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# On equilibrium between orography and atmosphere

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RESUMEN

En este trabajo son investigadas las condiciones del equilibrio entre la orografía de un planeta y su atmósfera. Un primer ejemplo relativamente simple es la solución del problema en una atmósfera barotrópica equivalente, mientras que el modelo cuasi-geostrófico de dos capas es tomado como segundo ejemplo. El problema puede también ser resuelto por modelos cuasi-geostróficos de múltiples capas.

El problema que se investiga aquí tiene por objeto encontrar el movimiento que puede existir en una atmósfera bajo condiciones de estado estacionario en equilibrio con la orografía del planeta. Una solución trivial es un estado de reposo, pero es demostrado que soluciones no triviales también pueden ser construidas. La investigación es básicamente diferente al clásico problema de encontrar la influencia de orografía sobre un flujo preexistente, el cual ha sido resuelto en numerosos casos.

En la Tierra con su fuerte gradiente de temperatura meridional creado por el calentamiento diferencial entre el Polo y el Ecuador y debido a los procesos de calentamiento diabáticos compensados parcialmente por un transporte meridional de calor sensible, la circulación general es en una primera aproximación determinada por los procesos de inestabilidad baroclínica, derivando su energía de los campos meridionales de temperatura y manteniendo a su vez las corrientes zonales mediante transporte de calor y cantidad de movimiento, principalmente a través de las ondas.

En las escalas globales la circulación es modificada grandemente por la existencia de continentes y océanos y por la orografía.

En otros planetas con menor calentamiento meridional diferencial que la Tierra, la orografía puede tener una mayor influencia en el patrón de flujo estacionario.

ABSTRACT

The equilibrium conditions between the orography of a planet and its atmosphere are investigated. A first relatively simple example is the solution of the problem in an equivalent barotropic atmosphere, while the two-level quasi-geostrophic model is taken as the second example. The problem can also be solved for multi-level, quasi-geostrophic models.

The problem under invesigation is to find the motion which can exist in an atmosphere under steady state conditions in equilibrium with the orography of the planet. A trivial solution is a state of rest, but it is shown that non-trivial solutions can be constructed as well. The investigation is basically different from the classical problem of finding the influence of orography on a preexisting flow which has been solved in numerous cases.

On the Earth with its strong meridional temperature gradient created by the differential heating between Pole and Equator and due to diabatic heating processes compensated partly by meridional transport of sensible heat the general circulation is in the first approximation determined by the processes of baroclinic instability deriving its energy from the meridional temperature field and in turn maintaining the zonal currents by heat and momentum transport primarily by the waves. On the global scales the circulation is modified greatly by the existence of continents and oceans and by orography.

On other planets, with smaller meridional differential heating than the Earth, the orogrphy may have a major influence on the stationary flow pattern.

## 1. Introduction

The investigations reported here started with a study of a picture from meteosat which showed quite clearly that a cloud band started at the northern edge of the Pyrenees in a southerly air stream crossing the mountains (Woetman, 1985), see Figure 1. The cloudband was turning to the right in an anticyclonic direction and had obtained a west–east direction with a slight bend to the south before the clouds disappeared. The Pyrenees are situated along the Spanish–French border with the highest peaks reaching a little less than 4000 m. One would thus expect that the mountains could have an influence on an airstream crossing them.

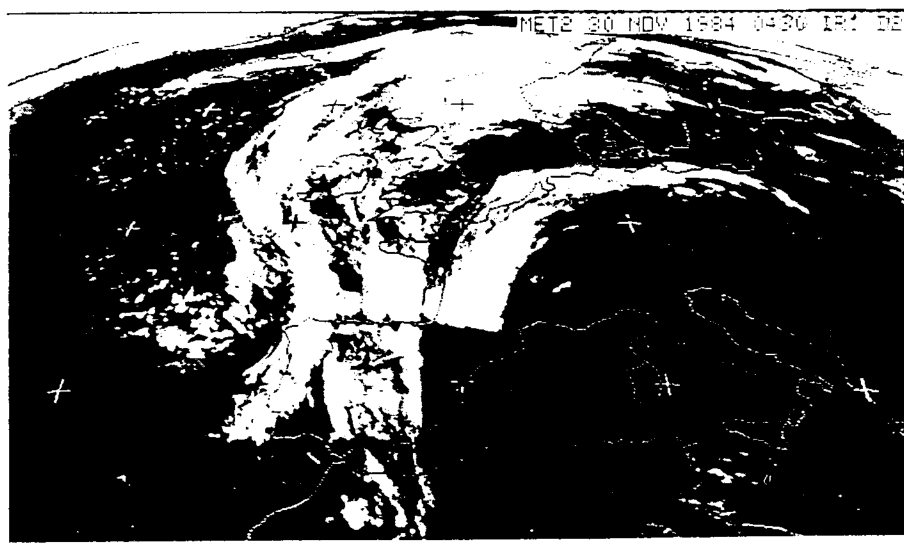


Fig. 1. Meteosat infrared picture (30 Nov. 1984, 04.30 GMT). Note the cloud starting at the Pyrenees and bending to the right going just south of Denmark.

Most investigations have dealt with the passing of a westerly air current over mountains having the major extension in the south–north direction such as the classical investigations by Charney and Eliassen (1949) and Bolin (1950). As a start it was natural to explore the passing of a narrow current across the mountains as done by Bolin (1950) simulating the Rocky Mountains in the USA. This approximate method assumes the conservation of potential vorticity in a homogeneous atmosphere. The additional assumption that the vorticity appears mainly as curvature and to a much lesser degree as shear permits the calculation of an approximate trajectory of a particle in the narrow current. In the case of a southerly current crossing a mountain with its major extention in the west–east direction one finds a marked difference from the classical westerly current. The reason is that the influence of the mountain and of the change of the Coriolis parameter work in the same direction when a southerly current crosses a west–east mountain range both trying to force the airstream in an anticyclonic direction. On the other hand, if a northerly current crosses the same mountain the Coriolis effect will create cyclonic vorticity while the mountain effect still works in the anticyclonic direction.

These considerations, treated in details in Section 2, lead to the question of how an extended mountain range going all the way around the globe would influence the atmosphere. However, the trajectory method is unsatisfactory due to the assumptions which must be made. It was thus desirable to use more accurate methods, but to restrict the investigation to the determination of stationary solutions. Most other investigations of the orographic effects on the atmospheric flow consider the influence on a given current. On the Earth it is a very reasonable problem since we know that some major aspects of the general circulation can be simulated by a model without mountains as done for example by Smagorinsky (1963) although such a model cannot account for the very long stationary waves. The reason is apparently that the differential heating between Equator and Pole and the Earth is sufficiently strong to create baroclinic instability creating baroclinic waves which in turn maintains the zonal currents in the atmosphere.

On a planet where the differential heating does not exist or at least is much less dominating than on the Earth the situation may be quite different. Because the baroclinic instability may be missing on such a planet it is important to compute the stationary flow in equilibrium with the orographic features of the planet. A trivial solution of no flow at all exists, but in addition to the solution it is also possible to compute other solutions which are much more complicated. That will be the main problem in the other sections of the paper. Section 3 will treat the relatively simple problem of an equivalent barotropic atmosphere, while Section 4 will contain an expansion to a standard two level, quasi-geostrophic model. These calculations will be carried out using numerical values for the various parameters taken from the atmosphere of the Earth. The calculations apply then supposedly to a planet of the same size as the Earth, with the same gravity, the same rotation rate and with the same basic hydrostatic stability. Having similar information from another planet will of course permit a repetition of the calculations.

It would appear that this particular view on orographic effects has not been given much attention. Otherwise, mountain effects are well covered in the meteorological literature. The review papers by Queney (1960) and Smith (1979) cover the early contributions, but since then there has been contributions by Buzzi and Speranza (1979); Hart (1979); Eliassen (1980); Frederiksen and Sawford (1981); Frederiksen (1982); Eliassen and Thorsteinsson (1984); Thorsteinsson (1988), and by Frederiksen and Frederiksen (1988, 1989 and 1991). The fluid dynamical aspects have been investigated by Fultz and Long (1951); Long (1953); Warren (1963); Jacobs (1964), and Jones (1970). The importance of the mountains for the general circulation of the Earth's atmosphere has been studied by Manabe and Terpstra (1974); Derome (1984), while the orographic influence on the atmospheric energy budget was investigated by Saltzman (1961). Lee cyclogenesis as influenced by the topography has been treated by Egger (1972, 1974), Hayes *et al.* (1987); Pierrehumbert (1985), and Hartjenstein and Egger (1990), while specific local effects are treated in many papers as for example Mesinger and Strickler (1982); Tibaldi and Buzzi (1983) for Mediterranean cyclogenesis, Reuter and Pichler (1964) for the Alps and Egger and Fraedrich (1987) for Antarctica.

The orographic effect has also played a major role in the investigations of blocking as a possible stable stationary state in low order systems as first tried by Charney and De Vore (1979) and later followed by other studies such as Charney and Strauss (1980); Trevisan and Buzzi (1980); Källén (1983, 1985). The references given here are by no means complete. Whole areas have been neglected in the references such as all the numerical aspects of incorporating the orographic effect in numerical prediction and climate models, the small scale influences by the mountains and the numerous studies which have been made of individual cases.

In the mathematical treatment we shall abstain from introducing frictional effects in the calculations. This is also consistent with the fact that energy conservation exists in the models considered here with the neglect of the heat sources.

## 2. Elementary considerations

The discussion in this section will be based on a simple model of the atmosphere. We shall assume that we are dealing with a homogeneous and incompressible atmosphere with a free surface. Such an atmosphere has conservation of potential vorticity which in this case is the ratio of the absolute vorticity and the depth of the atmosphere. i.e.,

$$\frac{\varsigma + f}{H} = const. \tag{1}$$

where $\varsigma$ is the relative vorticity, $f$ the Coriolis parameter and $H$ the depth of the atmosphere. Denoting the mountain height by $h(y)$ and the undisturbed height by $H_o$ we may write

$$\frac{\varsigma + f}{H_o - h} = \frac{f_e}{H_o} \tag{2}$$

where $f_e$ is the value of the Coriolis parameter at the point where the "parcel" meets the mountain. The parcel is in this case a column from top to bottom. (Fig. 2).

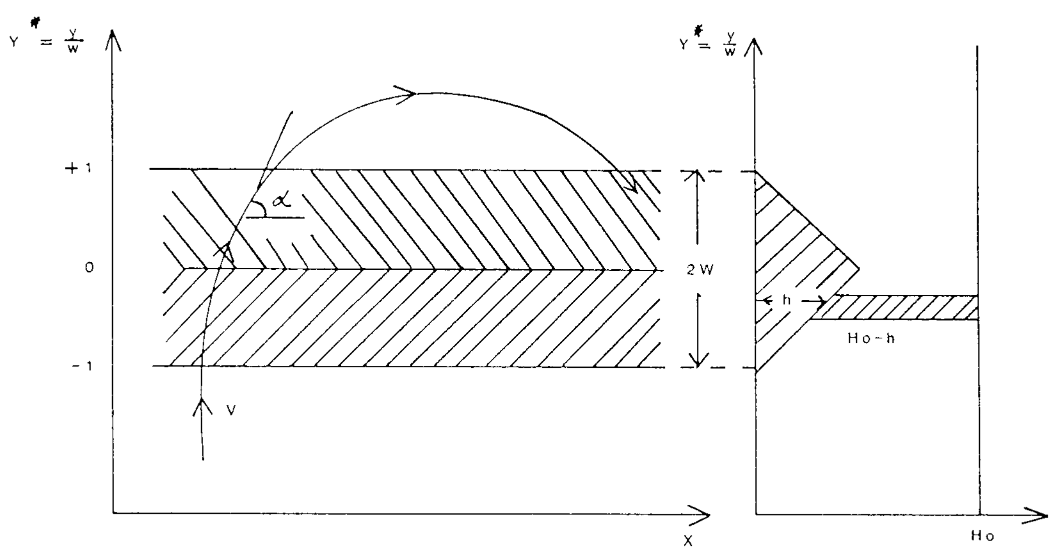



Fig. 2. A schematic figure showing the essential aspects of flow over a west–east oriented ridge. The notations are also shown.

As long as the parcel is over the mountain the denominator in (2) will be smaller than outside the elevation thus creating a tendency for anticyclonic vorticity. In (2) we have assumed that the parcel arrives at the mountain from the south and without relative vorticity. As it crosses the mountain it will gain anticyclonic vorticity partly from the decrease in the effective height and partly from the increase of the Coriolis parameter with latitude. If it has arrived at the mountain with sufficient speed, it will leave the northern edge of the mountain with anticyclonic vorticity, say $\eta_n$. As long as it is outside the mountain it will obey the equation

$$\varsigma + f = \eta_n \tag{3}$$

where only the change of the Coriolis parameter can change the relative vorticity. Eq. (3) is the basic equation for the so-called constant vorticity trajectory, and (2) is the equivalent problem, if the ground is not level. To use these two equations to calculate trajectories we make the additional assumptions that the relative vorticity is expressing itself in curvature, i.e.

$\varsigma = V/R$, where $V$ is the speed and $R$ the radius of curvature. Using the definition of the radius of curvature from geometry and denoting the angle of the tangent to the trajectory with the x-axis by $\alpha$ we may write (2) in the form

$$\cos(\alpha) = \frac{\beta(y - y_e}{V} + \frac{h(y)(f_o + \beta y_e)}{V H_o} \tag{4}$$

where we have introduced the beta-plane approximation, and where $y_e$ is the ordinate at the point where the parcel meets the mountain. Specifying $h(y)$ we may integrate (4) either analytically or numerically to obtain the direction of movement as a function of $y$. The trajectory may then be completed by noting that the following equation follows from the definition of the derivative:

$$\frac{dx}{dy} = \frac{\cos(\alpha)}{(1 - \cos^2(\alpha))^{1/2}} \tag{5}$$

Eq. (4) and (5) are the basic equations for the computation of the trajectory. They are most conveniently treated by numerical integrations in such a way that $\alpha$ is determined from eq. (4) followed by $x = x(y)$ from eq. (5). In very simple cases it is of course possible to obtain analytical solutions, but they are in any case quite complicated. Figure 2a shows the angle $\alpha$ as a function of the non-dimensional width $y^* = y/W$ where $W$ is the half width of the mountain which in this case has two linear slopes meeting at the height $h_m$. The parcel starts from the south with the velocity $V = 30$ ms$^{-1}$. The mountain is restricted to the region between -1 and +1 on the ordinate. At the end of the trajectory the parcel has returned to the mountain and crossed it once more. The curve has negative angles whenever the parcel moves southward. Figure 2b displays the analogous calculation on the beta plane calculated without paying attention to the mountain. This trajectory was started at $y^* = -1$ with the same speed (30 ms$^{-1}$). The

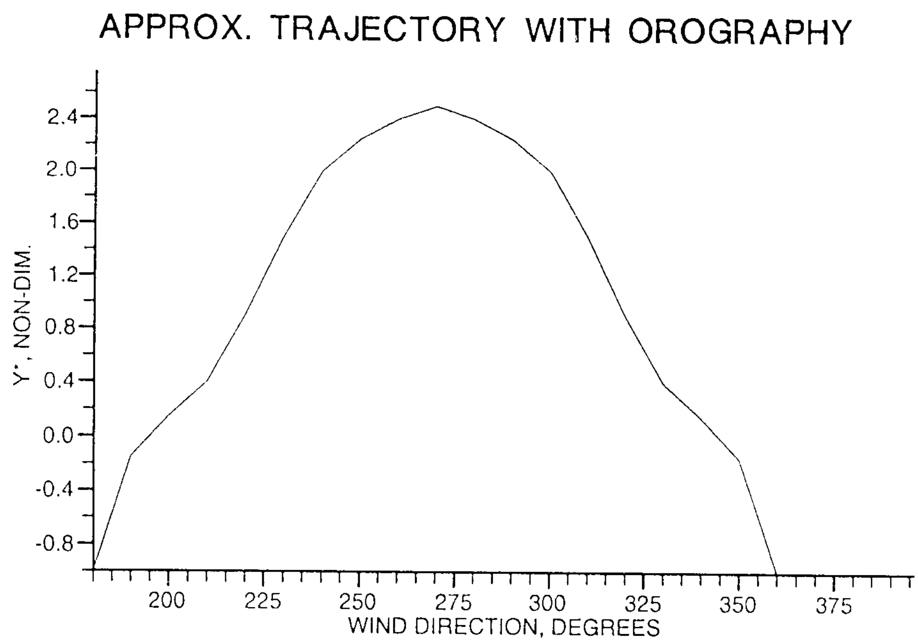



Fig. 2a. The wind direction in degrees for a parcel starting at the southern edge of the mountain.

anticyclonic effect of the mountain is clearly seen since the parcel with the mountain present has larger anticyclonic curvature and reaches a smaller distance to the north of the mountain.

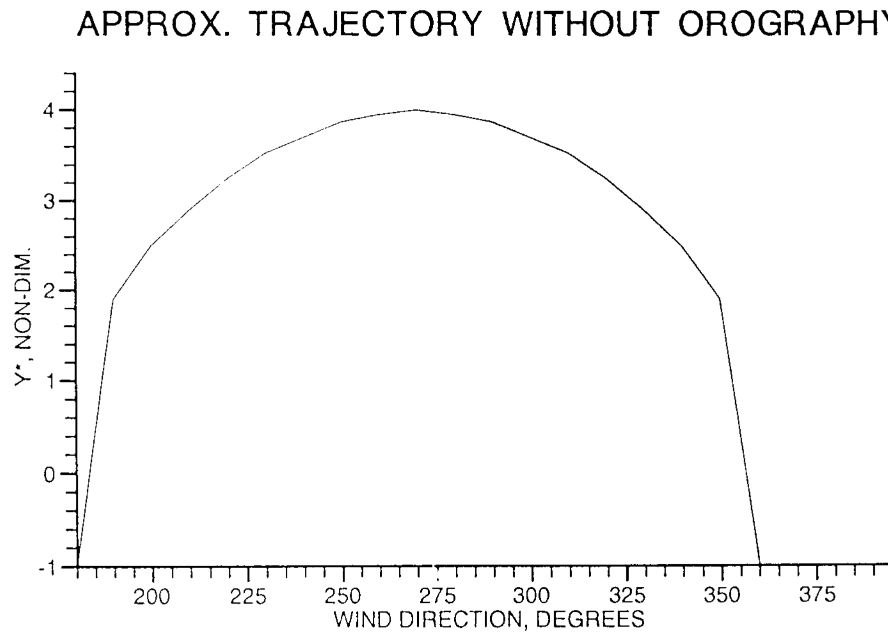



Fig. 2b. As Fig. 2a, but without the mountain.

Figures 3a and 3b show the trajectory itself in the non–dimensional coordinates $x_* = x/W$ and $Y^* = y/W$. The curve with the lower amplitude (3a) is influenced by the mountain while the other curve (3b) is the trajectory as it would be without the mountain. The effect of the mountain is thus to create a path with a lower amplitude and a longer wavelength. It is also seen that if the mountain were to reach all the way around the planet the oscillatory motion would continue with the low amplitude.

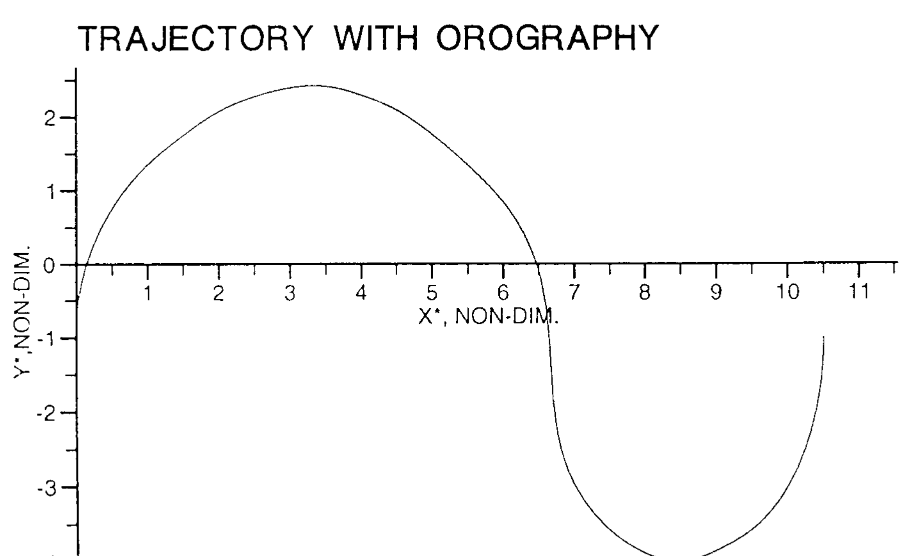



Fig. 3a. The trajectory of a parcel starting at the southern edge of the mountain.

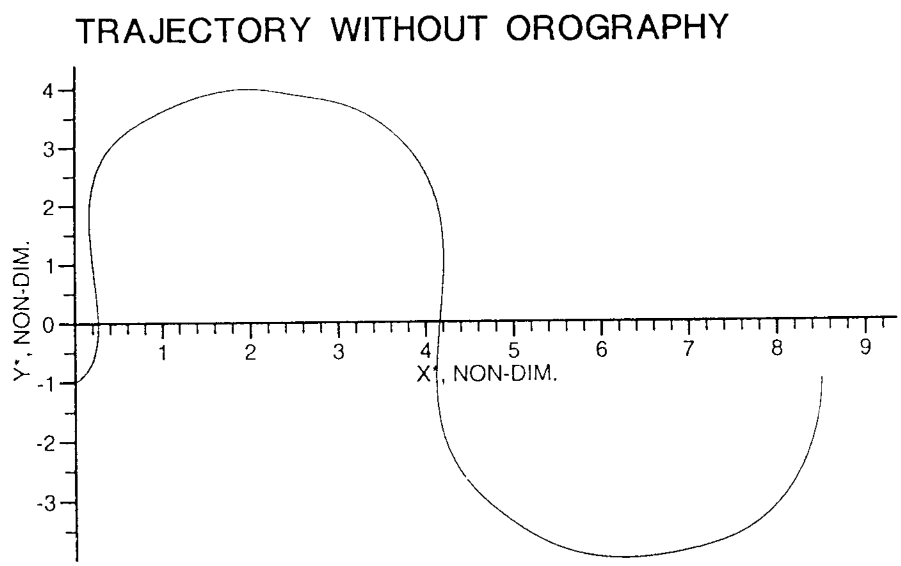



Fig. 3b. As Fig. 3a, but without the mountain.

In the simple case illustrated in Figure 2 and Figure 3 it is straightforward to integrate the equation for $\alpha$. In particular, when we calculate the angle of the wind direction at the northern edge of the mountain range we find that

$$\cos(\alpha_N) = 2\frac{\beta W^2}{V} + \frac{(f_o - \beta W)}{V} * \frac{h_m}{H_o} \tag{6}$$

Eq. (6) can be used to calculate the minimum speed required if the parcel should cross the mountain. The requirement is that the cosine function at the northern edge is less than unity. For a width of 200 km, a maximum mountain height of 4 km, the height of the atmosphere equal to 10 km we find a minimum speed of 4.3 $\mathrm{ms}^{-1}$.

Among the mountains of the Earth which can be considered in these elementary considerations are, in addition to the Pyrenees, the Alps and the mountains complexes forming the Himalayas in Asia. A qualitative understanding can be achieved using the methods developed in this section where an equation equivalent to (4) was used by Bolin (1950) in considering a narrow current crossing the Rocky Mountains. However, only a limited quantitative application can be made of the equations due to the severe assumptions of which the restriction to the curvature effect in the vorticity probably is the most severe. In the following sections we shall take a more general approach to the problems.

## 3. Mountains in an equivalent barotropic model

The orography effect enters an atmospheric model through the lower boundary condition. Considering the forced flow due to the mountains and requiring that the velocity component perpendicular to the lower surface shall vanish we may write the condition in the form:

$$W_m = \vec{v}_o \cdot \nabla h \tag{7}$$

The forced vertical velocity depends therefore on the slope of the lower surface an the horizontal wind at the level of the surface. In this section we shall make use of a coordinate system with pressure as the vertical coordinate. It is thus necessary to convert the vertical velocity in (7) to the appropriate quantity in this system. We do this by noting that

$$\frac{f_o}{p_o}\omega = -\frac{gf_o}{RT_o}\vec{v}_o \cdot \nabla h \tag{8}$$

where $g$ is the acceleration of gravity, $f_o$ a standard value of the Coriolis parameter, $R$ the gas constant for air and $T_o$ a standard value of the temperature close to the surface of the Earth. It has to be decided how one relates the surface wind to the other parameters in the model. As indicated in the title of this section we shall make use of the equivalent barotropic model in this section. Such a model is built on the assumption that the horizontal wind at the various levels is a function of pressure only, i.e.

$$\vec{v} = A(p)\vec{v}_a \tag{9}$$

where $A(p)$ is given by a mathematical expression or calculated from data. Using eq. (9) it is clear that

$$v_o = A(p_o)\vec{v}_a \tag{10}$$

where it as usual has been assumed that the wind at 1000 hPa is a representative value of the wind close to the surface of the Earth. Following hereafter the standard procedure for the derivation of the basic vorticity equation for the equivalent barotropic model (see for example Wiin–Nielsen, 1972) we may write

$$\frac{\partial \varsigma}{\partial t} + \vec{v} \cdot \nabla(\varsigma + f) = -\Gamma \vec{v} \cdot \nabla h \tag{11}$$

where $\Gamma$ is defined by the expression

$$\Gamma = A(p_o)\frac{gf_o}{RT_o} \tag{12}$$

Assuming stationary conditions and non–divergence in the horizontal wind we may write eq. (12) in the form:

$$J(\Psi, f + \nabla^2\psi + \Gamma h) = 0 \tag{13}$$

where $\Psi$ is the streamfunction. The simplicity of eq. (13) is due to the adopted model and to the neglect of any form of friction. If eq. (13) shall be satisfied it is necessary that the two components in the Jacobian are proportional to each other which may be written in the form:

$$f + \nabla^2\Psi + \Gamma h = \mu^2\Psi \tag{14}$$

where $\mu^2$ is the proportionality factor. We shall use eq. (14) in the following, and we shall apply it on the sphere. The proportionality factor would normally be determined "upstream" or where the current meets the mountain, but such a procedure is not applicable in this case where we are trying to obtain a solution over the complete sphere. We note, however, that eq. (14) should apply in each point of the sphere, and it must therefore also apply for the average over the sphere. Now, both the Coriolis parameter and the relative vorticity vanish when averaged over the sphere. Denoting the area average by an overbar we find from (14) that

$$\mu^2 = \frac{\Gamma \overline{h}}{\overline{\Psi}} = \frac{f_o \Gamma \overline{h}}{g \overline{z}} \tag{15}$$

In (15) it has been assumed that the streamfunction is related to the height field by the simple relation $\psi = gz/f_o$. Since the equivalent barotropic model normally is applied at the 500 hPa surface we shall use a standard value for the height of this surface to calculate the numerical value of the proportionality factor. The area average of the orography depends on the assumptions, but is should in any case be calculated with respect to the area extent of the mountains under consideration.

In the remaining part of this section it is the intention to specify the mountains in various ways by analytical expressions and to use eq. (14) to calculate the streamfunction as a function of longitude and latitude over the whole globe. The windfield described by the streamfunction will, due to the method of calculation, be in equilibrium with the assumed orography.

For a given specification of the orography by the function $h(\lambda,\ \varphi)$ we may calculate the proportionality factor and proceed to solve eq. (14). One may select to solve this equation by a use of finite differences on a suitable grid, or one may want to use an expansion of the streamfunction and the orography in series of associated Legendre functions. Both procedures have advantages and disadvantages. A single isolated mountain may be difficult to represent with accuracy using the spherical harmonic functions, but the saving factor is in this case that the calculated amplitude of the spherical harmonics decrease rather fast as the spherical planetary wave number, i.e. $n(n+1)$, increases. As long as we are satisfied with the large scale response to the orography the method of series expansion work well, and it has been adopted in all the calculations presented later in this paper. Before we enter the just announced program for the remaining part of this section we shall make a brief deviation to consider time-dependent solutions.

### *3.1 Influence of mountains on Rossby–Haurwitz waves*

The classical Rossby–Haurwitz solution of the non-divergent vorticity equation is obtained on the sphere by assuming a basic current with a constant angular velocity $\dot{\lambda}$· from the west. Solving the linearized vorticity equation using spherical harmonic functions one obtains the famous formula:

$$c = \dot{\lambda} - \frac{2(\Omega + \dot{\lambda})}{n(n+1)} \tag{16}$$

where the denominator in the last term is the square of the spherical wave number. One may ask how the wave speed is influenced by the orography of the Earth. This question cannot normally be answered by using linearization procedures because the mountains are time-independent. However, in the special case where the mountain height does not depend on longitude, it turns out that one may obtain solutions. We shall demonstrate this procedure by using eq. (14) including mountains. Denoting $\eta = \sin(\varphi)$ and using the same basic current as in the classical solution we linearize and obtain the equation:

$$\frac{\partial \varsigma}{\partial t} + \lambda \left( \frac{\partial \varsigma}{\partial \lambda} + \Gamma \frac{\partial h}{\partial \lambda} \right) + \frac{1}{a^2} \left( 2(\Omega + \lambda) + \Gamma \frac{\partial h}{\partial \eta} \right) \frac{\partial \Psi}{\partial \lambda} = 0 \tag{17}$$

where $\varsigma$ and $\psi$ are perturbation quantities. It is seen that the term containing the derivative of the mountain height does not fit into a perturbation scheme. Restricting ourselves to mountains depending on latitude only we may however proceed in the usual way. The streamfunction is developed in a series of spherical harmonic functions containing the factor exp $(\mathrm{im}(\lambda - ct))$ where $c$ is the phase speed. We find then that the problem reduces to a standard eigenvalue problem of the form:

$$\left( \lambda - \frac{2(\Omega + \lambda)}{n(n+1)} - c \right) \Psi(m,\ n) - Q(m,\ n) \sum_{m+1}^{n\ max} I(q,\ n) \Psi(m,\ q) = 0 \tag{18}$$

In this equation we have introduced the following notations:

$$Q(m,\ n) = \Gamma \frac{2n+1}{2} \frac{(n-m)!}{(n+m)!} \tag{19}$$

and

$$I(q,\ n) = \int_{\eta_o - \Delta}^{\eta_o - \Delta} \frac{dh}{d\eta} P_n^m(\eta) P_q^m(\eta) d\eta \tag{20}$$

Only numerical procedures can be used to solve the problem, but a standard eigenvalue procedure will suffice. The only remaining point is to specify the form of the mountain. To pick something which is rather easy to work with we have selected:

$$h = h_m \cos^2 \left( \frac{\pi}{2} \frac{\eta - \eta_o}{\Delta} \right) \tag{21}$$

In (21) $h_m$ is the maximum height at $\eta = \eta_o$, while $\Delta$ is the width of the mountain. In using the formulation given above it should be remembered that the longitudinal wave number is fixed in each calculation. The integrals entering the formulation were calculated by numerical means using a trapezoidal form with good resolution. Several calculations were made. The main result is that the influence of the mountains as specified by eq. (21) is of a minor nature only when $\Delta$ is small. Figure 4 shows the difference in the wave speed between the mountain case and the case of the wave speed of the non-divergent wind. Expressed in longitudes per day it amounts to less than one of these units. The order of magnitude does not change for relatively small changes in the width of the mountain range or in the height of the mountain.

This result depends, however, on the horizontal scale of the mountain, but, in agreement with earlier investigations (Bolin, 1950), the scale has to be large to have a major impact. We may see this by going to an extreme case. Suppose that we were to replace eq. (21) by the expression $h = h_m \eta$. This expression represents the largest possible scale on the planet. It has a constant derivative, and the integration has to be extended over the whole globe. A direct solution is possible in this case, because we can use the orthogonality of the spherical harmonic function. We get:

$$c = \lambda - \frac{2(\Omega + \lambda) + \Gamma h_m}{n(n+1)} \tag{22}$$

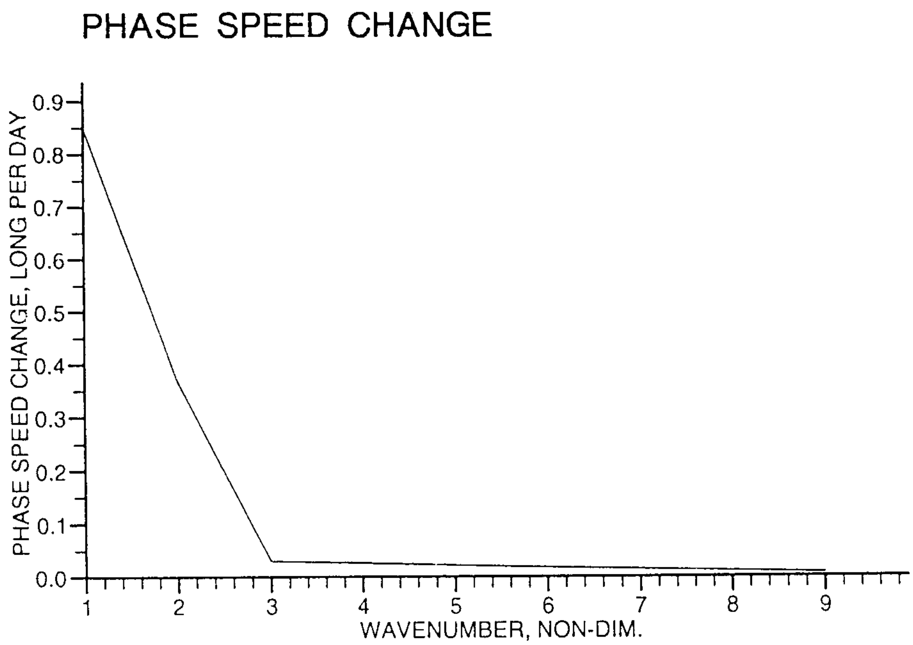



Fig. 4. Difference in phase speed, measured in longitude per day, with and without the mountain as specified by eq. (21).

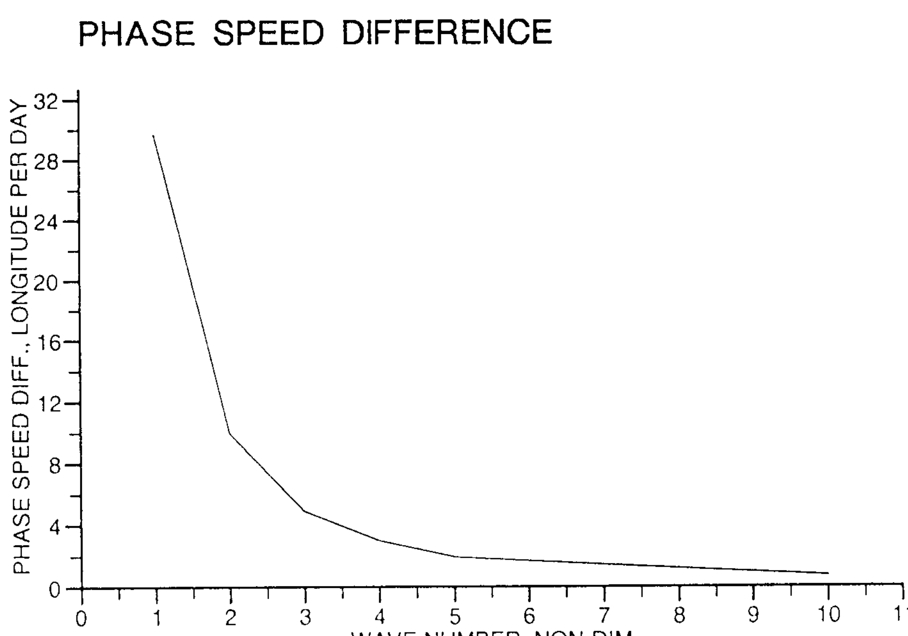



Fig. 5. As Fig. 4, but for a mountain described by $h = h_m \eta$.

Figure 5 shows the difference between the wave speeds with and without the mountains for $h_m = 1.0 \times 10^3$m expressed in the unit long/day. We find now a considerable difference for small values of the wave number $n$, but the difference decreases rapidly as $n$ increases. Another example has been computed. For this example we have selected the following height profile:

$$h = 27 h_m \eta^2 (1 - \eta^2) \tag{23}$$

where we have a vanishing height at both poles and at the Equator. In both hemisphere there is a single maximum which occurs at about 35 degrees of latitude. The result is shown in Figure 6 for the same value of the maximum height and for a longitudinal wave number 1 as in the previous figure. We note again a considerable reduction in the retrogression of the very long waves, but a vanishing influence on the small meridional scales.

We may thus conclude that on a planet with a large scale mountain whose position is a function of latitude only we find a considerable reduction of the wave speeds for the truly planetary motion, while the change in the same parameter for the small meridional scales is negligible.

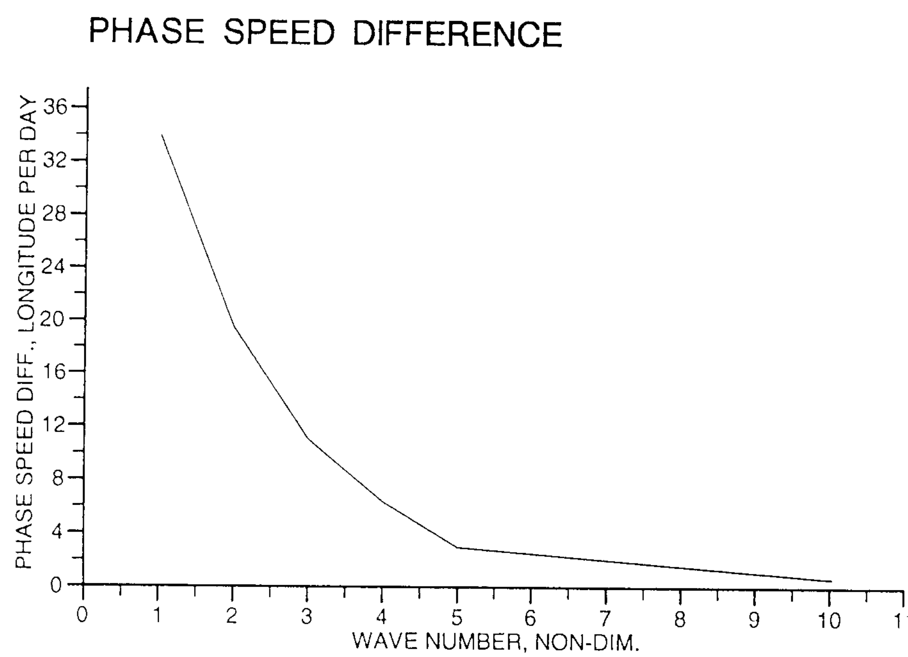



Fig. 6. As Fig. 4, but with the mountain specified by eq. (23).

*3.2 Equilibrium conditions*

We return to the problem of finding the stationary motion which is in equilibrium with the orography. As a first example we take the specification of the height given by eq. (21). In this case where the orography specification is independent of longitude it is seen that the resulting streamfunction can be a function of latitude only. The problem is solved by developing the orography as well as the streamfunction in series of ordinary Legendre polynomials and solving for the amplitudes of the components of the streamfunction. At the end the streamfunction is obtained as a function of latitude by summing the series. An example is shown in Figure 7. In this calculation it was assumed that the mountain was centered at 30 N with a width of about 300 km and a maximum height of 2000 m. $z_M$ was taken to be 5500 m and the coefficient $A_o$

as low as 0.1. A similar mountain is placed at 30 S. Considering first the region between the two mountains we observe that the region between the Equator and the mountain to the north has easterlies while the region north of the mountain in the Northern Hemisphere has westerlies. The rapid change from easterlies to westerlies takes place over the mountains. The wind field is a–symmetric around the Equator. The maximum zonal wind is only 3.8 ms$^{-1}$, but since it is proportional to the height of the mountain and to the coefficient $A_o$ it could easily be increased by a factor of 4 to 5.

A result of a similar nature can be obtained in a channel as seen in Figure 8. In this figure it has been tried to keep the conditions as close as possible to those of Figure 7 by selecting the same width, height and position of the mountain and selecting a channel width equal to the distance from Equator to Pole. From the previous examples it may thus be concluded that characteristic wind systems are necessary to stay in equilibrium with the orography as long as the height of the mountain is a function of latitude only.

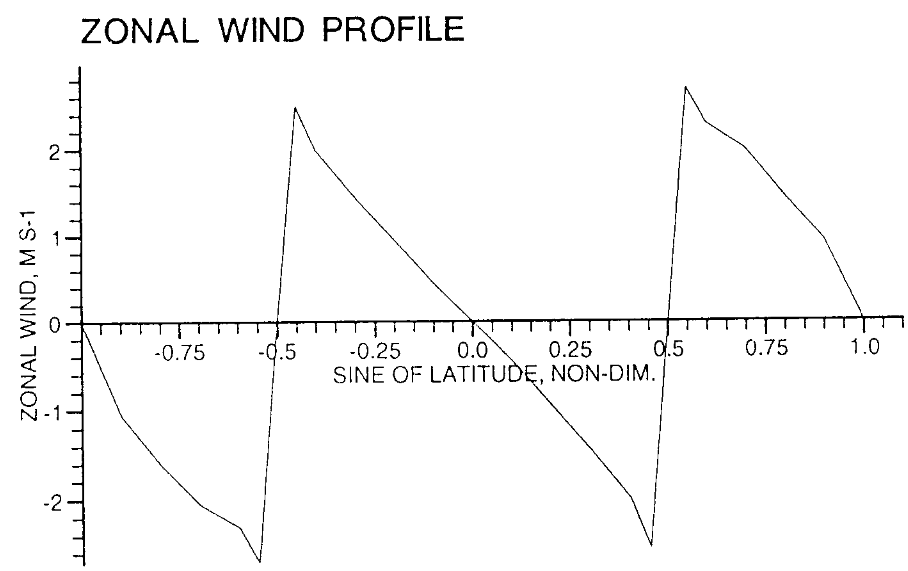



Fig. 7. The zonal wind profile as a function of latitude for two mountains centered at 30 S and 30 N with a width of about 300 km and a maximum height of 2000 m.

In the following we shall look at some examples of isolated mountains where the dependence on longitude comes in as well. To solve this problem it is necessary to extend the integrations over longitude and latitude and therefore to use the spherical harmonic functions. A calculation has been made for the following specification of the mountain:

$$h - h_m \cos^2\left(\frac{\pi}{2}\frac{\lambda - \lambda_o}{\Delta_1}\right)\cos^2\left(\frac{\pi}{2}\frac{\eta - \eta_o}{\Delta_2}\right) \qquad (24)$$

where the notation are analogous to those used earlier.

Using (24) we proceed to solve eq. (14) by developing the mountain heights and the unknown streamfunction in terms of spherical harmonics. The unknown amplitudes for the streamfunction are determined from the linear equation, and the field representation is obtained by summing the series. Since standard procedures can be used in the whole procedure, it will not be necessary to reproduce the details of the mathematics involved. In using (24) we have to specify the maximum height and the horizontal dimensions of the mountain as given by the other parameters. Most of the calculations have been carried out with the following values:

$$h_m = 4.0 \times 10^3 m;\ \Delta_1 = \Pi/6;\ \Delta_2 = \Pi/18 \tag{25}$$

Figure 9 shows the computed values of the zonal average of the streamfunction as a function of latitude. Figure 10 contains the corresponding zonal winds. The mountain was in this case centered at 30 N, and it is thus seen that the effect of the mountain in the equilibrium solution is to create westerlies north of the mountain and easterlies south of it.

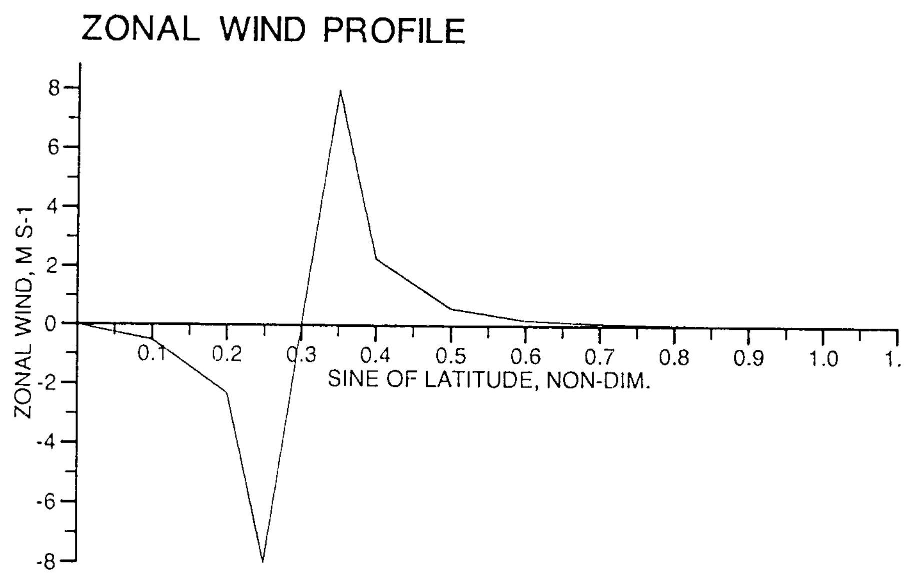



Fig. 8. An example similar to the one in Fig. 7, but calculated in a channel flow, where the mountain is centered at 0.3.

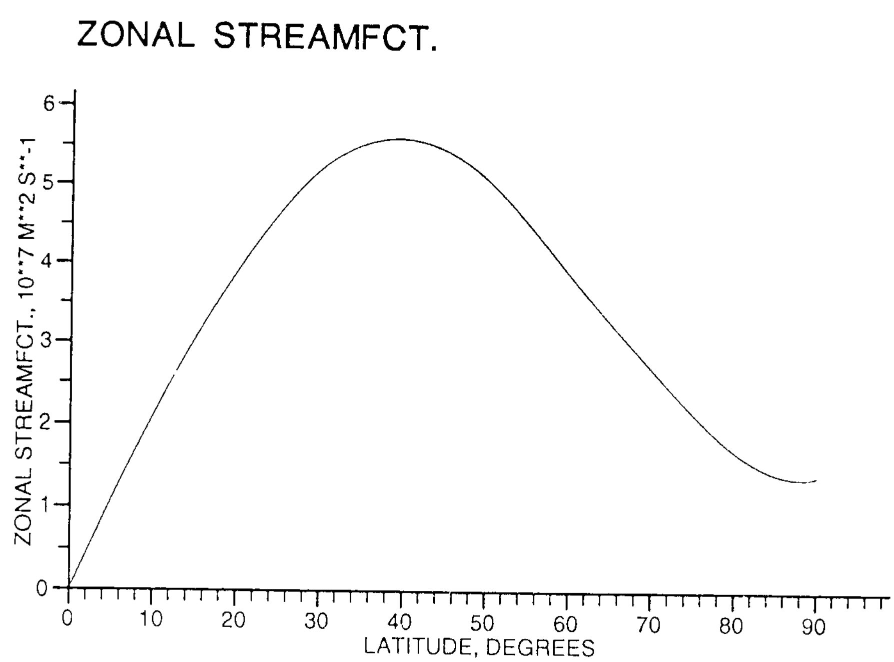



Fig. 9. The zonal streamfunction resulting from a mountain as specified in eq. (24) with parameters as given by (25) and centered at 30 N.

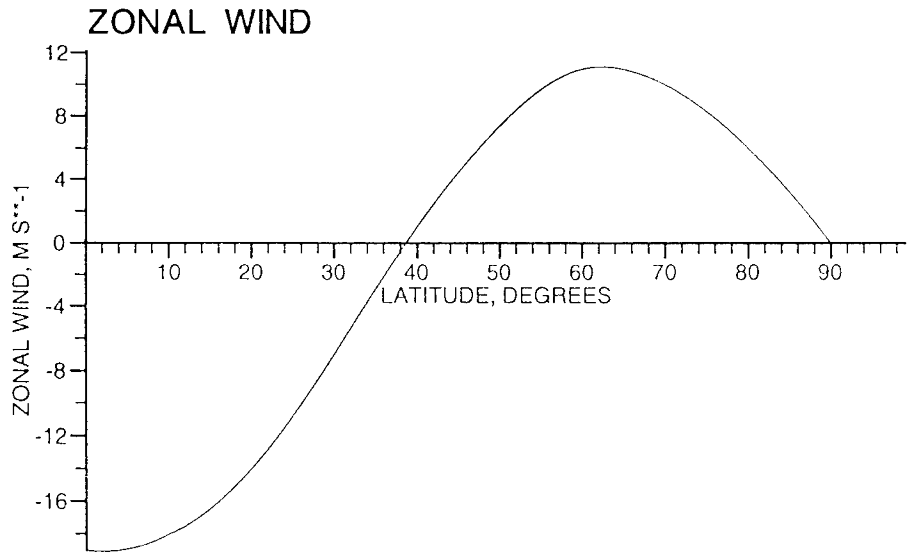



Fig. 10. The zonal wind corresponding to Fig. 9.

## 4. The two–level model

The considerations in Section 3 are easily extended to the normal two–level, quasi–geostrophic model. The reason is that, just as in the equivalent barotropic case, we can write the governing equations for the two–level case in a conservative form when we disregard the external heating. The neglect of the external heating is natural in our case where we are trying to isolate the equilibrium conditions between the orography and the atmosphere, but it is of course not justified for the Earth.

The parameterization of the orography effect follows the presentation given in Section 3, but also in this case it is required to relate the wind at the mountain level to the wind at one of the two levels. It seems natural to use the assumption that the surface wind is half of the wind at the lower level in the model, i.e. at 750 hPa. With this assumption we may write the two equations in the form:

$$f + \nabla^2\Psi_1 - q^2(\Psi_1 - \Psi_3)/2 = -\mu_1^2\Psi_1 \tag{26}$$

$$\Gamma h + \nabla^2\Psi_3 + q^2(\Psi_1 - \Psi_3)/2 = \mu_3^2\Psi_3 \tag{27}$$

where the standard parameters for the two–level model are

$$\Gamma = \frac{g f_o}{R T_o}; \quad q^2 = \frac{2 f_o^2}{\sigma P^2} \tag{28}$$

Equations (26) and (27) express the conditions for equilibrium between orography and the model atmosphere in a stationary state. The first problem is to obtain the numerical values for the two parameters which are introduced as proportionality factors between the conservative quantities at the two levels and the streamfunctions at the same levels. The procedure is the same as in Section 3. We average both of the equations over the whole globe and note that the

Coriolis parameter and the Laplacians average to zero. We get then

$$q^2(\overline{\Psi}_1 - \overline{\Psi}_3)/2 = \mu_1^2\overline{\Psi}_1 \tag{29}$$

$$\Gamma\overline{h} + q^2(\overline{\Psi}_1 - \overline{\Psi}_3) = \mu_3^2\overline{\Psi}_3 \tag{30}$$

where the overbar means a global average. These formulas are used to calculate the two proportionality factors. For this purpose we use a conversion to height by relating the streamfunction to the geopotential by

$$\overline{\Psi} = \frac{g\overline{z}}{f_o} \tag{31}$$

The resulting formulas are:

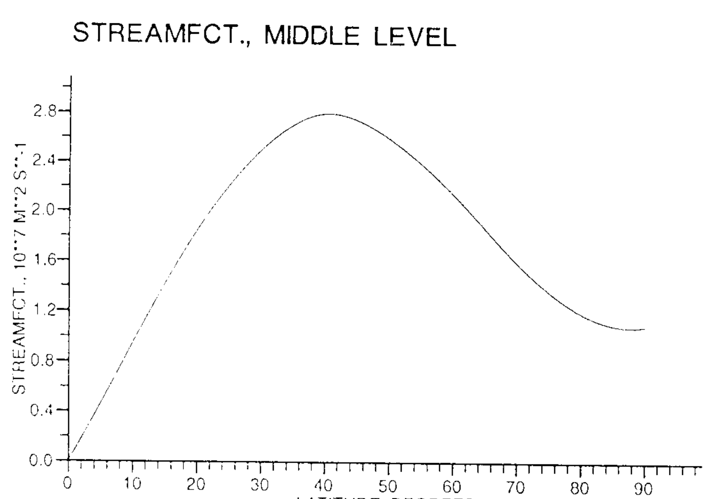
$$\mu_1^2 = q^2\frac{\overline{z}_1 - \overline{z}_3}{2\overline{z}_1} \tag{32}$$

$$\mu_3^2 = \Gamma\frac{f_o\overline{h}}{g\overline{z}_3} + q^2\frac{\overline{z}_1 - \overline{z}_3}{2\overline{z}_3} \tag{32}$$

Values taken for instance from the standard atmosphere are used to estimate the values of the geopotentials. By using such values and otherwise standard values for the other parameters it is seen that the first term in (32) is almost two orders of magnitude smaller than the second term, but there is no reason to disregard it since it is easily calculated from the given orography.

The remaining calculations are carried out by obtaining the spherical harmonic amplitudes for the orography from the specified or measured heights of the mountains followed by a calculation of the spherical harmonic amplitudes for the two streamfunctions from the equations (31) and (32). The field distributions of the two streamfunctions are finally obtained by summing the series of spherical harmonic functions. Since these two equations are linear, it is a standard matter to obtain these results.

11a



11b

11c

11d.

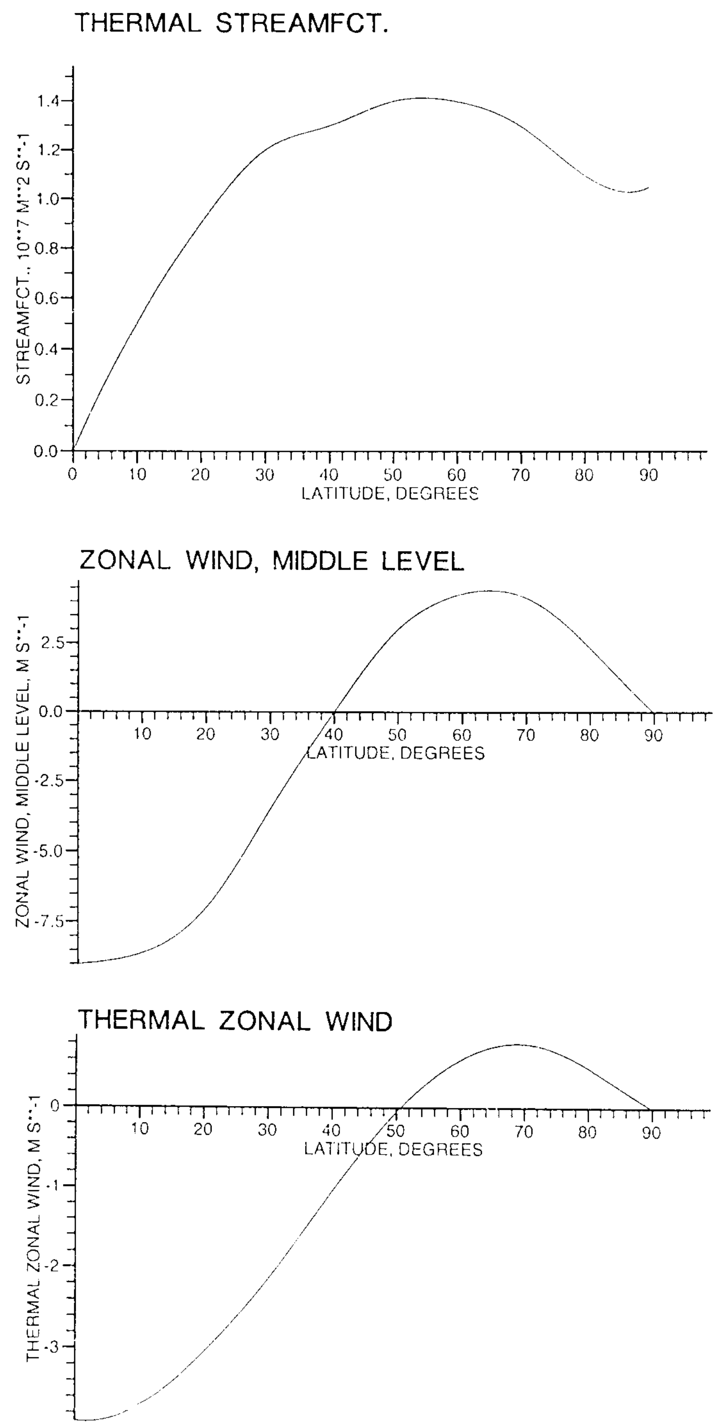
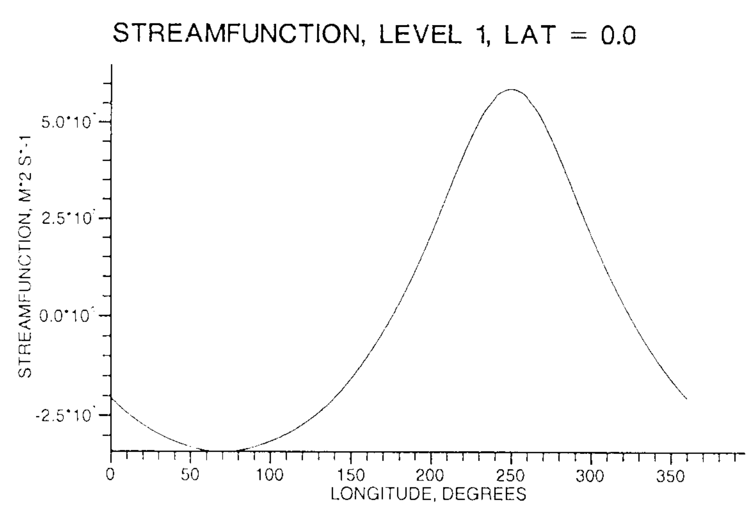



Fig. 11. (a) The middle level streamfunction for the mountain given by eq. (24); (b) the thermal streamfunction; (c) the zonal wind at the middle level; (d) the thermal zonal wind.

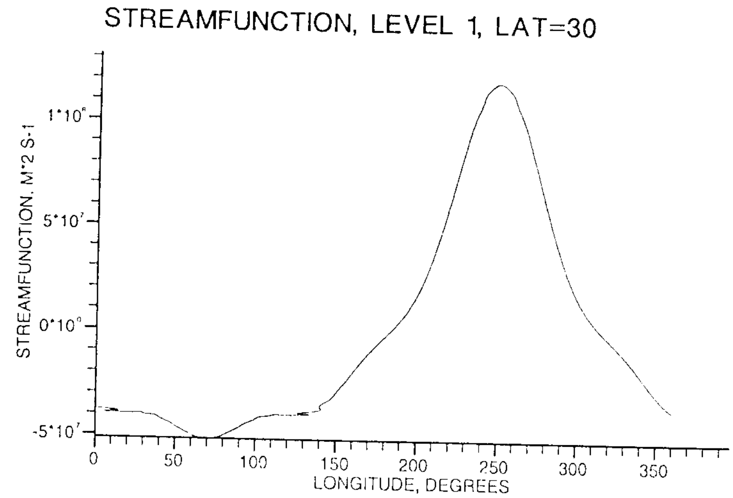
Our first calculation will use the same specification of an isolated mountain as in the equivalent barotropic calculation, see (24). Looking first on the zonally averaged part of the atmospheric state we see from Figure 11a that the streamfunction for the vertical mean flow has a maximum at the northern edge of the mountain resulting in a zonal wind profile as given in the lower part of Figure 11c. The maximum for the thermal streamfunction, see Figure 11b, is located somewhat further to the north at about 50 N. The corresponding profile of the thermal zonal wind is given in Figure 11d. Since the streamfunctions at the upper and lower levels are obtained as the sum and the difference, respectively, of the two curves in Figure 11 we see that the result at the two levels are of the same type as in the equivalent barotropic case resulting in easterlies to the south and westerlies to the north of the mountain. The strength of the zonal winds are somewhat smaller than in the earlier case, but this is due to the use of a rather large value (0.4) of $A_o$ in the barotropic calculation.

The eddy streamfunctions could have been presented again in the form of maps, but it has been preferred to show the profiles at selected latitudes. Figures 12 a, b, c, and d diplay the vertical mean of the streamfunction at selected latitudes (0, 30, 60 and 80 N) as a function of longitude. At each latitude we observe a ridge at the position of the mountain which is centered at 250 degrees of longitude. The minimum is found close to 70 degrees such that wave number one again is dominating. The largest values for the ridge are found at 30 N indicating that we obtain an anticyclonic circulation around the mountain. Removed 180 degrees of longitude from the mountain we observe a much less intense cyclonic circulation. An example of the corresponding curves for the thermal streamfunction are found in Figure 13. The amplitude is somewhat smaller, but the distribution is very similar to the one found in Figure 12. We can therefore conclude that the response will be larger at the upper level (the sum) than at the lower level (the difference) and also that the waves will be in phase in the vertical direction.

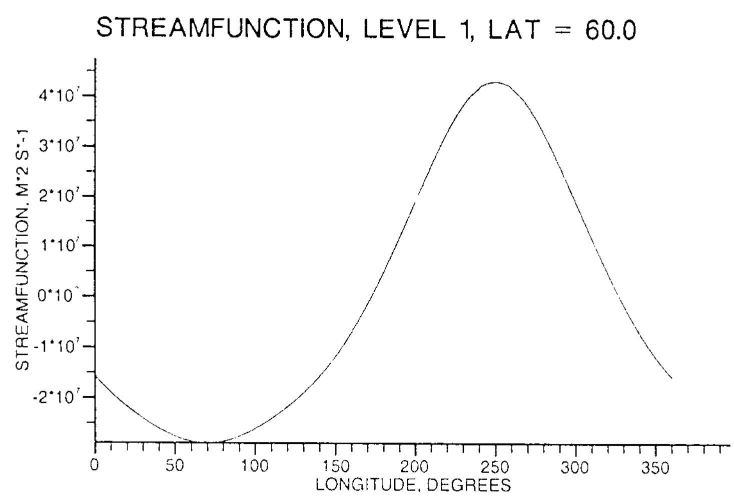
12 a



b




c




d

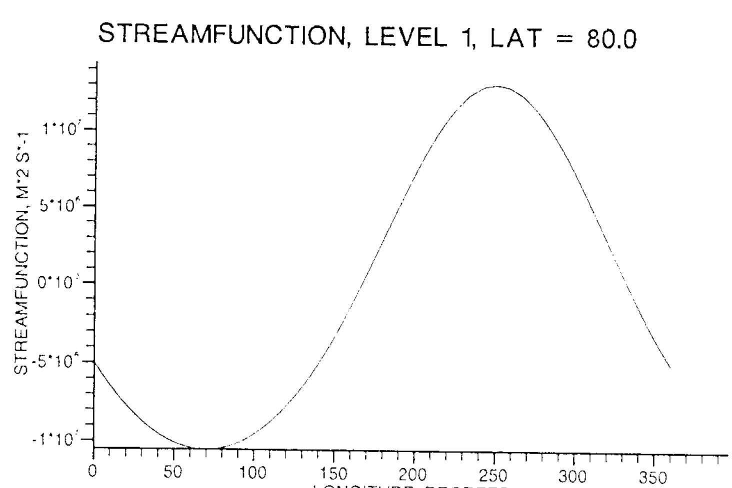



Fig. 12. (a) The streamfunction at the upper level at $\varphi = 0$. (b) The streamfunction at the upper level at $\varphi = 30$. (c) The streamfunction at the upper level at $\varphi = 60$. (d) The streamfunction at the upper level at $\varphi = 80$.

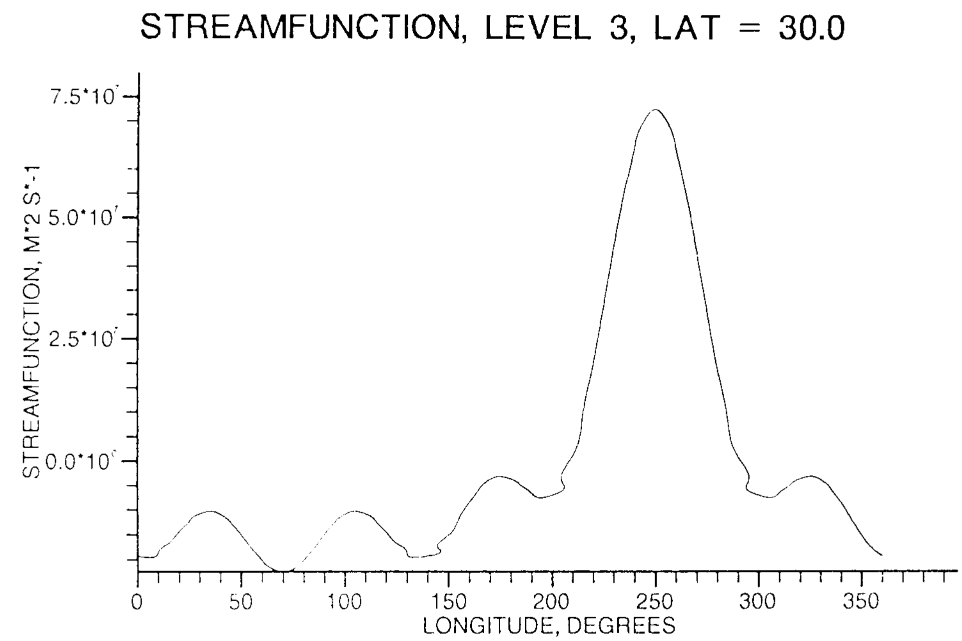



Fig. 13. The streamfunction at the lower level at $\varphi = 30$.

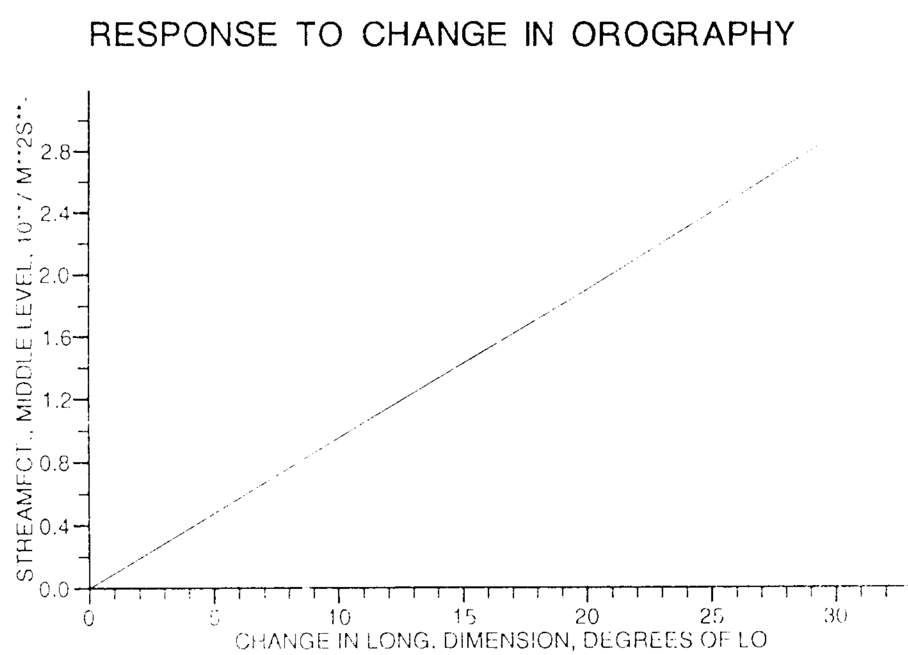



Fig. 14. The maximum value of the streamfunction at the middle level as a function of the longitudinal dimension of the mountain.

It is of interest to investigate the changes in the streamfunction due the dimensions of the mountain. Keeping the maximum height of the mountain in the same position (30 N) we may look at the changes in the streamfunction due to changes in the longitudinal and latitudinal dimensions. The latitude of the maximum value of the streamfunction is then also occurring at 30 N, and it is sufficient to consider this point. The value of the maximum is directly proportional to the maximum height of the mountain as one can see from the formulas. Figure 14 shows the increase in the maximum value of the streamfunction as the longitudinal dimension of the mountain increases. It is seen that an almost linear relation exists. Figure 15 displays in a similar way the dependence on increasing dimensions of the mountain in the latitudinal direction. Also here we note an almost linear response except for the very largest dimension where the mountain reaches from Equator to Pole.

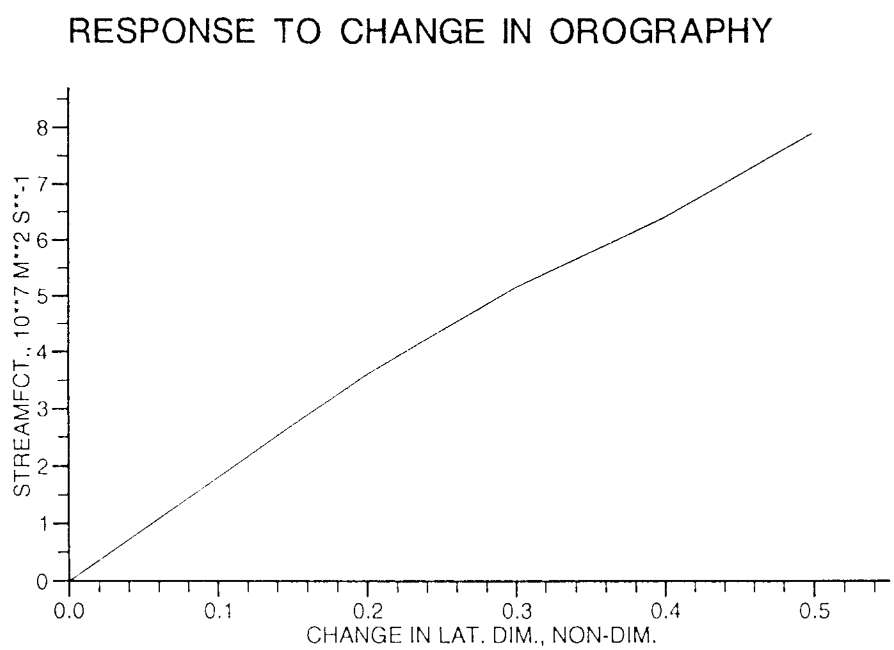



Fig. 15. The maximum value of the streamfunction at the middle level as a function of the latitudinal dimension of the mountain.

## 5. A global calculation

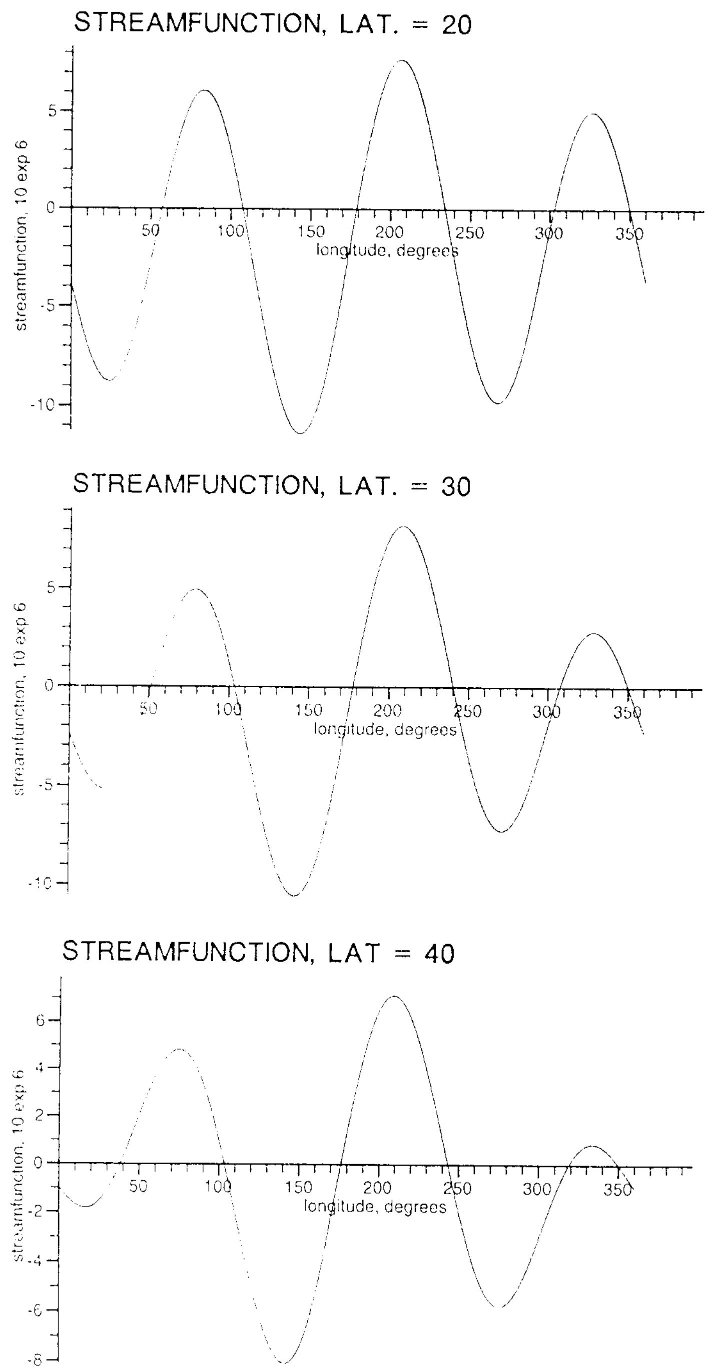
In this section we shall describe the results of a global calculation based on the orography of the Earth. The starting point is a representation of the orography in spherical harmonic amplitudes. Using the theory developed in the earlier sections we may compute the corresponding values of the equilibrium amplitudes of the streamfunctions, and the total field of the streamfunction is finally obtained by summing the double series of the spherical harmonic functions multiplied by the amplitudes. The present calculation is restricted to the equivalent barotropic case. It is on the largest scale only that we may assume that the assumption of stationarity may be approximately fulfilled. The results are therefore restricted to the values $m < 4$ and $n < 4$ for the associated Legendre functions included in triangular approximation.

The results are summarized in Figure 16 a to f where the eddy streamfunction in each case is shown as a function of longitude for a selected latitude. One notices a characteristic difference between the low and the higher latitudes. For the latitudes 20, 30 and 40 N the response shows three waves with troughs at about 3, 140 and 270 degrees of longitude. The first of these waves counting from Greenwich and eastward decreases in amplitude as latitude increases, and at 50 degrees north it has essentially disappeared. The same trough–ridge system is present in the observations and was also found by Charney and Eliassen (1949) in their calculation which applies to the channel centered around 45 N and with a width much smaller than the distance from the North Pole to the Equator.

16 a

b

c



d

e

f

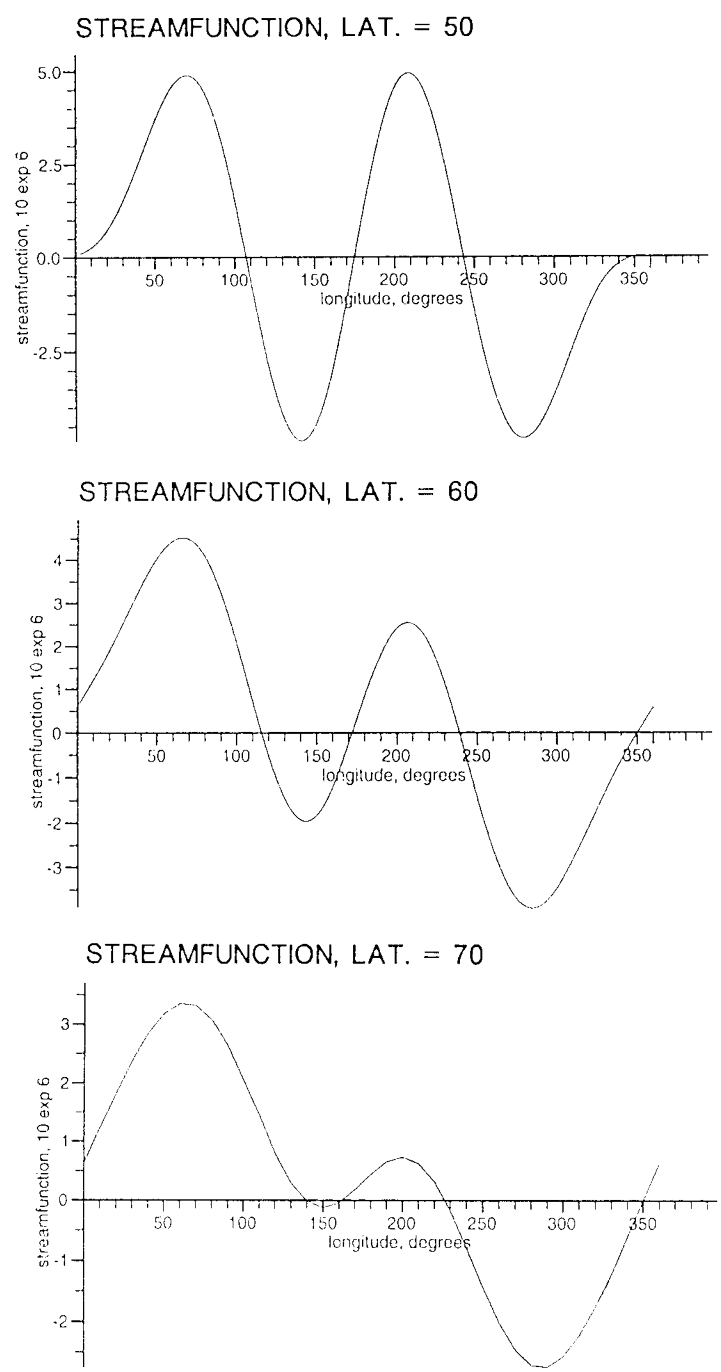



Fig. 16. (a) The streamfunction as computed from the equivalent barotropic model with $m < 4$ and $n < 4$. Latitude 20 N; (b) Latitude 30 N; (c) Latitude 40 N; (d) Latitude 50 N; (e) Latitude 60 N; (f) Latitude 70 N.

The second of the three waves corresponds to the marked trough observed in the western part of the Pacific ocean close to Japan with the ridge at about 200 E. The amplitude of this wave is large in the low and middle latitudes, but decreases markedly from 50 N and has a small amplitude only when the latitude has become 70 N. For the first two waves we observe only small changes in the positions of the trough and the ridge with respect to longitude.

The third of the three waves corresponds to trough–ridge system observed on the climatological maps at 500 hPa in the eastern part of North America and the Atlantic sector. The intensity of the trough decreases also in this case as latitude increases, but only to about 50 N. At 60 N the intensity is larger than at 50 N supposedly due to the effect of Greenland. We notice also for this trough that as latitude increases the trough is displaced eastward which is also in agreement with the presence of Greenland.

With repect to the computed values of the streamfunction we find that they are somewhat smaller than the observed climatological values. The latter are given in terms of geopotential heights, but a conversion is easily accomplished by multiplying the streamfunction values by $f_o/g$. For example, the observed depth of the trough close to Japan is -160 m while the computed value is 110 m only at 30 N. A much closer agreement is possible by using a larger value of the parameter $A(p_o)$ which in the present calculations was 0.1. Other authors have used a value as large as 0.4. However, the purpose of this study is not to vary the parameters to obtain the best possible agreement.

We find thus a qualitative agreement between the results of the equilibrium calculations and the observed features of the climatological maps restricting ourselves to the global scales. One may of course wonder why the agreement is qualitatively correct in the sence that the position of the various troughs and ridges agree with the climatological positions, since the standard theory involves both a given zonal current and a specification of the frictional dissipation. A direct comparison is not possible since it is known that the theory formulated by Charney and Eliassen (1949) is essentially a linear theory in which the zonal wind is specified, but more importantly this theory gives acceptable results only when the meridional scale (also specified) is relatively small (Derome and Wiin–Nielsen, 1971). The strength of the zonal wind and the intensity of the frictional coefficient have naturally also some influence, and these parameters can be used as tuning parameters for the solution. However, they have minor influence compared to the width of the channel.

On the other hand, in the present theory we need not specify such parameters, because the only parameters that are needed to calculate the proportionality factor are the mean height of the 500 hPa surface and the factor $A(p_o)$. It would thus appear that the present more general theory shows that the large scale stationary disturbances at 500 hPa can be explained in the first approximation as the perturbations necessary to stay in equilibrium with the orography. It is stressed that the theory can be a first approach only since the results apply to a climatological state. No seasonal variations can be incorporated.

## 6. Concluding remarks

The present paper is written primarily for pedagogical purposes. Much more complete investigations of the flow over various kinds of obstacles can be found in the literature cited in the references. It is felt, however, that it may be useful to present the problem of the equilibrium between the flow and the effects of orography in a pure form. It is for this reason that the investigation is kept as simple as possible, and it is deliberate that heat sources and dissipation has been neglected. The study is furthermore restricted to scales for which the quasi–geostrophic assumptions hold.

The main problem in the present investigation has been to determine the flow which is in equilibrium with the orographic influence. To isolate the influence of the mountains it has been necessary to neglect heat sources and dissipations. It has furthermore been assumed that the quasi–geostrophic assumptions can be applied. It has been stressed that although it is of interest to know the flow in the planetary atmosphere strictly due to the mountains, the reality of the solution will depend on how well the assumptions apply to the planet in question.

On the Earth we should not expect to observe the computed equilibrium flow because the differential heating between Pole and Equator is so large that baroclinic instability is dominating the atmospheric flow of the Rossby waves, while the differential heating is of about the same importance as the orographic effect on the planetary scale. A brief look at the other inner planets indicates that two of them, Mercury and Venus, have to be excluded. Mercury has almost no atmosphere, which makes it uninteresting in this connection, while Venus rotates so slowly that the quasi-geostrophic assumptions are unlikely to apply.

Some possibilities for application of the present theory seem to exist on Mars. It is generally assumed that the differential heating in a winter hemisphere is sufficiently strong to create baroclinic instability in the Martian atmosphere, but this may not be so in a summer hemisphere. The orography on Mars is sufficiently well known to permit a calculation of the equilibrium flow. It is furthermore known that the height of the orography is very large in places dominated by extinct vulcanoes, which may reach about 25 km into the Martian atmosphere.

For Mars it is furthermore known that the center of gravity does not coincide with the geometrical center of the planet. One may consider this peculiar feature as a special orographic effect that in the most simple case will give rise to an equilibrium flow that should be dominated by wave number one. The problem of determining the equilibrium flow on Mars will be reported in a future contribution.

The equivalent barotropic flow is in the first approximation independent of the heat sources. It may therefore be of some interest in future investigations to calculate the equilibrium flow for the whole globe using this model for a number of planets, where the orography is known.

In the present contribution the emphasis has been on the procedures to calculate the equilibrium flow on the sphere. This has been done for the equivalent barotropic atmosphere and for a two-level quasi-geostrophic model. Two procedures have been used. The first emphasizes the Lagrangian approach where an approximate trajectory is calculated. This has been done under the assumption that the vorticity expresses itself as curvature and not as shear. Trajectories over west-east oriented mountains have been emphasized because in that case it is possible to include the beta-effect in an easy manner. These trajectories have been compared with the trajectories which would exist if the mountains were removed, i.e., the well known constant vorticity trajectories, with the result that the effect of the mountains is to decrease the latitudinal extent of the paths.

The second and more general procedure is to calculate the streamfunction field created by the mountains under steady state conditions. For this purpose we have used mountains specified by relatively simple mathematical expressions. The zonal as well as the eddy parts of the flow has been considered. The results have been obtained for isolated mountains and for configurations reaching around the whole globe. In the resulting zonal flow we observe a westerly flow north of the mountain and an easterly flow to the south of the obstacle. The zonal flow is the only component created by a mountain which varies in latitude only. The steady state eddy flow has an anticyclonic circulation centered on the peak of an isolated obstacle with a weaker, but broader cyclonic flow removed about 180 degrees of longitude from the peak of the mountain.

The general description given above for the equivalent barotropic case applies also to the results obtained from the two-level model, but with the important addition that the thermal flow behaves in a similar way. We may therefore conclude that the effect of the mountains increases with altitude also in the equilibrium cases treated in the present study. The dimensions of the mountain are important for the strength of the resulting equilibrium flow. We find in this regard an almost linear dependence.

As indicated above the future work should be calculations of the equilibrium flow on various planets considering the real topography. A calculation of basic interest would be one in which the heating of the planet would be kept at a level low enough to prevent baroclinic instability. One can therefore imagine using a general circulation model for the Earth where the heating is reduced, but where the mountains are represented in the usual way to simulated the circulation which would exist if for some reason the temperature difference between Pole and Equator were greatly reduced as compared to the present values.

## REFERENCES

Bolin, B., 1950. On the influence of the Earth's orography on the general character of the westerlies, *Tellus*, **2**, 184-195.

Buzzi, A. and A. Speranza, 1979. Stationary flow of a quasi-geostrophic, stratified atmosphere past finite amplitude obstacles, *Tellus*, **31**, 1-12.

Charney, J. S. and A. Eliassen, 1949. A numerical method for predicting the perturbations of the middle latitude westerlies, *Tellus,* **1**, No. 2, 38-54.

Charney, J. G. and J. G. De Vore, 1979. Multiple flow equilibria in the atmosphere blocking, *J. Atmos. Sci.*, **36**, 1205-1216.

Charney, J. G. and D. M. Strauss, 1980. From drag instability, multiple equilibria and propagating planetary waves in baroclinic, orographically forced, planetary wave systems, *J. Atmos. Sci.*, **37**, 1157-1176.

Derome, J. F., 1984. On quasi-geostrophic, finite amplitude disturbances forced by topography and heating, *Tellus*, **36A**, 313-319.

Derome, J. F. and A. Wiin-Nielsen, 1971. The response of a middle-latitude model atmosphere to forcing by topography and stationary heat sources, *Mon. Wea. Rev.*, **99**, 564-576.

Egger, J., 1972. Numerical experiments on cyclogenesis in the Gulf of Genoa, *Beitr. Phys. Atmos.,* **45**, 320-346.

Egger, J., 1974. Numerical experiments on lee cyclogenesis, *Mon. Wea. Rev.*, **102**, 847-860.

Egger, J. and K. Fraedrich, 1987. Topographyic Rossby waves over Antarctica, *Tellus*, **39A**, 110-115.

Eliassen, A., 1980. Balanced motion of a stratified, rotating fluid induced by bottom topography, *Tellus*, **32**, 537-547.

Eliassen, A. and S. Thorsteinsson, 1984. Numerical studies of stratified airflow over a mountain ridge on the rotating earth, *Tellus*, **36A**, 172-186.

Frederiksen, C. S. and J. S. Frederiksen, 1988. The application of spectral techniques in a study of the stability of orographically forced planetary waves. Computational Techniques and Applications, CTAC-87, J. Noye and C. Fletcher, Eds., North Holland.

Frederiksen, C. S. and J. S. Fredericksen, 1989. Flow over topography and instability on beta-planes: the effects of different expansion representations. *J. Atmos. Sci.*, **46**, 1664-1686.

Frederiksen, C. S. and J. S. Frederiksen, 1991. Flow over topography and instability on the sphere. *J. Atmos. Sci.*, **48**, 2411-2425.

Frederiksen, J. S. and B. L. Sawford, 1981. Topographic waves in nonlinear and linear barotropic models, *J. Atmos. Sci.*, **38**, 69-86.

Frederiksen, J. S., 1982. Eastward and westward flows over topography in nonlinear and linear barotropic models, *J. Atmos. Sci*, **39**, 2477-2489.

Fultz, D. and R. R. Long, 1951. Two-dimensional flow around a circular barrier in a rotating spherical shell, *Tellus*, **3**, 61-68.

Hayes, J. L., R. T. Williams and M. A. Rennick, 1987. Lee cyclogenesis, Part I: Analytical studies, *J. Atmos. Sci.*, **44**, 432-442.

Hart, J. E., 1979. Barotropic quasi–geostrophic flow over anisotropic mountains, *J. Atmos. Sci.*, **36**, 1736-1746.

Hartjenstein, G. and J. Egger. 1990. Frontogenesis near steep orography, *Tellus*, **42A**, 259-269.

Jacobs, S. J., 1964. On stratified flow over bottom topography, *J. Marine Res.*, **22**, 223-235.

Jones, O. K., 1970. The flow of a stratified fluid over a vertical step, *Tellus*, **22**, 481-492.

Källén, E., 1983. A note on orographically induced instabilities in a two–layer model, *J. Atmos. Sci.*, **40**, 500-505.

Källén, E., 1985. On hysteresis–like effects in orographically forced models, *Tellus*, **37A**, 249-257.

Long, R. R., 1953. Some aspects of the flow of stratified fluids, I. A theoretical investigation, *Tellus,* **5**, 42-58.

Manabe, S. and T. Terpstra, 1974. The effects of mountains on the general circulation of the atmosphere as identified by numerical experiments, *J. Atmos. Sci.,* **31**, 3-42.

Mesinger, F. and R. F. Strickler, 1982. Effects of mountains on Genoa cyclogenesis, *J. Meteor. Soc. of Japan*, 60, 326-337.

Pierrehumbert, R. T., 1985. A theoretical model of orographically modified cyclogenesis, *J. Atmos. Sci.*, **42**, 1244-1258.

Queney, P., 1960. The airflow over mountains, *World Met. Org.* Technical Note, No. 34, Geneva.

Reuter, H. and H. Pichler, 1964. On the orographic influence of the Alps, *Tellus*, **16**, 40-42.

Saltzman, B., 1961. Note on the role of mountains in the energy budget of the atmosphere, *Tellus*, 13, 291-292.

Smagorinsky, J., 1963. General circulation experiments with the primitive equations, I. The basic experiment, *Mon. Wea. Rev.*, **91**, 99-164.

Smith, R. B., 1979. The influence of mountains on the atmosphere, *Adv. in Geophysics,* Academic Press, **21**, 87-230.

Thorsteinsson, S., 1988. Finite amplitude stratified airflow past isolated mountains on a f–plane, *Tellus*, **40A**, 220-236.

Tibaldi, S. and A. Buzzi, 1982. Effects of orography on Mediterranean lee cyclogenesis and its relation to European blocking, *Tellus*, **35A**, 269-286.

Trevisan, A. and A. Buzzi, 1980. Stationary response of barotropic weakly nonlinear Rossby waves to quasi–resonant orographic forcing, *J. Atmos. Sci.*, **37**, 947-957.

Warren, B. A., 1963. Topographic influences on the path of the Golf Stream, *Tellus*, **15**, 167-183.

Wiin–Nielsen, A., 1972. Dynamic meteorology, World Meteorological Organization, Technical Note, No. 364, 334 pp.

Woetman, N., 1985. As seen from METEOSAT–2 (in Danish: Set fra METEOSAT–2), Vejret, Vol. **7**, No. 1, 20-26.