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SHORT CONTRIBUTION

On the evaluation of the wet bulb temperature as a function of dry bulb temperature and relative humidity

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RESUMEN

Se propone un polinomio de grado tres para evaluar la presión de vapor de saturación (e_s), a fin de convertir la ecuación psicrométrica en una ecuación cúbica en Tw (temperatura de bulbo húmedo), puesto que no es posible expresar a Tw como una función directa de la temperatura de bulbo seco (T), la humedad realtiva (RH) y la presión (p). Dicha expresión para e_s es más precisa que otras semiempíricas reportadas en la literatura. Los resultados del cálculo de Tw mediante la ecuación cúbica se compararon con los valores arrojados por un método iterativo, resultando ser satisfactorios.

ABSTRACT

This paper proposes the use of a cubic polynomial to evaluate the saturation vapor pressure (e_s) in order to transform the psychrometric equation into a cubic equation in Tw (wet bulb temperature). This is done because it is not possible to write Tw as a direct function of dry bulb temperature (T), relative humidity (RH) and pressure (p). Statistical comparisons show that this polynomial is accurasier than other semi-epirical methods used to calculate e_s. Results of the third degree equation for Tw were compared whit resultant values of an iterative method and they were satisfactory.

1. Introduction

The Clausius-Clapeyron equation establishes a relationship between the saturation vapor pressure (e_s) and the dry bulb temperature (T). If the process involves only the vapor and water phases, the Clausius-Clapayron expression for temperatures between -10°C to 50°C is (Bindon, 1965):

$$e_s = \exp\left(\frac{21.4T + 494.41}{T + 273.15}\right) \tag{1}$$

where the e_s units are hectopascals (hPa).

It is possible to express the relative humidity (RH in %) in terms of the atmospheric pressure (p in hPa), the dry bulb temperature (T in °C), and the wet bulb temperature (Tw in °C) from the psychrometric relation:

$$e_{sw} - \frac{RH}{100} e_s = kp(T - Tw) \tag{2}$$

where e_{sw} is Eq. 1 evaluated in terms of Tw instead of T, and K is 1/0.622 times the ratio of the water vapor specific heat at constant pressure to the latent heat of condensation. The value of k however depends on the psychrometer type. Bindon (1965) has reviewed the k values of different psychrometer types obtained by various authors. A summary is presented in Table 1. The theoretical value of 6.53×10^{-4} °C^{-1} will be used in this paper for evaluation of psychrometric variables.

Table 1. Psychrometric constants for water (Kw) and ice (Ki) from some authors reported by Bindon (1965).

Kw (°C^{-1})	Ki (°C^{-1})	Ventilation	Table or slide rules
6.53×10^{-4}	5.703×10^{-4}	Almost 3 m/sec	Thermodynamic
6.62×10^{-4}		Adequate (greater that 2.5 m/sec)	Aspirations Psych. Preussischen Met. Ins., 1930.
6.56×10^{-4}	5.79×10^{-4}	Idem	Jelineks Psych. Jafeln, 1929.
12.0×10^{-4}	10.6×10^{-4}	Inadequate, Almost 0 m/sec	Jelineks Psych. Jafeln, 1929.
8.00×10^{-4}	7.06×10^{-4}	Inadequate, 1 to 1.5 m/sec	Jelineks Psych. Jafeln, 1929.
6.50×10^{-4}	5.80×10^{-4}	Idem	Meteorologie Nationale France, Nouvelles Tables, (1956).

If one attempts to evaluate Tw by sustituting Eq. 1 into Eq. 2 the problem is that Tw in the first member is affected by an exponential function and in the second by a linear function. So it is not possible to apply conventional algebraic methods in order to write Tw as a function of T and RH. This problem arises when explicit values of Tw are not given in the climatological records but T and RH values are readily available.

2. Vapor Pressure Evaluation

Equation 1 is not an exact expression for e_S. In comparison with experimental values reported by Byers (1959, p.158) the root mean squared error (RMSE) of Eq. 1 is 0.8 hPa between temperatures of -10°C to 50°C.

Here it is proposed the following polynomial expression:

$$e_S = aT^3 + bT^2 + cT + d \tag{3}$$

where $a = 6.6 \times 10^{-4}$, $b = 4.6 \times 10^{-3}$, $c = 4.58 \times 10^{-1}$, $d = 6.63$, and e_S units are hPa.

Other authors have proposed empirical or semi–empirical methods to evaluate e_S. Table 2 shows comparisons by means of the Pearson linear correlation coefficient (R) and the RMSE between experimental values of e_S (Byers, 1959) and some physical and empirical methods for each degree of temperature.

It is important to note that a non–exponential expression for e_S is an aid in the evaluation of Tw as a function of T and RH from Eq. 2. So, in view of the goodnes of the polynomial expression here proposed (Eq. 3), it is useful to find the function (f) for $Tw = f(T, RH, P)$.

Table 2. Comparison between the experimental values of e_s (Byers, 1959; p. 158), the Clausius – Clapeyron values and some empirical methods, from -10 to 50°C.

Author	Range (°C)	Pearson linear correl. coef.	RMSE (hPa)
Clausius-Clapeyron[1]	-10 to 50	0.9999	0.80
Adem, 1967[2]	-10 to 50	0.9998	0.80
Lowry and Lowry, 1989[3]	5 to 35	0.9996	0.70
Rosenberg, 1974[4]	-10 to 50	0.9982	1.00
Steadman, 1979[5]	-10 to 50	0.9997	0.60
Steadman, 1979	-10 to 40	0.9995	0.70
Equation 3	-10 to 50	0.9997	0.60

1) Equation 1.

2) $e_s = 6.115 + 0.42915\ T + 1.4206 \times 10^{-2}\ T^2 + 3.046 \times 10^{-4}\ T^3 + 3.2 \times 10^{-6}\ T^4$.

3) $e_s = 8.51 + 0.037\ T^2$.

4) $\log_{10} e_s = 0.82488 + 0.02604\ T$.

5) $e_s = 6.46 + 0.555\ T + 7.1 \times 10^{-4}\ T^3$

3. The Evaluation of Tw

3.1 Polynomial expression

By using Eq. 3 to calculate e_{sw} and Eq. 1 to calculate e_s, the psychrometric equation (Eq. 2) can be written:

$$Tw^3 + \frac{b}{a}Tw^2 + \frac{c+kp}{a}Tw + \frac{d-(RH/100)e_s - kpT}{a} = 0 \tag{4}$$

By sustitution of:

$$Tw = tw - \frac{b}{3a},$$

$$S = \frac{c+kp}{a} - \frac{1}{3}\left(\frac{b}{a}\right)^2$$

$$Q = \frac{2b^3}{27a^3} - \frac{b(c+kp)}{3a^3} + \frac{d-(RH/100)e_s - kpT}{a}$$

Eq. 4 can be rewritten so:

$$tw^3 + Stw + Q = 0 \tag{5}$$

The discriminant $[(Q/2)^2+(S/3)^3] > 0$ for atmospheric values near the ground. According the Cardano's method (Gellert *et al.*, 1975; p. 97-99) Eq. 5 has one real solution and two conjugate complex solutions. The real solution is:

$$Tw = \{-\frac{Q}{2} + [\frac{Q^2}{4} + \frac{S^3}{27}]^{1/2}\}^{1/3} + \{-\frac{Q}{2} - [\frac{Q^2}{4} + \frac{S^3}{27}]^{1/2}\}^{1/3} + \frac{b}{3a} \tag{6}$$

3.2 Iterative methods

In order to establish comparisons with the estimation made by means of the process of section 3.1, a computation program to evaluate Tw from Eq. 2 was implemented as follows:

$$Tw = T - \frac{e_{swo} - (RH/100)e_s}{kp}, \tag{7}$$

where T is the dry–bulb temperature in °C, RH is the relative humidity in %, e_s is the saturation vapor pressure (in hPa) found through Eq. 1, $k = 6.53 \times 10^{-4}$ C^{-1} and P is the atmospheric pressure in hPa. e_{swo} is the obtained value of Eq. 1 applied to an initial proposed value of $Tw_o = T - 5$ °C, wich can be approximated to correct Tw thus:

a) If $| Tw - Tw_o | \leq 0.1$ °C, then the evaluation of Tw is "correct";

b) If $(Tw - Tw_o) > 0.1$ °C, then a new value for Tw_o, i.e $Tw_o = T - 5$ °C $+N \times 10^{-3}$, is proposed where N is a whole number that indicates the order of iterative recurrence needed to obtain the condition (a).

c) If $(Tw - Tw_o) < -0.1$ °C, then $Tw_o = T - 5$ °C $-N \times 10^{-3}$ and the iteration runs until (a) is obtained.

4. Comparison Methods

The results of the application of the method showed in section *3.1* were compared and corrected by means of regular linear regression with the values of iterative recurrences of section *3.2*, for pressure levels of 1000, 950, 900, 850, 800, 750, 700 and 650 hPa, dry-bulb temperatures from -10 to +50°C for ranges of 1°C, and RH = 10, 20, 30,...90%.

Thus by substitution of constants and by the use of the above mentioned linear regression correlations the numerical expressions used in Eq. 6 were:

$$S = 662.23 + 0.97p \qquad (8a)$$

$$Q = 8264.65 - 1480.45(RH/100)e_s - 0.966pT \qquad (8b)$$

$$\frac{b}{3a} = -1^{\circ}C \qquad (8c)$$

For the 4392 points of comparison the correlation coefficient was of the order of 0.999 and the RMSE was 0.7 °C. Table 3 shows the goodness of fit as a function of the pressure levels.

Table 3. Goodness of fit of the polynomial method (Eqs. 6 and 8) for the evaluation of Tw at various pressure levels.

Pressure level (hPa)	Pearson linear Correl. Coef. (R)	Coef. of Determination (R^2)	RMSE (°C)
1000	0.9996	0.9991	0.4
950	0.9995	0.9990	0.5
900	0.9994	0.9988	0.5
850	0.9993	0.9985	0.6
800	0.9990	0.9980	0.6
750	0.9987	0.9974	0.7
700	0.9982	0.9966	0.9
650	0.9978	0.9956	1.0

5. Conclusions

Eq. 3 is a good expression for the evaluation of e_s for water-vapor phases between -10°C to 50°C. In view of the fact that this equation is not an exponential expression, it can be used to obtain Tw as a function of T, RH and p by means of the equations 6 and 8, with high goodness of fit, mainly at low pressure levels (below 850 hPa), where the RMSE is almost 0.5°C. The further advantage is that these procedures may be implemented with a pocket calculator.

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