Atmósfera (1995), 8, pp. 53-64

Orographically generated flow on Mars

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(Manuscript received July 5, 1994; accepted in final form September 14, 1994)

RESUMEN

El reporte discute el efecto sobre la atmósfera marciana de la distancia entre el centro de gravedad y el centro geométrico del planeta. Se obtiene una solución aproximada del problema por consideración de la superficie real como una orografía a gran escala con respecto a un planeta esférico de referencia.

La circulación resultante tiene un máximo de geopotencial en la región, donde el planeta real está arriba del planeta de referencia y un mínimo en el lado opuesto. Los vientos geostróficos resultantes tienen una rapidez de 10 a 30 ms^{-1}.

Un segundo problema es la determinación del flujo cuasi geostrófico estacionario que está en equilibrio con la orografía marciana. El cálculo está fundado en una especificación de la orografía marciana en funciones armónicas esféricas con una resolución triangular en 16 componentes. Esta es suficiente para el propósito presente, porque la contribución de los rasgos menores de la orografía a la configuración del flujo estacionario se vuelve despreciable.

ABSTRACT

The paper discusses the effect on the martian atmosphere of the distance between the center of gravity and the geometrical center of the planet. An approximate solution of the problem is obtained by considering the real surface as a large-scale orography with respect to a reference spherical planet. The resulting circulation has a maximum in the geopotential in the region, where the real planet is above the reference planet and a minimum in the geopotential on the opposite side. The resulting geostrophic winds have a strength of 10 to 30 $\mathrm{m\ s}^{-1}$.

A second problem is the determination of the stationary quasi-geostrophic flow which is in equilibrium with the martian orography. The calculation is based on a specification of the martian orography in spherical harmonic functions with a triangular resolution in 16 components. This resolution is sufficient for the present purpose because the contribution from smaller features in the orography to the stationary flow pattern become negligible.

1. Introduction

The present paper is a continuation of an earlier paper by Wiin-Nielsen (1994) on the equilibrium between a planet and its atmosphere. It was shown in the earlier paper how an equilibrium flow with the orography of the Earth can be calculated within the framework of quasi-geostrophic theory. Applications were made by using specific examples, and the paper continued by calculating the equilibrium solution for the Northern Hemisphere from an equivalent barotropic model, although the use of the theory was demonstrated also for a two-level quasi-geostrophic model. The computed quasi-geostrophic flow for the Earth is in reasonable agreement with the time-averaged observed flow at 500 hPa. In this paper we shall be concerned with Mars, its orography and its atmosphere.

According to Bills and Ferrari (1978) the geometrical center of the planet Mars is located at a position of $(r_o,\ \lambda_o,\ \varphi_o)$ relative to the center of gravity of the planet. The position of the centre of gravity will have an influence not only on the rotation of the planet itself, but also on the atmosphere around the planet. The first purpose is to give an estimate of the stationary circulation in the martian atmosphere created by the difference between the geometrical center and the center of gravity.

For this purpose we shall make use of the standard meteorological treatment of a planet approximated by a sphere, where gravity, i.e. the sum of the gravitational force and the centrifugal force, for practical purposes may be considered as a constant in the lower part of the atmosphere.

The problem may be considered as an effect of orography by considering the deviations from an idealized (spherical) shape of the planet rotating around an axis going through the center of gravity. The distance and orientation of the vector connecting the center of gravity with the geometrical center will - relative to the idealized shape - create negative orographical heights on one side of the planet and positive values on the other side. The dynamical effect of such a large-scale orography may be considered by paying attention to the forced vertical motion created by the orography and computed from the horizontal wind and the gradient of the slope of the orography.

The general problem of computing the influence of the orography of Mars on the circulation of the martian atmosphere is more difficult than the corresponding problem on the Earth because some isolated mountains on Mars are very tall and steep forcing the flow to go around the mountains rather than over them. The normal assumption made on the Earth that the main effect of the mountains is a forced vertical velocity that can be computed from the surface winds and the slope of the orography is therefore not applicable to these isolated features of the martian orography. On the other hand, if we restrict our interest to the global stationary waves, we know from the earlier calculations on Earth that only the large-scale orographical features will have a major influence on the stationary flow. These large-scale features of the Martian orography have moderate slopes, and we may therefore for this purpose apply quasi-geostrophic theory.

Applications of general circulation models adapted to Mars have been made by Leovy and Mintz (1969), but this calculation did not include the influence of orography. A later simulation study by Mass and Sagan (1976) emphasized on the other hand the influence of orographic features on the general circulation of Mars, since Gierasch and Sagan (1971) had estimated an increase in the windspeed by a factor 2-3 in certain localities. By comparing model integrations with and without orography they demonstrated that the orography generated winds may be as large as 40-60 m s^{-1}. A simulation by Pollack *et al.* (1976) made use of an improved version of the model applied by Leovy and Mintz (1969). One of the improvement was the addition of orography in the model. The results show very clearly the influence of orography since the computed winds are considerably larger than in the previous calculations. Webster (1977) has made of study of the low-latitude circulation on Mars using a simple, linear baroclinic model from which he calculates the response of the Martian atmosphere to the steady-state influence of the orography. He argues that the orography possesses both mechanical and thermal influences, where the thermal influence is a result of the temperature anomalies introduced by the mountains.

The most recent contribution comes from Joshi *et al.* (1994). They emphasize that the rather steep orography on Mars in certain situations may create western boundary currents. Such currents are mostly known in the oceans on Earth, but they are observed also in the atmosphere, the main example being the East African Jet. These boundary current depend not only on the orography, but also on the parameterization of the surface drag. The second purpose of this investigation is to provide an approximate determination of the stationary flow pattern forced by mechanical influence of the Martian orography. We shall restrict the calculation to an equivalent barotropic atmosphere, although it could easily be expanded to a two-level baroclinic, quasi-geostrophic model.

2. The problem

The axis of rotation of the planet goes through the center of gravity. The reference planet is defined as follows. We select a coordinate system, where the z-axis is along the axis of rotation, while x- and y-axes are in the plane perpendicular to the axis of rotation and going through the center of gravity. The coordinates of the *geometrical center* of the planet in the just defined system are denoted $(r_o,\ \lambda_o,\ \varphi_o)$. We shall describe the planet in the selected system. It is also approximated by a sphere with its center in the geometrical center of the planet. The two spheres will intersect each other. The real planet is described with respect to the reference sphere by giving the distance in the radial direction between the two spheres. In this way we may consider the shape of the real planet as orography on the reference sphere. In the selected coordinate system the equation for the reference sphere is:

$$x^2 + y^2 + z^2 = a^2 \tag{2.1}$$

where a is the radius of the planet. We may of course also describe the surface of the reference planet by using longitude and latitude as follows:

$$x = a \cos\varphi \cos\lambda$$

$$y = a \cos\varphi \sin\lambda$$

$$z = a \sin\varphi \tag{2.2}$$

The equation for the real planet in the reference system is:

$$(x - r_o \cos\varphi_o \cos\lambda_o)^2 + (y - r_o \cos\varphi_o \sin\lambda_o)^2 + (z - r_o \sin\varphi_o)^2 = a^2 \tag{2.3}$$

We insert the coordinates of an arbitrary position vector in the reference system in the equation for the real planet, i.e. (2.3). Let the coordinates be:

$$x = R \cos\varphi \cos\lambda$$

$$y = R \cos\varphi \sin\lambda$$

$$z = R \sin\varphi \tag{2.4}$$

We may solve eq. (2.3) for R. The solution is:

$$R = r_o T \pm (a^2 - r_o^2(1 - T^2))^{1/2} \tag{2.5}$$

where T is a notation for

$$T = \cos\varphi\cos\varphi_o\cos(\lambda - \lambda_o) + \sin\varphi\sin\varphi_o \tag{2.6}$$

We note finally that the height of the orography is:

$$h = R - a = r_o T - a + (a^2 - r_o^2(1 - T^2))^{1/2} \tag{2.7}$$

With respect to the two signs we can observe that the minus sign will have to be removed since R has to be a positive quantity. This has been done in (2.7).

The final formula has been used in the form in which it is written. We observe, however, that since $r_o << a$ in our case we could write (2.7) in the form:

$$\frac{h}{a} \approx \frac{r_o}{a}T - 1/2\frac{r_o^2}{a^2}(1 - T^2) \tag{2.8}$$

Having determined the orography we shall next address the question of how it may influence the atmospheric circulation of the planet. For the moment we shall be satisfied by considering the steady state circulation. This problem has been treated by Wiin-Nielsen (1994) for both an equivalent barotropic atmosphere and a two layer baroclinic atmosphere. We consider the first case where the circulation due to orography may be calculated from the equation:

$$J(\varsigma + f + \Gamma h, \Psi) = 0$$

$$\Gamma = \frac{g f_o}{R T_o} \tag{2.9}$$

It is thus seen that in order to apply (2.9) to a given planet it is necessary to know the gravity on the planet, its rotation rate, the composition of the planetary atmosphere resulting in a numerical value of the gas constant and a characteristic value of the surface temperature.

The solution of (2.9) is obtained by the same treatment as given by Wiin-Nielsen (*loc.cit.*) where one makes use of the fact that if (2.9) shall have more than the trivial solution of no motion it is necessary that the two quantities entering the Jacobian are proportional to each other giving

$$\varsigma + f + \Gamma h = q^2\Psi \tag{2.10}$$

where q^2 is the proportionality factor. It is determined from the fact that (2.10) should apply everywhere on the sphere and thus also for the average over the planet. Noting that both the relative vorticity and the Coriolis parameter average to zero, we find from (2.10) that the proportionality factor is determined from the following expression:

$$q^2 = \frac{\Gamma\overline{h}}{\overline{\Psi}} = \frac{\Gamma f_o}{g}\frac{\overline{h}}{\overline{z}} \tag{2.11}$$

where the overbar means an area average. In particular, the average of z means the averaged height of the isobaric surface corresponding to the equivalent barotropic level. In the atmosphere of the Earth this surface is normally approximated by the 500 hPa isobaric level. Assuming that the same principles will hold for the martian atmosphere we need an estimate of the mean height of the isobaric surface that has a pressure which is half of the surface pressure on the planet. On the Earth it is a reasonable assumption to use a standard value of the surface pressure equal to 1000 hPa, but on Mars we have only limited information of the surface pressure. It appears that the relative horizontal variation is larger on Mars than on the Earth, and that the pressure itself varies between 2 and 10 hPa. To estimate the height of the equivalent barotropic level on Mars we shall also need to have additional information about the vertical structure of the atmosphere. It is costumary to express the vertical variation of the temperature field in the form:

$$T = T_o - \gamma z \tag{2.12}$$

in which T_o is a representative value of the surface temperature and γ is the lapse rate. According to Hess and Panofsky (1951) the *adiabatic* lapse rate is

$$\gamma = 3.7 \times 10^{-3} km^{-1} \tag{2.13}$$

but this value is incorrect because it is based on a composition of the atmosphere assuming that nitrogen is the main component. Knowing that the atmosphere is almost 100% CO_2 we find a value of the specific heat for constant pressure of 661.36 J kg^{-1} K^{-1} giving an adiabatic lapse rate of 5.6×10^{-3} K m^{-1}. It is difficult to obtain a value of the actual lapse rate in the martian atmosphere, but as we shall see later the results are insensitive to the assumed value. It has been assumed that the actual lapse rate is 2/3 of the adiabatic rate, i.e. 3.74×10^{-3} K m^{-1}. We may estimate the averaged height of the equivalent barotropic level by hydrostatic considerations. For the temperature at the desired level we find:

$$T = T_o\left(\frac{p}{p_o}\right)^{\frac{R\gamma}{g}} \tag{2.14}$$

where $p/p_o = 1/2$. Knowing the temperature we find the mean height from

$$\bar{z} = \frac{T_o - T}{\gamma} \tag{2.15}$$

The evaluation of the mean height of the orography is straightforward. We can obtain the desired quantity by making a numerical integration of (2.7) over the whole sphere. As expected we find a very small value of the mean height because the values of h are positive on one side and negative on the other side of the planet. The spherical coordinates of the geometrical center of the planet are:

$$r_o = 2.5 \times 10^3 m$$

$$\lambda = 92.29 \ degrees \ (eastlong.)$$

$$\varphi = -62.0 \ degrees \tag{2.16}$$

in the system centered at the center of gravity.

3. Procedure

Eq. (2.9) is solved for the streamfunction by expanding the right hand side in spherical harmonic functions. The coefficients are calculated from the specification given in eq. (2.7). Let the coefficients for the orographical field be denoted $Hc(m, n)$ and $Hs(m, n)$. Knowing these coefficients we may calculate the coefficients for the streamfunctions denoted by $Sc(m, n)$ and $Ss(m, n)$. For these purposes it may be an advantage to write eq. (2.9) in the form:

$$\nabla^2\Psi - q^2a^2\Psi = -\Gamma a^2 h \tag{3.1}$$

We get then

$$Sc(m,n) = \frac{\Gamma a^2}{n(n+1)+q^2a^2} hc(m,n) \tag{3.2}$$

where Γ is given by eq. (2.9) and q^2 by (2.11). The other coefficient obeys an analogous formula. To estimate the first factor we have used the following values of the parameters:

$$g = 3.71ms^{-2}$$

$$f_o = 10^{-4}s^{-1}$$

$$R = 189Jkg^{-1}K^{-1}$$

$$T_o = 217k \tag{3.3}$$

Using furthermore $a = 3.397 \times 10^6$ m we find that

$$\Gamma a^2 = 1.0439 \times 10^5 ms^{-1} \tag{3.4}$$

while the other coefficient becomes:

$$a^2q^2 \approx 2.4 \times 10^{-4} \tag{3.5}$$

The value given in (3.5) is small because the averaged height of the orography is small, while the value of the averaged height of the equivalent barotropic level is of the order of magnitude of 7000 m. It is thus seen that the value is completely negligible compared to the factor $n(n+1)$. For this reason we find that the numerical value of the lapse rate going into the determination of the equivalent barotropic level is rather unimportant in this calculation.

4. Results

The values of the coefficients of the streamfunction were calculated for a number of spherical harmonics. Some results are given in Table 1, and they indicate very clearly that only the components (0,1) and (1,1) have significant values, while the rest are so small that they may be neglected.

TABLE 1

m	n	Sc	Ss
0	1	-1.15×10^{8}	0.0
0	2	7.14×10^{3}	0.0
0	3	3.41×10^{3}	0.0
1	1	-1.22×10^{6}	3.05×10^{7}
1	2	1.77×10^{2}	-4.42×10^{3}
1	3	-1.11x10-3	2.75x10-2
2	2	-5.86×10^{2}	-4.69×10^{1}
2	3	5.84×10^{-4}	-1.06×10^{-5}

Having only two simple components it becomes possible to do the whole calculation by applying eq. (2.8) with T given by eq (2.6). This is done by calculating the coefficients of the spherical harmonics directly. They are:

$$h(0,1) = \frac{3}{4\pi}\int_{-1}^{1}\int_{o}^{2\pi} h(\lambda,\varphi)P_1(\mu)d\lambda d\mu \tag{4.1}$$

$$hc(1,1) = \frac{3}{8\pi}\int_{-1}^{1}\int_{o}^{2\pi} h(\lambda,\mu)P_1^1(\mu)\cos(\lambda)d\lambda d\mu \tag{4.2}$$

$$hs(1,1) = \frac{3}{8\pi}\int_{-1}^{1}\int_{o}^{2\pi} h(\lambda,\mu)P_1^1(\mu)\sin(\lambda)d\lambda d\mu \tag{4.3}$$

The three integrals may be calculated without difficulties, and they give the following results:

$$h(0,1) = r_o \sin(\varphi_o)$$

$$hc(1,1) = 2/1 r_o \cos(\varphi_o)\cos(\lambda_o)$$

$$hs(1,1) = 1/2 r_o \cos(\varphi_o)\sin(\lambda_o) \tag{4.4}$$

These approximate results may for the two components be compared with the values given in Table 1. It will be seen that there is an excellent agreement. We shall therefore in the following use the approximation (4.4). We consider first the zonal component (0,1). Calculating the zonal wind from the (0,1) component we get

$$u_z = 33.76(1-\mu^2)^{1/2} \tag{4.5}$$

It is thus seen that this component of the zonal wind has a maximum at the equator and decreases to zero at the two poles. On the other hand, the u and v components of the contribution by the (1,1) component are:

$$u(1,1) = 9.0\cos(\lambda + 267.71)\mu$$

$$v(1,1) = 8.99\cos(\lambda - 2.29) \tag{4.6}$$

where it is seen that the u-component has a zero value at the equator, while the v-component is independent of latitude as it always is for this particular component. Figure 1 and Figure 2 show the isolines of the streamfunction on the sphere. By considering the total streamfunction from the two components it may be calculated that the minimum value of the streamfunction (-1.152×10^8) is found for $\lambda = 272.29$ degrees and $\varphi = 75.11$, while the maximum (1.152×10^8) is found at (92.29, -75.11) as verified by the figures.

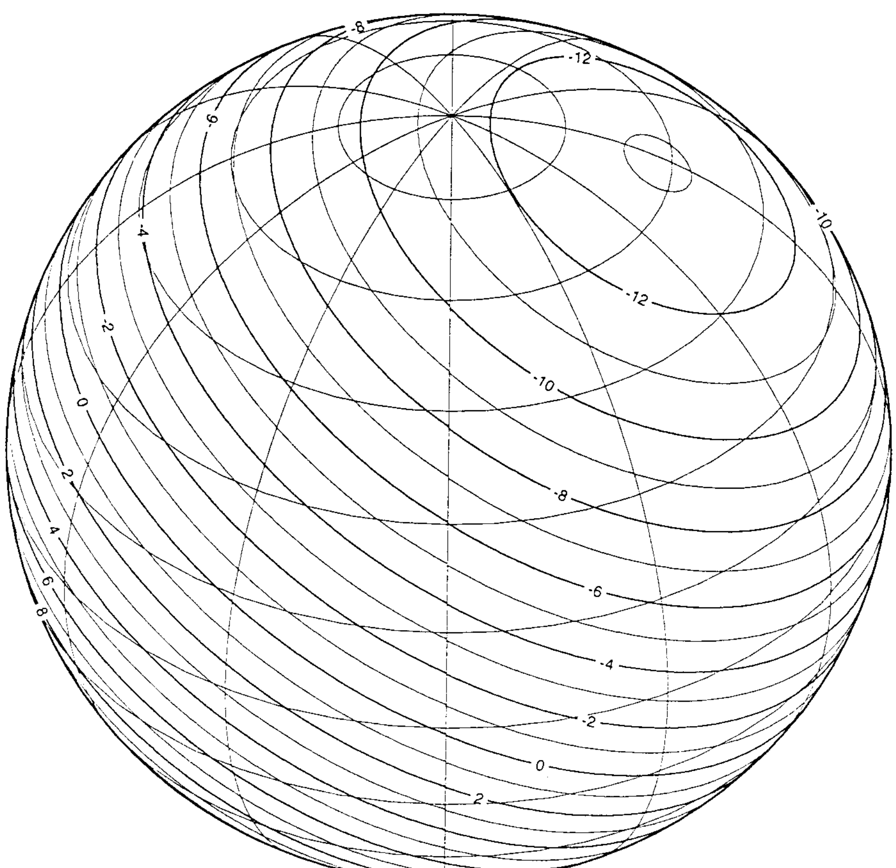

Fig. 1. The circulation due to the difference between the center of gravity and the geometrical center with emphasis on the Northern Hemisphere. The center of circulation represents a minimum.

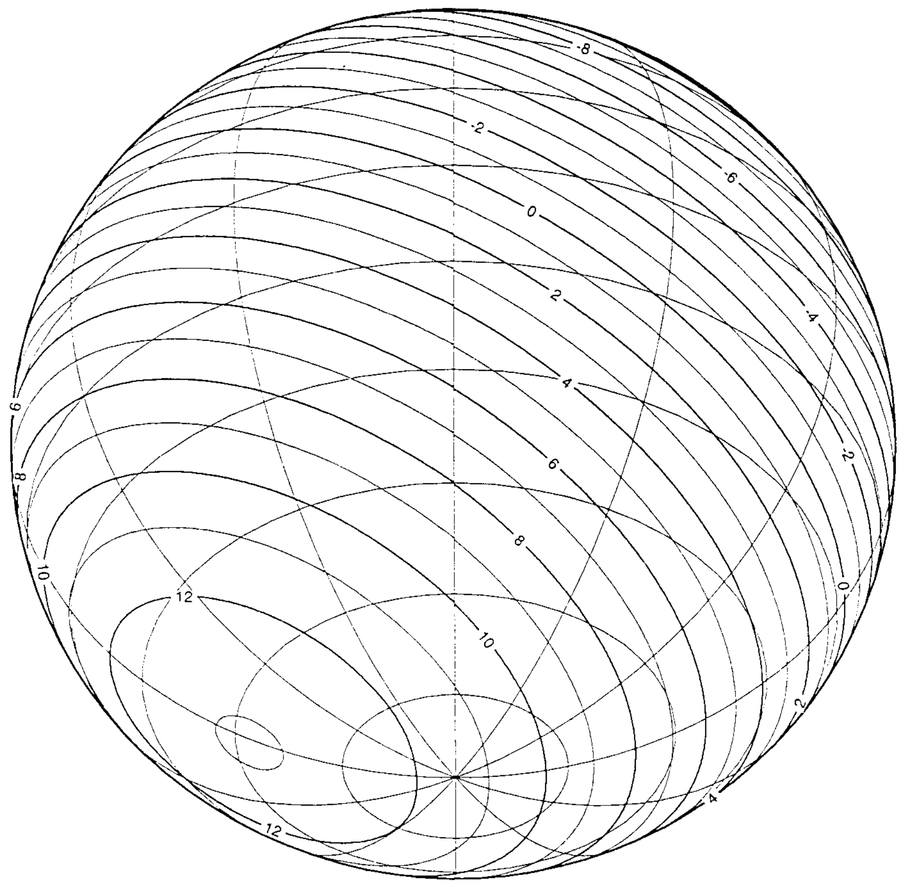

Fig. 2. As Figure 1, but viewing the Southern Hemisphere, where the center of circulation is a maximum.

5. The general influence of orography on the atmosphere of Mars

It may be of some interest to make a first estimate of the influence of the orography of Mars on the circulation of the atmosphere. An estimate of the orography expressed in the amplitudes of the spherical harmonics has been carried out by Bills and Ferrari (1978). They include 16 components in both directions on the sphere. Considering the available data and their uneven distribution over the surface of Mars, it is unavoidable that considerable uncertainty exists in the results. In any case, the chosen resolution will have a minimum wavelength of about 1000 km in the middle latitudes of the planet and some 1300 km at the equator. It is thus clear that the tallest mountains that may have a height of 27 km and a width of only 600 km are not well represented in the data. As mentioned earlier in this paper the flow around these tall individual mountains cannot be handled by the model employed in the previous section. On the other hand, the large-scale mountains resolved by the data may be handled by quasi-geostrophic theory which will be used in the same form as in section 4.

An inspection of the numerical values of the amplitude of the spherical harmonic components given by Bills and Ferrari (*loc.cit.*) reveals that the zonal component (0,2) has a particularly large value (-6180 m). The reason for this is that the orography is given in the form:

$$R(\lambda,\phi) = R_o[1 + \sum_{n=1}^{16}\sum_{m=0}^{n}(C(m,n)\cos(m\lambda) + D(m,n)\sin(m\lambda))P_n^m(\mu)] \quad (5.1)$$

in which the Legendre functions are normalized by the factor:

$$N(m,n) = [(2-\delta(m,0))(2n+1)\frac{(n-m)!}{(n+m)!}]^{1/2} \quad (5.2)$$

in which $\delta(m,n)$ is the Kronecker delta. $R(\lambda,\varphi)$ in (5.1) is the radial distance from the origin. While (5.1) is a correct way to express the orography, we shall express it relative to the mean radius of the planet. The equatorial central distance of the planet is $r_e = 3397$ km and the oblateness is $f = 0.0059$. From these numbers we get that the polar central distance r_p is

$$r_p = r_e(1-f) = 3377\ km \quad (5.3)$$

The mean radius is then found from the formula:

$$R_m = (r_e^2 r_p)^{2/3} = 3370\ km \quad (5.4)$$

We may thus correct the zonal components of the spherical harmonics. This is done by calculating the distance h between the sphere and the ellipsoid. It is:

$$h = \frac{r_e}{(1+\frac{2f}{1-2f}\sin^2\varphi)^{1/2}} - R_m \quad (5.5)$$

in which we have introduced the oblateness by the approximate relation $e^2 = 2f$, valid for small values of the ellipticity. The spherical harmonic components of the above expression have been calculated. It is obvious that it is symmetrical around the equator and there will thus be only even components. In addition, the distance h will have some similarity to the second Legendre polynomial with a negative coefficient. It is therefore not surprising that the main component is the second. Using the same scaling as Bills and Ferrari (*loc.cit.*) where the components are multiplied by 10^6 we find that C(0,2)= -1757 and C(0,4)= 4, while all the other components vanish for practical purposes.

The original components have been corrected according to the above results. The orography in this part of the study will thus not agree with the one displayed by Bills and Ferrari (*loc.cit*). Thereafter we have calculated the streamfunction using the same procedure as in Section 3. From the data available it is not possible to estimate the global mean height of the orography. We are therefore forced to make the assumption that the term containing q^2 is negligible. It is less valid here than in Section 3, especially for the small values of n. The neglect of the q^2 term will result in values of the streamfunction that may be too high.

The orographical field computed from all spherical components is shown in Figure 3, where 180 deg. long. is in the middle of the figure, while 0 deg. is at the edge. We notice in particular the large mountain heights at the equator at about 120 deg. east. with low values to the west with a minimum around 30 deg. east. Considerable slopes exist therefore in the eastern hemisphere close to equator. In the Southern Hemisphere one notices the deep depression in the western hemisphere with a center located at about 300 deg. east and 40 deg. south.

The streamfunction, which really is only a rescaled geopotential, calculated as a response to the orographical forcing is shown in Figure 4. It is much simpler in structure than the orography. The main response is on the first few wave numbers. The major anticyclonic feature across the equator in the eastern half is clearly related to the large ridge in the orography. The center of the anticyclone is located to the west of the ridge. The cyclonic circulation in the western hemisphere is connected with the low orographic heights in the same region.

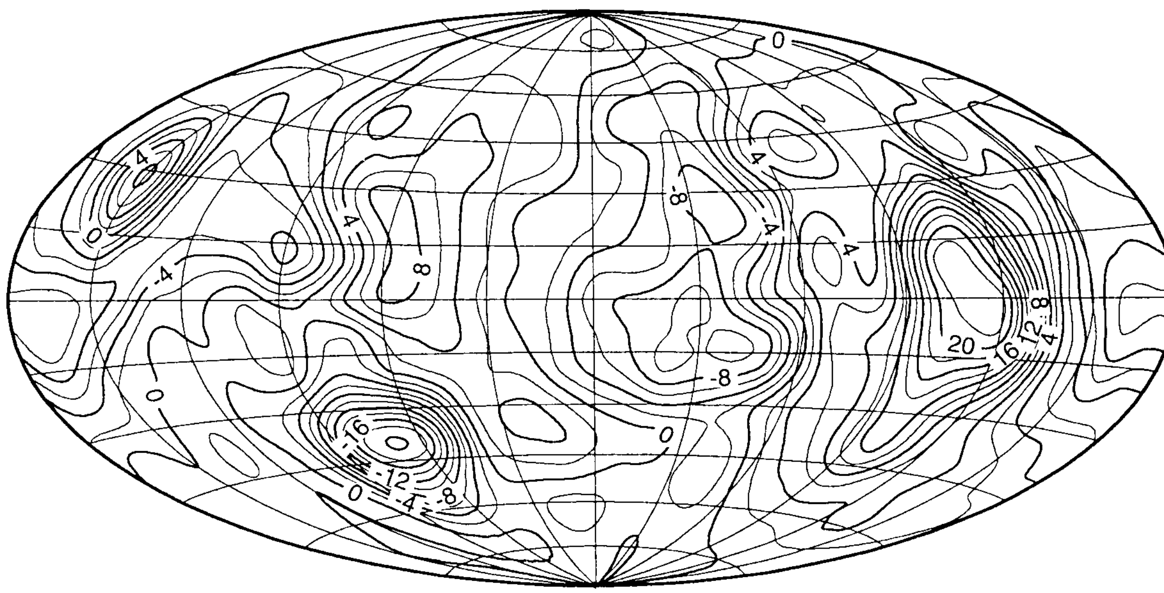

Fig. 3. The orography of Mars as obtained from the addition of all spherical harmonic components with triangu-lartruncation at wave number 16.0 degree along the edge, 180 degrees in the middle.

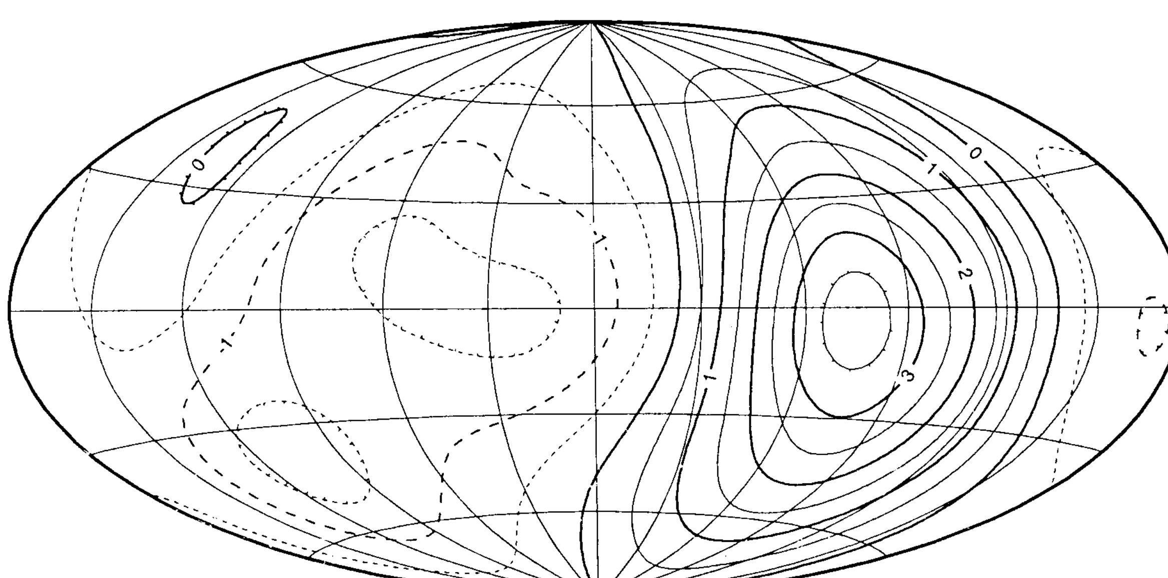

Fig. 4. The eddy streamfunction for the circulation in equilibrium with the orography. Map projection as in Figure 3.

6. Concluding remarks

The investigation in this note deals with a part of the general circulation of the Martian atmosphere. We have concentrated on the influence on the atmosphere of the displacement of the center of gravity from the geometrical center of the planet. In addition, we have restricted the investigation to the stationary problem. In spite of the fact that the displacement is only 2.5 km as compared to the radius of the planet (close to 3400 km), it is nevertheless a significant orographic disturbance on the planetary scale.

Due to the position of the geometrical center of the planet (see (2.16)) the orography is not strictly speaking expressible in the lowest order harmonics, but, as shown in Table 1, the calculation indicates that only the components (0,1) and (1,1) have significant amplitudes in the derived components of the streamfunction. The remaining part of the calculations contains the two components only. As seen from Figure 1 and Figure 2 showing two different representations of the streamfunction we find low values of the streamfunction in the Northern Hemisphere of the planet, centered at 272.29 and 75.11 in longitude and latitude, respectively. High values are found in the Southern Hemisphere at 92.29 long. and -75.11 lat.

The zonal winds connected with the components have a maximum of more than 30 m s^{-1} at the equator and vanish at the Poles. The velocities connected with the planetary wave component (1,1) are considerably smaller, but amount to about 9 m s^{-1}.

The motion in equilibrium with the large-scale orography is computed in Section 4 based on a triangular 16 component spherical harmonic representation. Such a representation is inadequate to resolve all features of the Martian orography, but our calculation displayes the larger scales of the orographically forced circulation of the planet.

Acknowledgements

The authors thank Mr. Torben Frohn for considerable assistance with the graphics in this note.

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