simple Wave-CISK type of heating, without considering the condition of positive heating. Here $\eta(p)$ is the normalized heating function whose vertical integral is unity.

The thermodynamic energy equation with the inclusion of moisture becomes

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial p}\right) - v' f \frac{\partial U}{\partial p} + \sigma\omega' = \frac{K}{p}\sin\left(\frac{\pi p}{p_o}\right)\frac{\partial v'_L}{\partial y} \tag{8}$$

The constant K is given by

$$K = \frac{RmrLq\Delta pN}{p_o C_p} \tag{9}$$

where N is the normalization constant $\left(\frac{\pi}{2}\right)$. We have considered here a simple profile of the form $\sin\left(\frac{\pi p}{p_o}\right)$ as the vertical distribution of heating which is quite consistent with the oberved profiles for the monsoon and tropical areas. Schaack *et al.* (1990) estimated the three-dimensional distribution of atmospheric heating over different parts of the globe from the ECMWF GWE level IIIb data set. The vertical profiles of time-averaged heating for July over India show maximum heating concentrated around 400 mb to 500 mb.

We now define the streamfunction ψ' such that $v' = -\frac{\partial \psi'}{\partial p}$ and $\omega' = \frac{\partial \psi'}{\partial y}$ from equation (6).

Eliminating u' between equations (3) and (4) we get

$$\frac{\partial^2 v'}{\partial t^2} + \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial y}\right) - f\left(\frac{\partial U}{\partial y} - f\right)v' - f\omega'\frac{\partial U}{\partial p} = 0 \tag{10}$$

Eliminating ψ' between equations (8) and (10) and substituting for v' and ω' in terms of ψ', we finally get

$$\left\{-\frac{\partial^2}{\partial t^2} - f\left(f - \frac{\partial U}{\partial y}\right)\right\}\frac{\partial^2 \psi'}{\partial p^2} - \beta\frac{\partial U}{\partial p}\frac{\partial \psi'}{\partial p} - 2f\frac{\partial U}{\partial p}\frac{\partial}{\partial p}\left(\frac{\partial \psi'}{\partial y}\right)$$

$$-f\frac{\partial^2 U}{\partial p^2}\frac{\partial \psi'}{\partial y} - \sigma\frac{\partial^2 \psi'}{\partial y^2} - \frac{K}{p}\sin\left(\frac{\pi p}{p_o}\right)\frac{\partial^2}{\partial y^2}\left(\frac{\partial \psi'}{\partial p}\right)_L = 0 \tag{11}$$

We assume solutions of the form

$$\psi' = \Psi(y,\, p)e^{i\nu t} \tag{12}$$

where $\Psi(y,\, p)$ is the perturbation streamfunction amplitude and ν is the frequency of oscillation. Substituting equation (12) into equation (11) we get

$$\left\{\nu^2 - f\left(f - \frac{\partial U}{\partial y}\right)\right\}\frac{\partial^2 \Psi}{\partial p^2} - \beta\frac{\partial U}{\partial p}\frac{\partial \Psi}{\partial p} - 2f\frac{\partial U}{\partial p}\frac{\partial}{\partial p}\left(\frac{\partial \Psi}{\partial y}\right)$$

$$-f\frac{\partial^2 U}{\partial p^2}\frac{\partial \Psi}{\partial y} - \sigma\frac{\partial^2 \Psi}{\partial y^2} - \frac{K}{p}\sin\left(\frac{\pi p}{p_o}\right)\frac{\partial^2}{\partial y^2}\left(\frac{\partial \Psi}{\partial p}\right)_L = 0 \tag{13}$$

Our purpose is to solve equation (13) for ν the frequency and Ψ to get the zonally symmetric eigen modes and its $(y - p)$ structure.

3. Method of solution

We simplify the equation (13) by introducing discrete levels both in the vertical as well as the

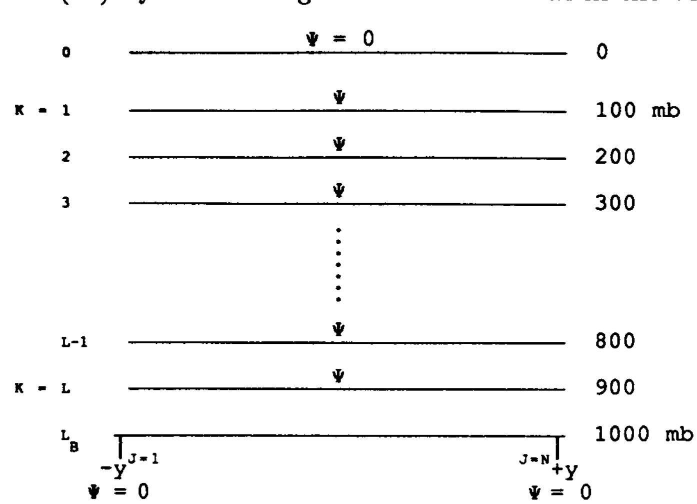

Fig. 1. Schematic diagram of the multi-level model.

meridional direction and replacing the vertical and meridional derivatives by finite differences. We have considered 10-levels in the vertical and a latitudinal domain from 39°S to 39°N with a resolution of 2°. Figure 1 represents the schematic structure of the vertical levels chosen.

The finite differenced form equation (13) at any interior grid point is given by

$$\begin{aligned}
&\frac{1}{\Delta p^2}\left(-f\left(f-\frac{\partial U}{\partial y}\right)\right)(\Psi_{J,\,K+1}-2\Psi_{J,\,K}+\Psi_{J,\,K-1})\\
&+\frac{1}{2\Delta p}\left(-\beta\frac{\partial U}{\partial p}\right)(\Psi_{J,\,K+1}-\Psi_{J,\,K-1})\\
&+\frac{1}{2\Delta p^2\Delta y}\left(-2f\frac{\partial U}{\partial p}\right)(\Psi_{J+1,\,K+1}-\Psi_{J+1,\,K-1}-\Psi_{J-1,\,K+1}+\Psi_{J-1,\,K-1})\\
&+\frac{1}{2\Delta y}\left(-f\frac{\partial^2 U}{\partial p^2}\right)(\Psi_{J+1,\,K}-\Psi_{J-1,\,K})\\
&+\frac{1}{\Delta y^2}(-\sigma)(\Psi_{J+1,\,K}-2\Psi_{J,\,K}+\Psi_{J-1,\,K})\\
&+\frac{1}{2\Delta p\Delta y^2}\left(-\frac{K}{p}\sin\left(\frac{\pi p}{p_o}\right)\right)(-\Psi_{J+1,\,L-1}+2\Psi_{J,\,L-1}-\Psi_{J-1,\,L-1})\\
&-\nu^2\left(\left(-\frac{1}{\Delta p^2}\right)(\Psi_{J,\,K+1}-2\Psi_{J,\,K}+\Psi_{J,\,K-1})\right)=0 \qquad (14)
\end{aligned}$$

Here f and β are functions of latitude and are evaluated at all latitude grid points. The vertical and lateral boundary conditions used are

$$v=-\frac{\partial\Psi}{\partial p}=0 \qquad \Psi|_{\pm Y}=constant=0$$

$$\omega=\frac{\partial\Psi}{\partial y}=0; \qquad \Psi\Big|_{\substack{p=0\\ p=L_B}}=constant=0 \qquad (15)$$

where $+Y$ and $-Y$ are the northern and southern boundaries respectively. Equation (14) with the boundary conditions, equation (15) applied at latitudes $j{=}1,\,2,\ldots,\,N-1,\,N$ and pressure levels $K{=}1,\,2,\ldots,\,L-1,\,L$ together can be put into the form

$$\left(\tilde{A}-\nu^2\tilde{B}\right)\tilde{Z}=0 \qquad (16)$$

where $\tilde{A}$ and $\tilde{B}$ are real square matrices, ν^2 is the eigenvalue and $\tilde{Z}$ is the eigenfunction. The eigenfunction

$$\tilde{Z}=(\Psi_{1,\,1},\,\Psi_{2,\,1},\ldots,\Psi_{N,\,1},\,\Psi_{1,\,2},\Psi_{2,\,2},\ldots,\,\Psi_{N-2},\ldots,\,\Psi_{1,\,L},\,\Psi_{2,\,L},\ldots,\,\Psi_{N,\,L})^T \qquad (17)$$

the superscript T stands for the transpose. The coefficients of the matrices $\tilde{A}$ and $\tilde{B}$ are simple and hence will not be provided. Equation (16) is a generalized eigenvalue problem and we have solved this equation with ($K \neq 0$ in equation (14)) and without ($K = 0$ in equation (14)) cumulus heating. The eigenvalues and eigenvectors are computed using the EISPACK eigen system package (Smith *et al.*, 1974).

4. Results of the stability analysis

We have conducted the stability analysis with and without cumulus heating. The basic flow of July has been taken from Ramage and Raman's Atlas (1972) as representative mean flow (Fig. 2). The static stability σ - profile is taken from the observations (Fig. 3). In the dry model as well as moist model, both unstable and neutral modes are obtained. The unstable modes have periods around 0.5 day to 2 days with doubling times around 0.5 day to 1.5 days. The neutral modes have periods ranging from 0.5 day to 5 days. The growth rates have slightly higher values when cumulus heating is considered.

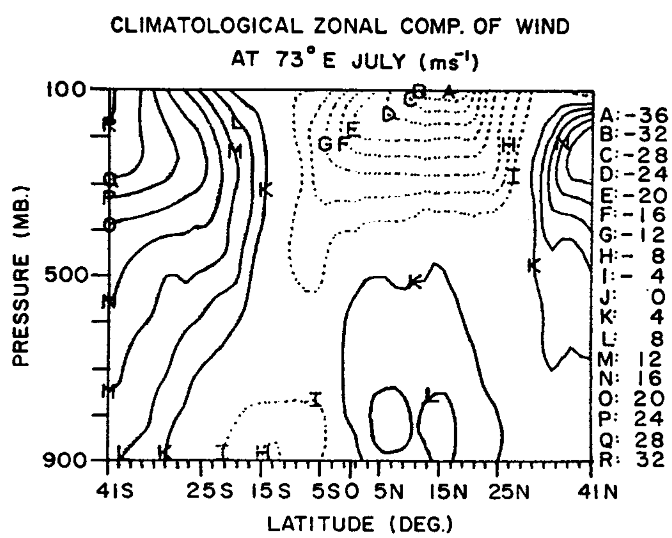

Fig. 2. Basic flow for July

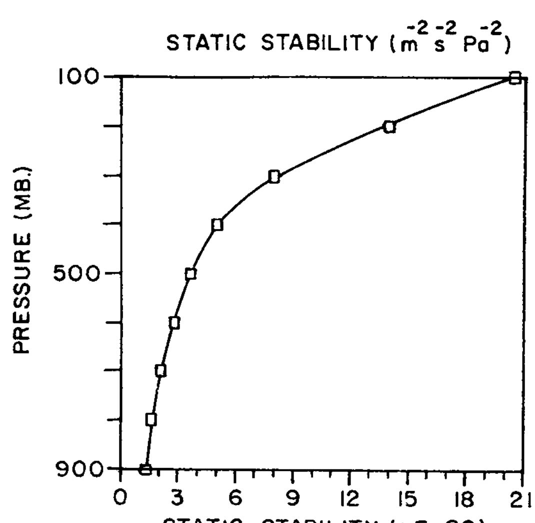

Fig. 3. Static stability at different levels.

The amplitude structure of Ψ of these unstable modes shows several cells of horizontal dimension around 6° latitude. The maximum amplitude is centered in the lower and middle troposphere. Figure 4 shows the streamfunction amplitude structure in the $y-p$ plane of an unstable mode with a period of one day and a doubling time of 1.2 days. It may be noted that meridional overturnings in the $y-p$ plane with small horizontal dimension are seen from 35°S to 35°N. The dynamical significance of this kind of structure is not completely clear. However, the cellular kind of structure in the $x-p$ plane as a result of symmetric instability has been reported by Sun (1984) to explain the formulation and evolution of the rainbands along the Baiu front over Eastern Asia during spring and summer. Thus, there may be a possibility that the zonal monsoon flow exhibits symmetric instability.

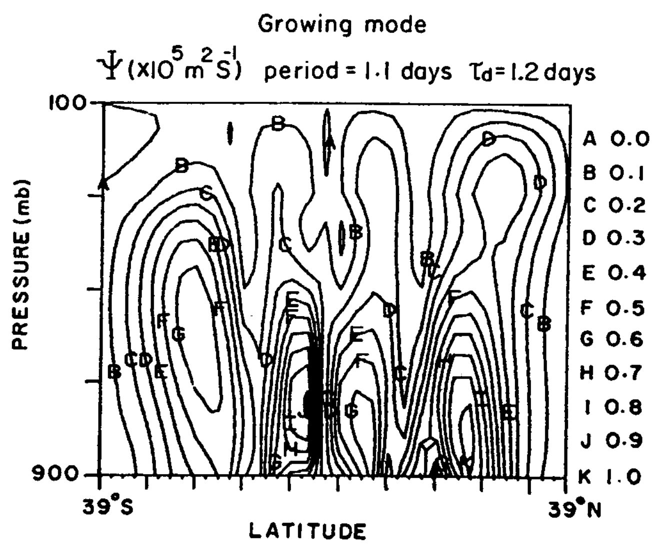

Fig. 4. Pressure-Latitude cross sections of streamfunction for a growing mode with doubling time 1.2 days and period 1.1 days.

The analysis also reveals basically two classes of neutral modes, one having a cell structure with ascent in the Northern Hemisphere and descent in the Southern Hemisphere with respect to the equator and the other having ascent near the equator and descent in both the hemispheres.

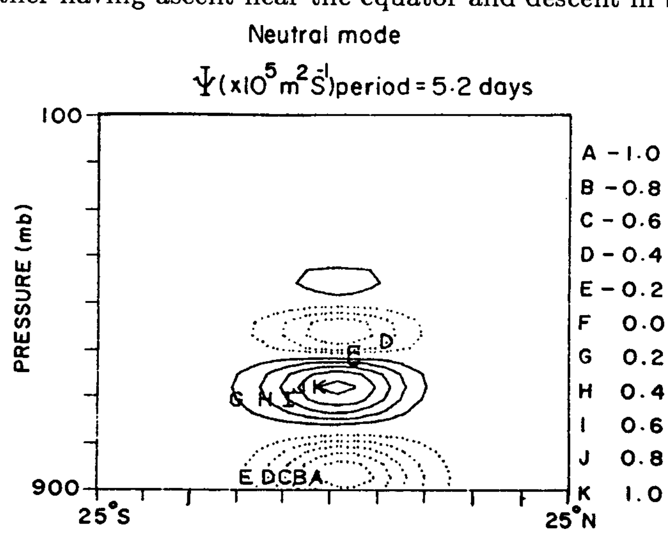

Fig. 5. Pressure-Latitude cross sections of streamfunction for a neutral mode with period 5.2 days.

In the former class of neutral modes, period ranges from 0.5 day to 5 days. These modes have structures with several cells in the $y-p$ plane. Figure 5 shows the streamfunction of one such mode with a period of 5 days. Several cells in the vertical with the streamfunction amplitude confined to the equatorial region can be clearly seen. Figures 6a, b, c present respectively the streamfunction, meridional velocity and vertical p-velocity in the latitude-pressure plane of a neutral mode with period of 2 days. The amplitude distribution of Ψ has a single cell structure across the equator (Fig. 6a). The v-distribution (Fig. 6b) has southerlies in the lower troposphere and northerlies in the upper troposphere. Upward motion north of the equator upto 35°N and downward motion south of the equator upto 35°S are seen in the ω-distribution (Fig. 6c).

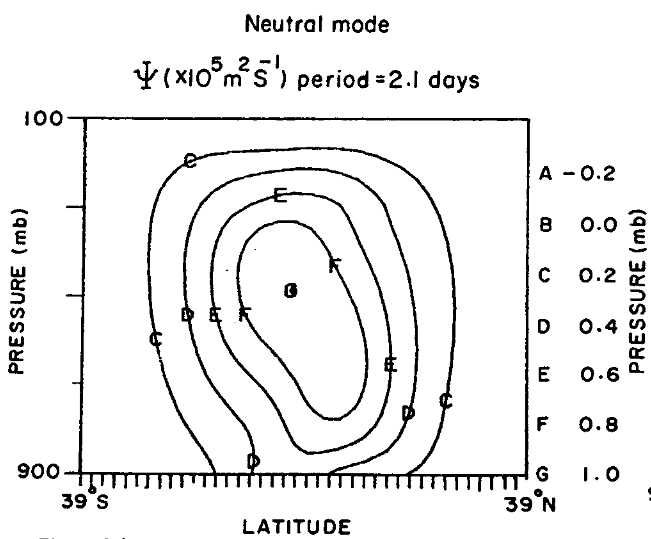

Figure 6 (a)

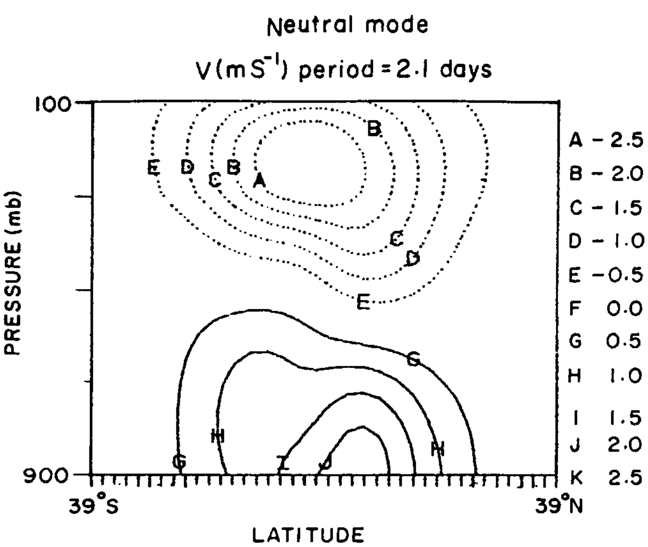

Figure 6 (b)

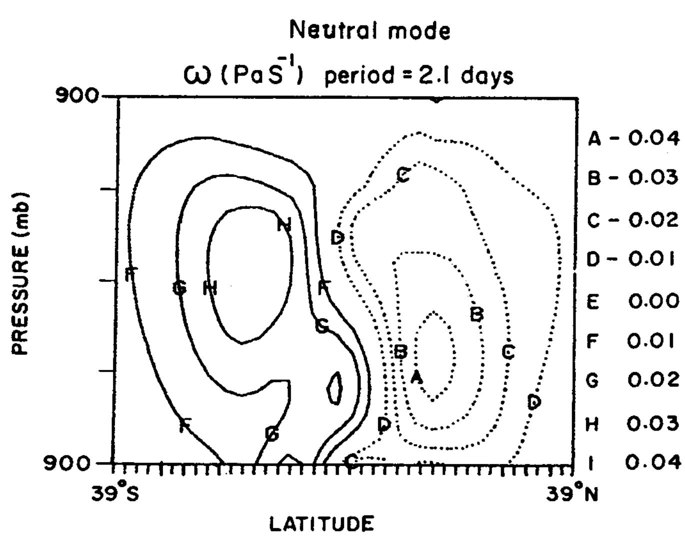

Figure 6 (c)

Fig. 6. Pressure-Latitude cross sections of (a) streamfunction, (b) meridional velocity, (c) vertical p-velocity for a neutral mode with period 2.1 days.

In another class of neutral modes, the period ranges from 0.5 day to 3 days. Figures 7a, b, c show respectively the streamfunction, meridional velocity and vertical p-velocity in the latitude-pressure plane of one such neutral mode with period 1.2 days. Vertical motion exists in the equatorial region from 20°S to 20°N and downward motion outside this region in both hemispheres (Fig. 7c). We have thus shown that the zonal monsoon flow near the equatorial regions with both latitudinal and vertical shear exhibit symmetric instability.

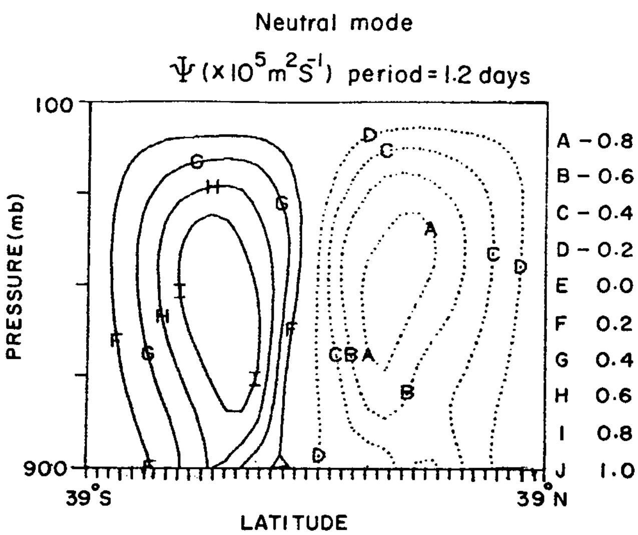

Figure 7 (a)

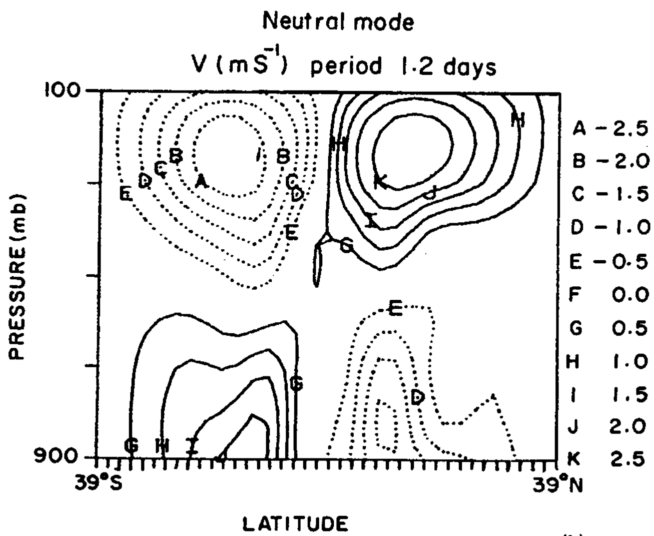

Figure 7 (b)

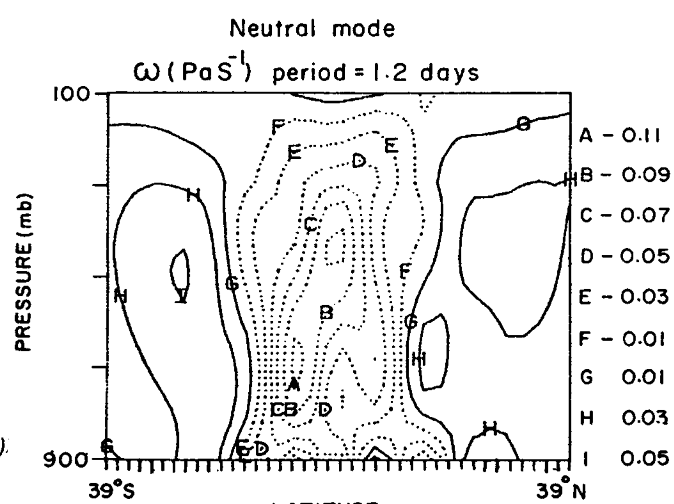

Figure 7 (c).

Fig. 7. Pressure-Latitude cross sections of (a) streamfunction, (b) meridional velocity, (c) vertical p-velocity for a neutral mode with period 1.2 days.

5. Concluding remarks

We have examined zonally symmetric perturbation equations in the equatorial region to investigate whether there are many unstable equatorial eigen modes in the monsoon region in the dry as well as moist models. In the dry model as well as moist model both unstable and neutral modes were obtained. The unstable modes have periods of around 1 to 2 days with doubling times of around 1 to 2 days. The amplitude structure of such unstable modes show several cells with horizontal dimension around 6° latitude and with maximum amplitude centered in the lower and middle troposphere. Two classes of netural modes with periods 0.5 day to 5 days, one having symmetric structure and the other with antisymmetric structure with respect to the equator were obtained. Thus we have shown that zonal flows in the monsoon region exhibit symmetric instability.

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