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Discontinuous autooscillations in a coupled atmosphere–ocean –continent ice model

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RESUMEN

Se estudia la variabilidad climática que surge de las interacciones no lineales entre la atmósfera, el océano y el hielo continental. Se obtienen soluciones analíticas y numéricas de las ecuaciones de evolución. Estas soluciones atestiguan la existencia de auto-oscilaciones discontinuas en el sistema climático. Un rasgo prominente de estas auto-oscilaciones es la alternancia de variaciones lentas y "a saltos" de la intensidad de dirección de la circulación termohalina.

Las variaciones lentas ocurren para un lapso de cerca de 10×10^3 años en el régimen de circulación normal (presente), y para uno de 50×10^3 años en el de circulación anormal (invertida). La transición de un régimen a otro es llevada a cabo de una manera prácticamente instantánea (en un lapso de cerca de 10^3 años) y se caracteriza por variaciones "a saltos" de la diferencia en salinidad. Los promedios oceánicos de temperatura y salinidad, así como los de masa de hielo continental, sufren las oscilaciones asimétricas, lentas y sin saltos.

ABSTRACT

Climatic variability arising from nonlinear interaction between atmosphere, ocean and continental ice is studied. Analitical and numerical solutions of the evolution equations are presented. These solutions testify to the existence of discontinuous autooscillations of the climatic system. A prominent feature of these autooscillations is the alternation of slow and jump-wise variations of ocean thermohaline circulation intensity and direction.

The slow variations occur for a time of about 10×10^3 years in the regime of normal (present) circulation and for a time of about 50×10^3 years in the regime of abnormal (reverse) circulation. Transition from one regime to another is performed practically instantly (for a time of about 10^3 years) and is stipulated by jump-wise variations of the salinity differences. The ocean-averaged temperature and salinity as well as continental ice mass undergo the slow asymmetric oscillations without jumps.

1. Introduction

It has been established in the previous authors report (Kagan and Maslova, 1990) that the thermohaline circulation in a three-layer ventilated ocean has 16 steady states but neither limit cycles nor other closed phase trajectories. It follows that the model cannot exhibit self-sustained oscillations. In the present paper we show that the interaction between the ocean, the atmosphere and the continental ice leads to the loss of stability of all above mentioned steady states and gives rise to discontinuous auto-oscillations with a period of about 60 kyr (1 kyr = 10^3 years)

The most remarkable feature of the oscillations is as follows. The slow variations of the thermohaline circulation are periodically interrupted by rapid transitions from one state to another. The direction of the circulation changes through these transitions to the reverse. On the contrary, the ocean average temperature and salinity as well as continental ice mass are subjected to smooth variations only.

The model investigated in the present paper is governed by the system of equations describing the evolution of the ocean thermohaline circulation, the evolution of the continental ice mass and the atmosphere heat budget. The model is presented in the Section 2. The presence of two large parameters –(1) the ratio of the rates of the heat exchange at the ocean–atmosphere interface and the heat transport in the ocean, (2) the ratio of the continental ice time scale to time scale of the ocean thermohaline circulation– allows us to undertake a qualitative exploration of the solution by means of the perturbation theory. The asymptotic properties of the solution are described in Section 3. Section 4 presents the results of numerical integration of the full system. We consider also the model sensitivity to the variations of the involved parameters and the paleontological data, confirming the possibility of nearly–discontinuous autooscillations. Conclusion follows in Section 5.

2. The model

We begin with equations describing the evolution of the ocean thermohaline circulation. As in the previous paper (Kagan and Maslova, 1990) the ocean is represented by three water masses (surface, intermediate and deep, Fig. 1) each being not isolated from atmosphere but exchanging by heat and moisture with it (i.e., ventilated). Each water mass has the volume V_j, the area of

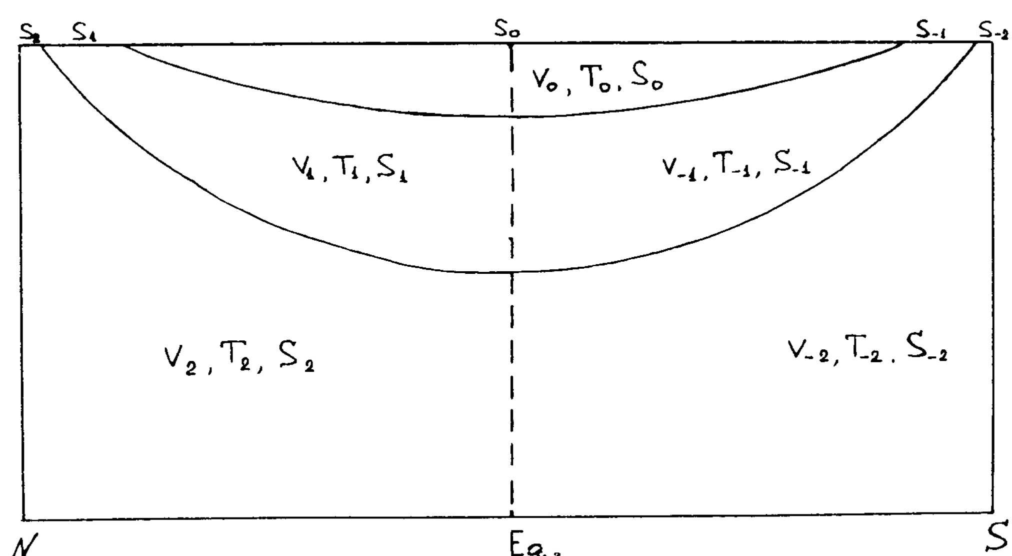


Fig. 1. A schematic cross–section of the ocean model. The water mass j has volume V_j, the area of ventilation s_j, the salinity S_j and the temperature T_j.

ventilation s_j, the salinity S_j and the temperature $T_j (j = 0, 1, 2)$. Here and elsewhere indices $j = 0, 1, 2$ correspond to surface, intermediate and deep water masses respectively. Symmetry about equator is assumed for simplicity. The temperature and the salinity in each water mass are assumed to be constant and water masses themselves immiscible. Following Stommel (1961) we use the hydraulic approximation and the linear equation of sea water state to approximate advective exchange (q_j, $j = 1, 2$) by neighbour water masses. We investigate here a modification of the previous model, including the diffusive exchange by heat and salt with a constant intensity κ. The model equations describing the heat and salt budgets in the various water masses are

$$V_j \frac{dT_j}{dt} = \sum_{k=1,2} A_{jk}(\mid q_k \mid + \kappa) X_k + s_j Q_j / \rho_w c_w$$

$$V_j \frac{d}{dt} S_{j/S^*} = \sum_{k=1,2} A_{jk}(\mid q_k \mid + \kappa) Y_k / S^* + s_j F_j \tag{2.1}$$

$$q_k = c \alpha_T X^* (X_k / X^* - \delta Y_k / S^*).$$

The first terms in the right side of the equations (2.1) express the advective and diffusive transports of heat and salt while the second terms represent the effect of the ocean–atmosphere interaction. The notation is as follows: A_{jk} are elements of the matrix

$$(A_{jk}) = \begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{pmatrix},$$

$X_k = T_{[k/2]} - T_k$, $Y_k = S_{[k/2]} - S_k$, $k = 1, 2$ – the differences of the temperature and salinity between the water masses, $[k/2]$ is the integral part of the number $k/2$; c, X^*, S^* – the reference values of the volume transport, the difference of the ocean temperature and salinity respectively; $\delta = \frac{\alpha_s}{\alpha_T} \cdot \frac{S^*}{X^*}$, α_T, α_s –the thermal and the salinity expansion coefficients, ρ_w –the density of the sea water, c_w –the heat capacity of the sea water; Q_j, F_j –the fluxes of heat and salt at the ocean–atmosphere interface.

We assume that salt flux at the ocean surface is determined by the evaporation – precipitation excess and the continental ice degradation, that the precipitation is controlled by the local evaporation and that the latter is determined by the difference of the ocean–atmosphere temperature. Thus we are led to the following parameterization

$$Q_j = Q_{1j} - \lambda_T (T_j - T_{aj}),$$

$$F_j = \lambda_E (T_j - T_{aj}) + C_j - (-\dot{M} / \rho_w s) \tag{2.2}$$

where Q_{1j} is the flux of absorbed solar radiation, λ_T, λ_E, C_j are constant coefficients. T_{aj} is the average air surface temperature in the domain of ventilation with number j, $-\dot{M}$ is the rate of the continental ice mass degradation, $s = \sum_j s_j$ is the area of the ocean surface.

To find the planetary average air surface temperature T_a we use the atmosphere heat budget equation. Assuming that the atmosphere responds to change of external forcing much faster than the ocean and the continental ice we use the last equation in its quasistationary form:

$$\overline{H}(\overline{T}_a,\ s_I) - \sum Q_j s_j / p = 0 \tag{2.3}$$

where $\overline{H}(\overline{T}_a,\ s_I)$ is the difference between the absorbed and the outgoing infrared radiation at the top boundary of the atmosphere, s_I is the ice area; p is the area of the hemisphere. The second term in the left side of (2.3) expresses the heat flux at the ocean surface. The dependence of $\overline{H}$ on $\overline{T}_a$ and s_I is taken to be linear.

To describe the meridional distribution of the air surface temperature T_a we adopt two–mode approximation (North *et al.*, 1981) with the coefficients depending on $\overline{T}_a$ and the intensity of the thermohaline circulation in the intermediate/deep water mass system (Spelman and Manabe, 1984), i.e.,

$$T_a(x) = \overline{T}_a - [b_1 - b_2(q_2 - q_2^*) - b_3(\overline{T}_a - \overline{T}_a^*)]P_2(x) \tag{2.4}$$

where $b_j (j = 1, 2, 3)$ are constants, $x = \sin\varphi$, φ being the latitude, $P_2(x)$ is a Legendre polynomial, here and elsewhere the subscripts * refer to stationary values of functions. A similar two–mode approximation is used also for the meridional distribution of the absorbed solar radiation (North *et al.*, 1981):

$$Q_1(x) = (1 - \chi)(1 - \alpha_o)[\overline{H}(\overline{T}_a,\ s_I) + H_2P_2(x)] \tag{2.5}$$

The absorption coefficient χ of solar radiation, the ocean surface albedo α_o and the coefficient H_2 are assumed to be constants. To obtain the functions T_{aj}, Q_{1j} we use the s_j–averaging of the functions $T_a(x)$, $Q_1(x)$.

Now we turn to our method of evaluation the ice area s_I. As usually it is assumed that the sea ice boundary coincides with the -10°C isotherm. Adopting for simplicity the linear relation between the mass and the area of the continental ice we can represent the radiation flux $\overline{H}$ as a function $\overline{H}(\overline{T}_a,\ M)$. According to the law of water conservation in the climate system we shall accept that the mass budget of the continental ice is subjected to the equation

$$\frac{dM}{dt} = \sum_j s_j \rho_w F_j \tag{2.6}$$

It is well known that the continental ice degradation is governed by mechanical and thermic factors. In the present model it is assumed that the basic mechanical factor is the iceberg discharge with a constant rate $-\dot{M}_1$ and the basic thermic factor is the ablation. Therefore $-\dot{M} = -\dot{M}_1 + (-\dot{M}_2)$ where $-\dot{M}_2$ is the ablation rate. The heat budget equation at the continental ice surface leads to the relation

$$-\dot{M}_2 = pL_f^{-1} \int_{s_a} \Lambda(x)Q(x)dx$$

where L_f is the heat of ice melting, $\Lambda(x)$ is the nondimensional zonal land extent normalized by the length of latitude circle, s_a is the ablation area. The boundaries $x = x_s$, $x = x_I$ of the ablation zone s_a are given by the equalities (North *et al.*, 1981)

$$x_s = 1/\sqrt{3}\left[1 + 2b_1^{-1}(T_a + \lambda_T^{-1}Q_1(x_s))\right]^{1/2}$$

$$s_I^L = p\int_{x_I}^1 \Lambda(x)dx$$

The first equality determines the snow line that is attached to the 0°C isotherm of the surface temperature $T_a(x)$; the second expresses the relation between the area s_I^L and the meridional extent x_I of the continental ice with the mass M. We recall that the linear relation between M and s_I^L is suggested. From the relations (2.6), (2.4), (2.2) we obtain the following linear approximation for the degradation rate of the continental ice mass

$$-\dot{M} = b_4 + b_5(q_2 - q_2^*) + b_6(\overline{T}_a - \overline{T}_a^*) \tag{2.7}$$

where $b_j(j = 4,5,6)$ are constant coefficients. Let us note that the constant b_4 coincides with the degradation rate $-\dot{M}^*$ in a stationary state. This constant is uniquely determined by the equation

$$\sum_j s_j \rho_w F_j^* = 0$$

Using the equations (2.2), (2.5), (2.7) we obtain the set of coupled differential equations (2.1)–(2.6) for the functions M, T_j, S_j $(j = 0,1,2)$ describing the evolution of the atmosphere–ocean–continental ice system.

3. Asymptotic solutions

Despite the simplifications the complexity of the system (2.1)–(2.6) remains considerable. Thus we now turn to qualitative study. We first switch to scaled variables defined by

$$(\overline{T}'_a,\ T'_a,\ T'_j) = (\overline{T}_a - \overline{T}_a^*,\ T_a - \overline{T}_a^*,\ T_j - \overline{T}_a^*)/X^*,$$

$$S'_j = (S_j - S^*)/S^*,\ M' = (M - M^*)/M^*,$$

$$(q'_k,\ \kappa') = (q_k,\ \kappa)/q^*,\ t' = tq^*/V_o$$

where $q^* = c\alpha_T X^*$ (we recall that subscript * denotes stationary values of the variables.) The resulting system is

$$\frac{dT_j}{dt} = \sum_{k=1,2} A_{jk}\frac{V_o}{V_j}(|\ q_k\ | + \kappa)X_k - \frac{s_j V_o}{s_o V_j}\lambda(T_j - T_{aj}) + \tilde{Q}_{1j} \tag{3.1}$$

$$\frac{dS_j}{dt} = \sum_{k=1,2} A_{jk}\frac{V_o}{V_j}(|\ q_k\ | + \kappa)Y_k + \tilde{F}_j \tag{3.2}$$

$$\frac{dM}{dt} = \sum_{j=0,1,2} \frac{V_j}{V_M}(\tilde{F}_j - \tilde{F}_j^*) \tag{3.3}$$

$$q_k = X_k - \delta Y_k \tag{3.4}$$

$$\lambda^{-1}\left[\tilde{H}-\sum_{j=0,1,2}\frac{V_j}{V_o}\frac{s_o}{s}\tilde{Q}_{1j}\right]-\sum_j\frac{s_j}{s}(T_j-T_{aj})=0 \tag{3.5}$$

where the strokes of scaled variables are omitted and the following combinations of initial parameters are involved

$$\tilde{Q}_{1j}=Q_{1j}\frac{V_o}{V_j}s_j/\rho_w c_w q^* X^*,\quad \tilde{F}_j=F_j\frac{V_o}{q^*}\frac{\bar{s}_j}{V_j}$$

$$\tilde{H}=Hs_o p/s\rho_w c_w q^* X^*,\quad s=\sum_{j=0,1,2}s_j$$

$$V_M=M^*/\rho_w,\quad \lambda=\lambda_T s_o/\rho_w c_w q^*$$

Let us note that the equations imply the relations

$$\frac{d\overline{T}_V}{dt}=\frac{s}{s_o}\frac{V_o}{V}\tilde{H}(\overline{T}_a,\ M),\ \frac{d}{dt}(\overline{S}_V-\frac{V_M}{V_o}M)=0 \tag{3.6}$$

where $V=\sum_j V_j$ is the ocean volume,

$$\overline{T}_V=\frac{1}{V}\sum_j V_jT_j,\ \overline{S}_V=\frac{1}{V}\sum_j V_jS_j$$

are the ocean–averaged temperature and salinity respectively.

The system (3.1)–(3.5) contains the parameter λ, characterizing the ratio of the rates of the heat exchange at the ocean–atmosphere interface and the heat transport in the ocean. We adopt the rather reasonable assumption that this parameter is large (Welander, 1986). The limit $\lambda\to\infty$ in the above equation leads to the system

$$T_j=T_{aj} \tag{3.7}$$

$$\frac{dM}{dt}=\sum_j\frac{V_j}{V_M}(\tilde{F}_j-\tilde{F}_j^*)\equiv f_M \tag{3.8}$$

$$\frac{dS_j}{dt}=\sum_{k=1,2}A_{jk}\frac{V_o}{V_j}(|\ q_k\ |+\kappa)Y_k+\tilde{F}_j\equiv f_{S_j} \tag{3.9}$$

$$\frac{d\overline{T}_V}{dt}=\frac{sV_o}{s_oV}\tilde{H}(\overline{T}_a,\ M)\equiv f_{T_V} \tag{3.10}$$

$$q_k=X_{ak}-\delta Y_k \tag{3.11}$$

$$X_{ak}=c_k+d_k\overline{T}_a+e_k(q_2-q_2^*) \tag{3.12}$$

$$\overline{T}_V = c_V + d_V \overline{T}_a + e_V (q_2 - q_2^*) \tag{3.13}$$

where the constants c_k, d_k, e_k, c_V, d_V, e_V are nondimensional combinations of the previous parameters.

It should be noted that the equation (3.1) contains the small parameter λ^{-1} in a singular way. Due to this fact there exists an "initial layer" where the reduced system (3.7)–(3.13) is not valid. However it can be proved that each solution of the full system satisfies the relation

$$T_j = T_{aj} + \mathrm{O}(1/\lambda) + \mathrm{O}(\exp\{-a\lambda t\}) \tag{3.14}$$

with a positive constant a. It follows that each solution of the full system is attracted to the solution of a reduced system and the initial layer is of width $\mathrm{O}(\lambda^{-1} \ln \lambda)$. Therefore the reduced system can be used to investigate the long time behaviour of the full system.

It follows from the equations (3.9)–(3.14) that

$$f_M = -\tilde{b}_5 (q_2 - q_2^*) - \tilde{b}_6 \overline{T}_a$$

where $\tilde{b}_5 = b_5 (V_o/V_M)/\rho_w$, $\tilde{b}_6 = (V_o/V_M) b_6 X_*/\rho_w q^*$.

The parameter $\tilde{b}_5 \equiv \varepsilon$ describes the ratio of the relaxation time of the ocean thermohaline circulation to the continental ice relaxation time. Introducing the "slow time" $\tau = \varepsilon t$ and new variables Y_1, Y_2, $\overline{T}_V$, M we obtain the system

$$\varepsilon \frac{dY_j}{d\tau} = f_{Y_j} \qquad j = 1, 2 \tag{3.15}$$

$$\varepsilon \frac{dT_V}{d\tau} = f_{T_V} \tag{3.16}$$

$$\frac{dM}{d\tau} = -(q_2 - q_2^*) - d\overline{T}_a \tag{3.17}$$

where $f_{Y_j} = f_{S_{[j/2]}} - f_{S_{[j]}}$, $d = \tilde{b}_6/\tilde{b}_5$.

In the limit $\varepsilon \to 0$ we have

$$f_{Y_j} = 0, \; j = 1, 2 \tag{3.18}$$

$$\tilde{H}(\overline{T}_a, \; M) = 0 \tag{3.19}$$

$$\frac{dM}{d\tau} = -(q_2 - q_2^*) - d\overline{T}_a \equiv f_M \tag{3.20}$$

The equations (3.18) imply

$$Y_j = (| \, q_j \, | + \kappa)^{-1} \psi_j, \qquad j = 1, 2 \tag{3.21}$$

where the constants ψ_j are linear functions of C_j

It follows from the equations (3.11), (3.13), (3.21) that

$$\kappa\varphi(\kappa^{-1}q_j,\ \beta_j) - c_j - e_j(q_2 - q_2^*) = d_j\overline{T}_a, \tag{3.22}$$

the function φ and the constants β_j being defined by

$$\varphi(\kappa^{-1}q_j,\ \beta_j) = \kappa^{-1}q_j + \beta_j(|\ \kappa^{-1}q_j\ | + 1)^{-1},\ \beta_j = \delta\psi_j/\kappa^2$$

Since $\tilde{H}$ is a linear function of M we can write the equation (3.19) in the form

$$M = -\left(\frac{d\tilde{H}}{dM}\right)^{-1}\left[\tilde{H}(\overline{T}_a,\ M) - \left(\frac{d\tilde{H}}{dM}\right)M\right] \equiv g_M \tag{3.23}$$

with the right side depending on $\overline{T}_a$ only.

We suggest that the continental ice mass decreases as the average surface temperature $\overline{T}_a$ increases, thus, $dg_M/d\overline{T}_a < 0$. Hence the equation (3.23) has the unique solution given by $\overline{T}_a = g_T(M)$ where g_T is the inverse of g_M. Then the equations (3.22), (3.20) imply the system for the functions $M,\ q_2$

$$\frac{dM}{d\tau} = -(q_2 - q_2^*) - dg_T(M) \equiv f(M,\ q_2) \tag{3.24}$$

$$M = g_M(d_2^{-1}[\kappa\varphi(\kappa^{-1}q_2,\ \beta_2) - c_2 - e_2(q_2 - q_2^*)] \equiv h(M,\ q_2)$$

The phase portrait of the system (3.24) is outlined in Figure 2, where the arrows show the direction of the phase point movement. It can be seen, that there exists only one point of

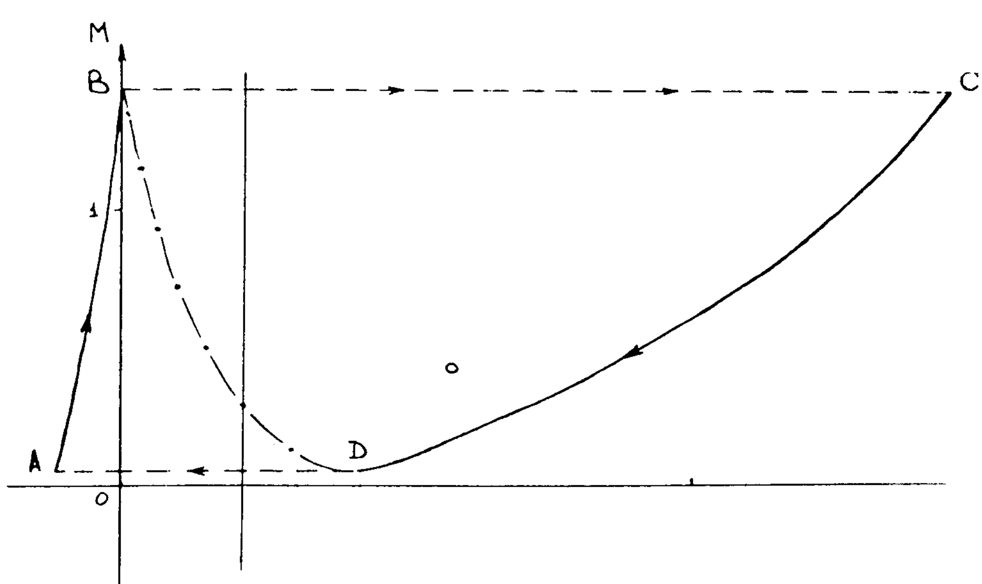


Fig. 2. The phase plane of the system (3.24). The solid curves mark the continuous time evolution, the dashed lines mark the jump-wise changes. The arrow shows the direction of the phase point movement. The circle represents the present state of the system.

intersection of the curve $h(M,\ q_2) = M$ and the straightline $f(M,\ q_2) = 0$ corresponding to a unique steady solution of the system (3.24). This steady state is unstable for reasonable values of q_2^*. It is also essential that any trajectory sets up in a finite time at one of the extremal points of the curve $h(M\ q_2) = M$, but there are no trajectories outgoing from these points.

Rigorously speaking to describe the behavior of the trajectories in a vicinity of the extreme it is necessary to investigate the full system. Yet it seems reasonable to suggest that it happens a rapid reorganization of the ocean thermohaline circulation accompanied by slow change of continental ice mass. It follows that the jumps $B \to C$, $D \to A$ take place. This conjecture can be proved. Indeed the full system has a stable limit cycle in a vicinity of the curve $ABCD$. The asymptotic formula $\mathbf{v} = (\varepsilon^{-1}h,\ f)$ for the phase velocity vector $\mathbf{v}$ is valid. The limit $\varepsilon \to 0$ in the full system gives rise to the discontinuous oscillations with the smooth variations of the ocean thermohaline circulation on the parts AB and CD of the limit cycle $ABCD$ interrupted by instant transitions $B \to C$, $D \to A$.

Let us note that the steady state O is the saddle point. It has a stable manifold with dimension 1. Hence the trajectory may set up in the point O, but this event occurs with a small probability. There exists another possibility of the disappearing of the limit cycle. It happens if $q_2^* > q_{2m}$; q_{2m} is the point where the function h reaches its minimum. If this inequality is valid, the unique steady state is stable and any trajectory sets up at the point corresponding to this state.

We return now to the case $q_2^* < q_{2m}$, when the discontinuous oscillations occur. The period τ_o of the limit cycle is determined by the time of the phase point movement along the continual parts of the cycle. Therefore $\tau_o = \tau_+ + \tau_-$, where τ_+ and τ_- are the life times of the normal $(q_2 > 0)$ and abnormal $(q_2 < 0)$ ocean thermohaline circulation respectively. These quantities are given by

$$\tau_+ = \oint_{AB} \frac{dM}{f(q_2,\ M)} \qquad \tau_- = \oint_{CD} \frac{dM}{f(q_2,\ M)} \tag{3.25}$$

Once the functions q_2, M are determined, the equation (3.22) can be used to find the intensity of the thermohaline circulation in surface/intermediate water masses. In general case the equation has three solutions given by

$$q_1^{(m)} = \kappa u^{(m)}(\gamma,\ \beta_1),\ m = 1, 2, 3 \tag{3.26}$$

where $\gamma = [c_1 + e_1(q_2 - q_2^*) + d_1 g_T(M)]/\kappa$

$$u^{(1)}(\gamma,\ \beta_1) = \frac{1+\gamma}{2} - \sqrt{\frac{(1-\gamma)^2}{4} + \beta_1}, \qquad \beta_1 > \gamma$$

$$u^{(2)}(\gamma,\ \beta_1) = -\frac{1-\gamma}{2} - \sqrt{\frac{(1+\gamma)^2}{4} - \beta_1}, \qquad \beta_1 < (1+\gamma)^2/4$$

$$u^{(3)}(\gamma,\ \beta_1) = \frac{\gamma-1}{2} + \sqrt{\frac{(1+\gamma)^2}{4} - \beta_1}, \qquad \beta_1 < (1+\gamma)^2/4,$$

the functions $u^{(j)}$ being the solutions of the equation $\varphi(u,\ \beta_1) = \gamma$.

It follows that the degenerate system has three limit cycles $I^{(m)}$ (periodic solutions) determined by the functions $\{q_1^{(m)}(\tau),\ q_2(\tau),\ M(\tau)\}$, $m = 1,2,3$, two cycles ($I^{(1)}$, $I^{(3)}$) being stable while the third ($I^{(2)}$) is unstable. The circulation in the surface/intermediate layer is normal within the cycles $I^{(2)}$, $I^{(3)}$ and abnormal within the cycle $I^{(1)}$. The fast transition from one stable cycle to another are possible but they haven't been obtained in the numerical simulations with chosen values of parameters.

4. Numerical results

Let us now confront the theoretical predictions with the numerical simulation of the full system (3.1)–(3.5) for the following values of the involved parameters: $b_1 = 19,2^{\circ}$C, $b_2 = 10^{-14}$ m^{-3} year $^{\circ}$C, $b = +1,0$, $q = 5,6 \cdot 10^{14}$ m^3/year, $X^* = 10^{\circ}$C, $S^* = 35$ o/oo, $s_o/s = 0,8$, $s_1/s = 0,16$, $s_2/s = 0,04$, $V_o/V = 0,09$, $V_1/V = 0,16$, $V_2/V = 0,75$, $\alpha_o = 0,1$, $\beta_2 = 16,5$, $\kappa = 0,6 \cdot 10^{14}$ m^3/year, $\lambda_T = 45$W/m^2 $^{\circ}$C, $\lambda_E = 0,36$ m/year $^{\circ}$C, $\chi = 0,3$.

The results are presented in the Figure 3. It can be seen that the model exhibits self-sustained oscillations with the period $\tau_o = 62$ kyr. The continental ice mass M decreases monotonically and rapidly enough as long as the thermohaline circulation state is normal ($q_1 > 0$, $q_2 > 0$). M reaches its minimum at the moment of the fast transition to the abnormal

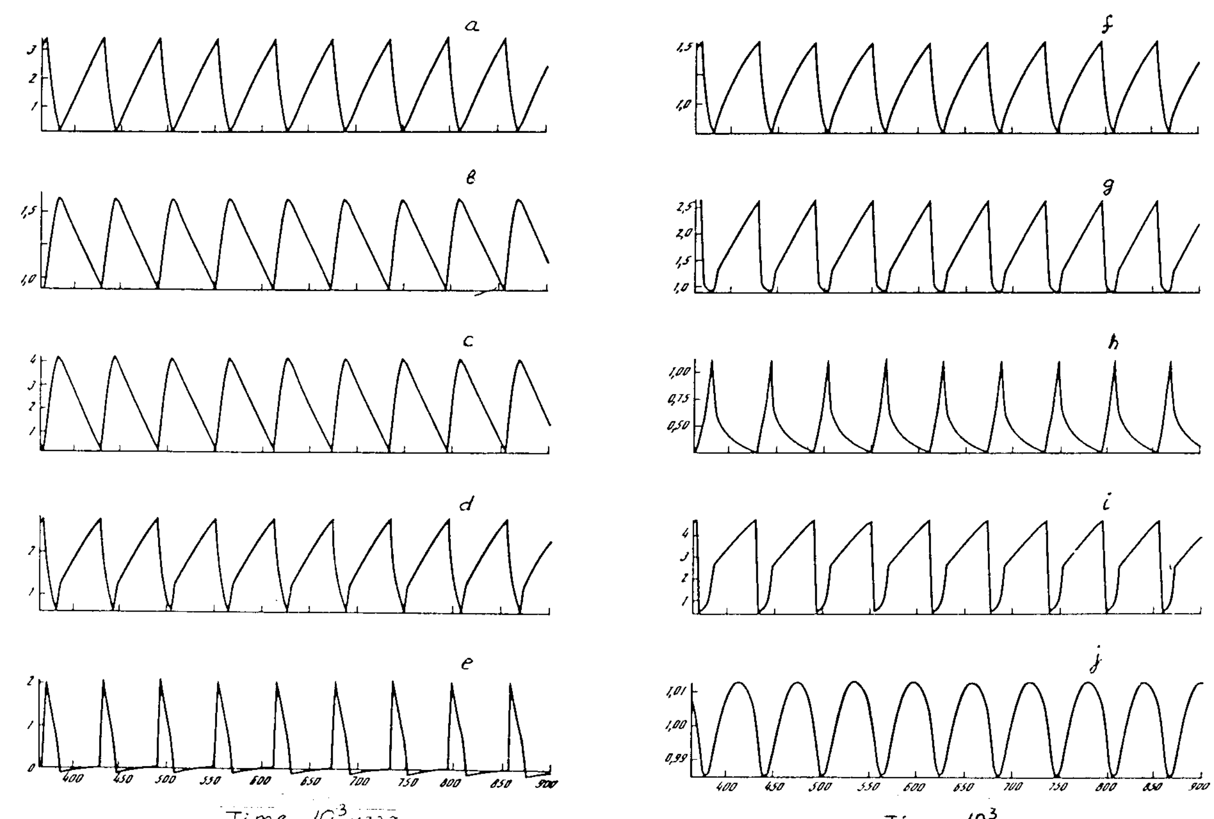
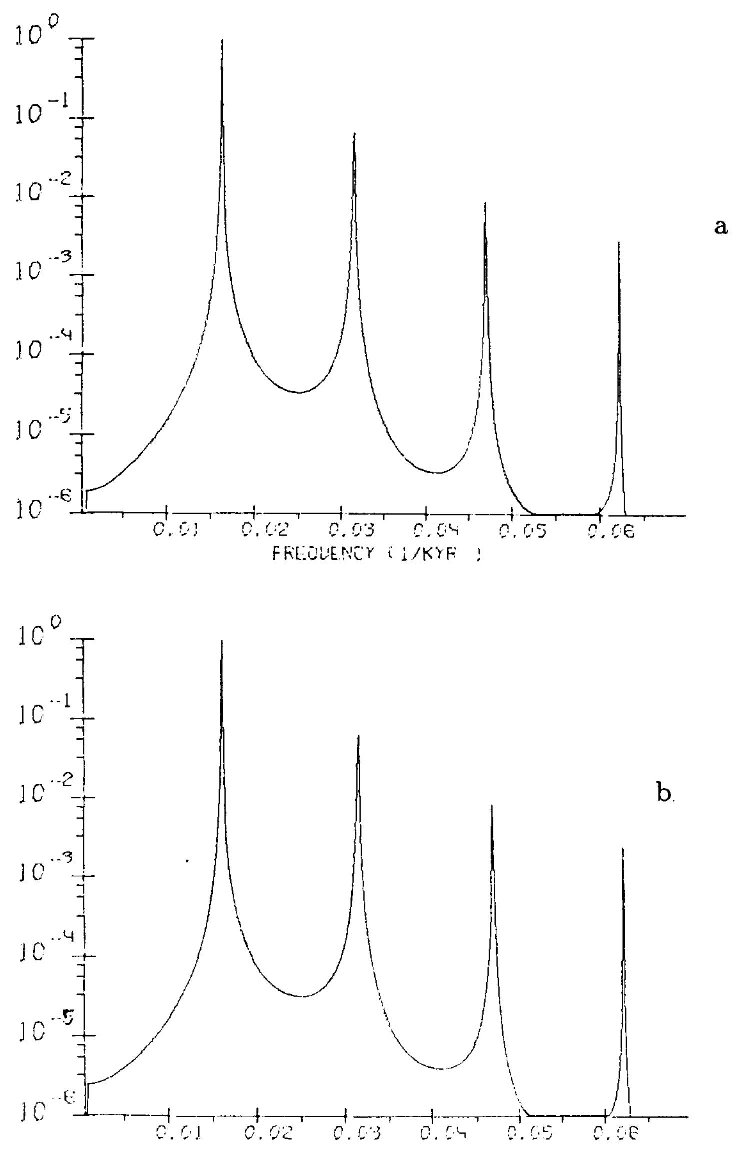


Fig. 3. Time evolution of the climatic characteristics: a – the continental ice mass M; b – the surface-averaged ocean temperature T_s; c – the volume-averaged ocean temperature T_V; d and e – the intensity of the thermohaline circulation in the surface/intermediate (q_1) and intermediate/deep (q_2) water mass system; f, g, h and i – the temperature and the salinity differences in the surface/intermediate and intermediate/deep water mass system respectively; j – the volume averaged ocean salinity S_V. Time unit is 1 kyr = 10^3 years. All climatic characteristics are normalized by their present values.

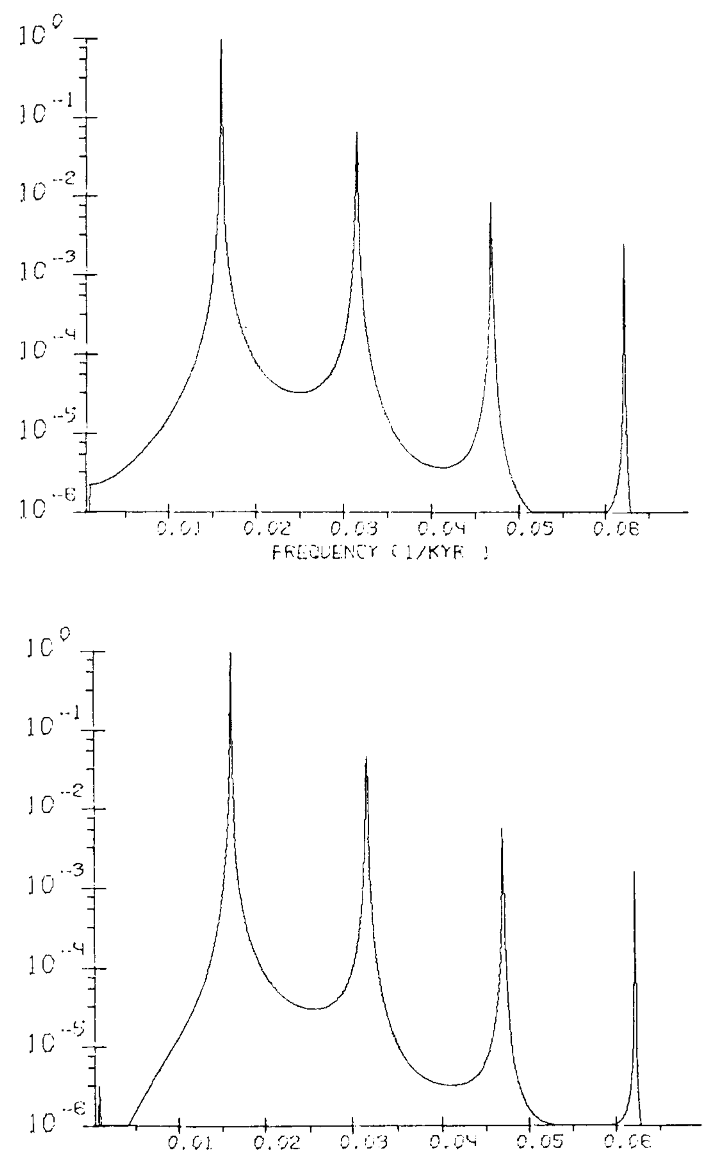
($q_1 > 0$, $q_2 < 0$) thermohaline circulation. Then a slow increase of the continental ice mass starts. It coninues until the thermohaline circulation returns to its normal (present) state. The gaps in the continuous variations of the ocean thermohaline circulation are determined uniquely by the gaps of the salinity differences. On the other side the gaps are absent in the oscillations of the ocean–averaged temperature, the ocean–averaged salinity and the continental ice mass. The remarkable feature of the oscillations is the asymmetry. According to the numerical results the time scale of the abnormal circulation is greater than the time scale of the normal one. We notice also a decrease of the ocean–averaged temperature and opposite changes in the intensity of the thermohaline circulation (its amplification in the surface/intermediate water masses and attenuation in the intermediate/ deep water masses) in the stage of the continental ice growth. These results are verified by paleontological data indicating that the deep ocean temperature decreased by 3–7°C at the glacial maximum (Yansen and Veum, 1990). On the other side the deep water production either markedly weakened or ceased and the intermediate–water production strengthened in this time (Streeter and Shackleton, 1979; Boyle and Keigwin, 1987).

(Fig. 4)



There exist the paleontological evidences that are in agreement with another feature of the present solution, namely the rapid changes in the thermohaline circulations. According to our results the time scale of the normal circulation is 15×10^3 years, the time scale of the abnormal circulation is 47×10^3 years while the duration of the transition from one type of circulation to the other is about 1×10^3 years. Indicators of the abrupt transitions are the change in the relative abundance of benthic foraminiferal taxa with time scale of order 1×10^3 years (Streeter and Shackleton, 1979) and the variations of Cd/Ca, δ^{13} C and δ^{18} O ratios in benthic foraminiferal shells with time scale of the order $0{,}5 \times 10^3$ years (Boyle and Keigwin, 1987). The exhaustive survey and analysis of the evidence for the abrupt reorganization of the ocean thermohaline circulation can be found in the paper of Broecker and Denton (1989).

(Fig. 4)



c

d

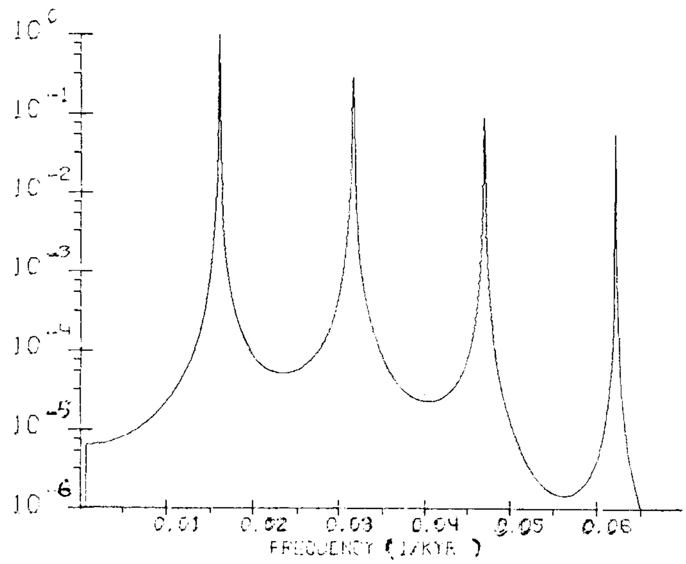


Fig. 4. Spectra of the climatic characteristics: a – the continental ice mass (M); b – the surface-averaged ocean temperature (T_s); c – the volume averaged ocean temperature (T_V); d – the volume-averaged ocean salinity (S_V); e – the intensity of the thermohaline circulation (q_2) in the intermediate/deep water mass system. Frequency unit is kyr^{-1}.

The above mentioned asymmetry in the oscillations implies the richness of the spectrum which contains not only main eigenoscillation with frequency $f_o = 0,0156\ \mathrm{kyr}^{-1}$, but also the combination modes with frequencies $f_1 = 0,0312\ \mathrm{kyr}^{-1}$, $f_2 = f_o + f_1 = 0,0468\ \mathrm{kyr}^{-1}$, $f_3 = f_2 - f_o + f_1 = 0,0624\ \mathrm{kyr}^{-1}$ and some their side harmonics with lower and higher frequencies (Fig. 4). Let us note that the first–mode frequency f_1 is determined by the difference between the time scales of the normal and abnormal circulations.

5. Sensitivity to changes in model parameters

Now we discuss the sensitivity of the model results to changes in model parameters. For the sake of simplicity we consider the problem for the degenerate system (3.24). Moreover, we assume that $e_2 = d = 0$ and the function g_M is linear, i.e. $g_M = k^{-1}\overline{T}_a$, where $k^{-1} = \frac{dg_M}{d\overline{T}_a}$ for $\overline{T}_a = \overline{T}_a^*$. These simplifications do not change qualitative features of the solution since the constants e_2, d are small ($e_2 = 0,3$, $d = 0,05$) and the function g_M is nearly linear in our numerical experiments. Under these assumptions we obtain

$$\frac{dM}{d\tau} = -(q_2 - q_2^*)$$

$$M = k^{-1}d_2^{-1}[\kappa\varphi(\kappa^{-1}q_2,\ \beta_2) - c_2] \tag{4.1}$$

The solution of the system (4.1) depends on six parameter q_2^*, k, d_2 κ, β_2, e_2. It is useful to perform the scaling transformations

$M' = \kappa^{-1}(c_2 + kd_2M)$, $u' = \kappa^{-1}q_2$, $u^* = \kappa^{-1}q_2^*$, $\tau' = kd_2\kappa^{-2}\tau$ and represent the system (4.1) in the form

$$\frac{dM}{d\tau} = (u^* - u'), \qquad M' = \varphi(u', \beta_2) \tag{4.2}$$

displaying two dimensionless parameter u^* and β_2 only. The limit cycle exists if the relations

$$\beta_2 > 1, \qquad 0 < u^* < (\beta_2^{1/2} - 1)$$

are fulfilled. The amplitude a' and the period τ_o' of the limit cycle are

$$a' = \varphi\,|_B - \varphi\,|_D = (\beta_2^{1/2} - 1)^2, \quad \tau_o' = \tau_-' + \tau_+' \tag{4.3}$$

where τ_-' and τ_+' are the time scales of the abnormal and normal circulations respectively.

Returning to the system (4.1) we conclude that it has the limit cycle with the amplitude $a = k_1(\beta_2^{1/2} - 1)^2$ and the period $\tau_o = k_2(\tau_-' + \tau_+')$ where $k_1 = \kappa(kd_2)^{-1}$, $k_2 = \kappa^2(kd_2\tilde{b}_5)^{-1}$.

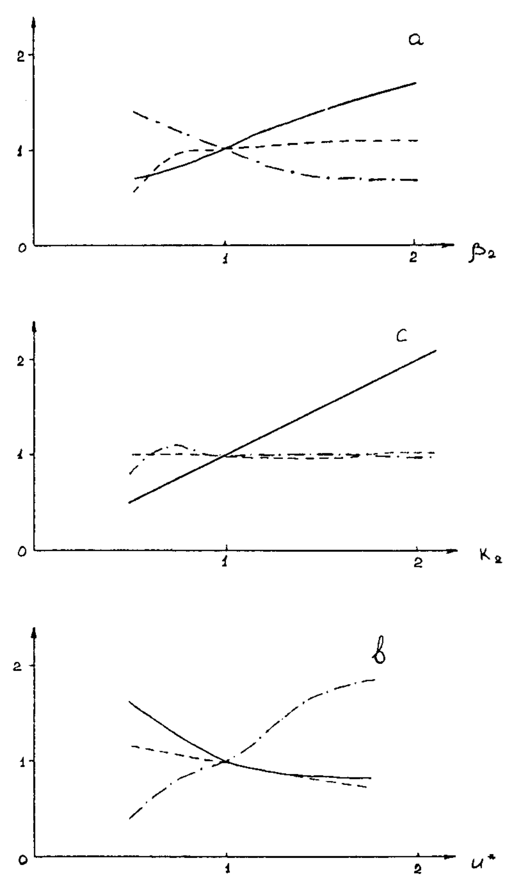


Fig. 5. Sensitivity of the limit cycle characteristics to the variations of the model parameters: a – the period (solid line) of the limit cycle and the life-time of the abnormal circulation (dash-dotted) as functions of the parameter β_2; b and c – the same as in fragment a, but for the parameters u^* and k_2 respectively

It occurs that only four parameter k_1, k_2, β_2, u^* are essential. The limit cycle disappears if the vector with components k_1, k_2, β_2, u^* reaches the boundary of the domain

$$\beta_2 > 1,\ 0 < u^* < \beta_2^{1/2} - 1,\ k_1 > 0,\ k_2 > 0 \tag{4.4}$$

It follows that the effects of the diffusion, the dependence of the pole–to–equator air–surface temperature difference on the heat transport and the haline constituent of the ocean thermohaline circulation play the crucial role in arising of the limit cycle. On the contrary, small variations of the parameters inside the domain (4.4) imply only small variations in the limit cycle parameters.

The numerical investigation of the full system (3.1)–(3.5) confirm these statements. It follows from Figure 5a that the period of oscillations τ_o increases as β_2 increases. The change in τ_o is related mainly with the increase of the abnormal circulation time scale τ_-. On the contrary, the increase of u^* results in the decrease of τ_- and the corresponding diminution of period τ_o (Fig. 5b). The Figure 5c shows the model response to the k_2 increase: the ratios τ_+/τ_o, τ_-/τ_o do not change, while the period τ_o grows.

6. Conclusions

It is shown both analytically and numerically that discontinuous autooscillations can be raised in the atmosphere–ocean–continental ice model. The important property of these oscillations is the occurrence of the fast transitions between two different regimes corresponding to normal and abnormal thermohaline circulations in the intermediate/deep water masses, i.e., to a temperature–controlled and salinity–controlled one respectively. The system under consideration spends about 10 kyr in the normal (present) thermohaline circulation regime, then a jump to abnormal (reverse) circulation follows. The duration of the second regime is approximately 50 kyr. The cycle is closed by the jump to the normal regime. The slow changes in the thermohaline circulation are accompanied by slow changes in the ocean–averaged temperature and salinity, as well as the continental ice mass. The time of transition from one regime to the other is of the order 1 kyr.

The jumps in the ocean thermohaline circulation are determined uniquely by the jumps of the salinity differences in the intermediate/deep water masses. The ocean–averaged temperature and salinity and the continental ice mass do not undergo any jumps. The continental ice mass increases (decreases) monotonically in the abnormal (normal) regime.

The occurrence of the abrupt transitions between the different thermohaline circulation regimes by itself is not unexpected (Broecker and Denton, 1989) although the phenomena can be seen unusual from the view point of the classical theory of the ocean thermohaline circulation.

But what is actually surprising is the Fourier spectrum of the solution. According to the numerical results it is characterized by the peaks at 62, 32, 21 and 16 kyr. The presence of the harmonics with these periods may lead to resonant amplification of the climate variations of astronomical origin. We emphasize that the model exhibits self–sustained oscillations mainly as a result of the circulation intensity – salinity difference feedback. As an outgrowth of these results we are led to reconsider the role of the other feedbacks (in particular, the atmospheric CO_2) in the climate dynamics.

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