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Mathematical modeling of the distribution and transport of the water vapor over Mexico

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RESUMEN

Tanto métodos estadísticos como modelación matemática se usan para estudiar la distribución y evolución de varias características climáticas en el campo de la humedad sobre México. El análisis estadístico provee estimaciones locales de la humedad basadas en la aplicación del método de Kriging y la interpolación de Shepard. Se formula un modelo matemático de transporte de la humedad en la región de la República Mexicana, que usa σ-sistema de coordenadas con el fin de tomar en consideración la topografía de la región. El modelo es balanceado y bien puesto según Hadamard, es decir, cualquiera de sus soluciones es única y estable con respecto a perturbaciones en los datos iniciales y condiciones de frontera. En la ausencia de fuentes y sumideros de la humedad, el modelo tiene dos leyes de conservación.

ABSTRACT

Both statistical methods and mathematical modelling are used to study the distribution and evolution of various climatic characteristics in the humidity field of Mexico. The statistical analysis provides local humidity estimates based on Kriging's method and Shepard's interpolation. A mathematical limited area model of humidity transport (taking account of topography) is formulated for a region of Mexico in the σ-system of coordinates. The model is balanced and well-posed according to Hadamard, that is, both of its solutions are unique and stable to perturbations in the initial and boundary conditions. In the absence of the humidity sources and sinks the model has two conservation laws.

1. Introduction

A knowledge of the humidity distribution and water vapor transport over Mexico is of more than theoretical interest, since this country has vast expanses of arid zones. Agriculture, cattle-breeding activity, and the development of ecosystems are some of the most vital activities which depend directly upon a knowledge of the distribution and transport of humidity. The first studies of the humidity field were concentrated on the aridity estimations using total precipitation and maximal temperature data (Soto Mora and Jáuregui, 1965). Subsequent researches have evaluated climatic monthly mean values of water vapor content over Mexico, using the data from eight aerological stations extending between 1965 and 1974 (Jáuregui, 1986). Similarly, attempts have been made to estimate the humidity content and vapor balance in the troposphere over Mexico (Reyes and Cadet, 1986, 1988) and study urban humidity contrasts (Jáuregui and Tejeda, 1997). A climatic diagnosis of the global humidity field has recently been carried out by Kalnay *et al.* (1996), using a numerical model, atmospheric remote sonding and satellite images.

The present work is devoted to the application of statistical methods and mathematical modeling in a study of the distribution and evolution of such climatic characteristics of humidity in Mexico. Parameters investigated include specific, relative and absolute humidity, total water vapor content in a troposphere column, dew-point temperature and atmosphere temperature, pressure and winds. This permits us to explain the climatic water vapor anomalies over Mexico. A statistical analysis described in the first part of this work and based on Kriging's method (Journel and Huijbregts, 1978) provides local humidity estimates. The second part briefly describes the application of Shepard's interpolation in a finite-dimensional subspace (Kincaid and Cheney, 1994). These two methods were successfully applied in a study of the tropical mountain climate of Mexico, with the aim of evaluating a potential site for the Large Millimetric Telescope (the joint USA-Mexico project, Davydova, 1995, 1997; Davydova *et al.*, 1996; Torres *et al.*, 1996, 1997). In the last part, a balanced model of the humidity transport taking account of topography is formulated for a region of Mexico in the σ-system of coordinates. Due to special boundary conditions, the model is well-posed according to Hadamard, that is, both of its solutions are unique and stable to perturbations in the initial and boundary conditions. In particular, if the humidity sources and sinks (including humidity flux through the boundary) are absent then the model has two conservation laws.

2. Kriging's method

The Kriging's method allows one to determine optimal estimates of unknown variables in areas whose size is small compared to that of the homogeneous zone where the variable is located (Journel and Huijbregts, 1978).

Let $Z(x)$ be an unknown two-dimensional random function (e.g., the total water vapor, or relative humidity) with the mathematical expectation $E\{Z(x)\}=m$, and covariance

$$E\{Z(x+h)(Z(x)\} - m^2 = C(h), \quad \forall x \tag{1}$$

Here h is a vector, and the same notation x is used both for the radius-vector and for the domain point. Denote as

$$Z_V(x) = \frac{1}{V}\int_{V(x)} Z(x)dx \tag{2}$$

the average value of $Z(x)$ in a homogeneous zone $V(x)$ of the point x of measure V. Also assume that each available observation $Z_\alpha(x)$ is considered as the average (2) of $Z(x)$ over a vicinity V_α of point x (α=1, 2, ..., n), whilst,

$$E\{Z_\alpha\} = m, \quad \forall\alpha \tag{3}$$

Given this, the linear combination

$$Z^* = \sum_{\alpha=1}^{n} \lambda_\alpha Z_\alpha \tag{4}$$

of n observations with $\sum_{\alpha=1}^{n} \lambda_\alpha = 1$ represents the unbiased estimate, since

$$E\{Z^*\} = m \sum_\alpha Z_\alpha = m = E\{Z_V\} \tag{5}$$

The optimal weight coefficients λ_α are chosen with the aim of minimizing the variance

$$E\{[Z_V - Z^*]^2\} = E\{Z_V^2\} - 2E\{Z_V Z^*\} + E\{Z^{*2}\} \tag{6}$$

Using Lagrange's technique,

$$\partial/\partial\lambda_\alpha[E\{[Z_V - Z^*]^2\} - 2\mu(\sum_\beta \lambda_\beta - 1)] = 0, \tag{7}$$

then leads to Kriging's system of *(n+1)* linear equations for unknowns λ_α ($\alpha = 1,\ldots,n$) and Lagrange parameter μ:

$$\begin{cases} \sum_{\beta=1}^{n} \lambda_\beta \overline{C}(V_\alpha, V_\beta) - \mu = \overline{C}(V_\alpha, V), & \forall\alpha = 1,2,\ldots,n, \\ \sum_{\beta=1}^{n} \lambda_\beta = 1. \end{cases} \tag{8}$$

Here $\overline{C}(V_\alpha, V$ and $\overline{C}(V_\alpha, V_\beta)$ are the averages of the covariance function *C(h)* (Journel and Huijbregts, 1978). The minimum of the variance is

$$\sigma^2 = E\{[Z_V - Z^*]^2\} = \overline{C}(V,V) + \mu - \sum_{\alpha=1}^{n} \lambda_\alpha \overline{C}(V_\alpha, V) \tag{9}$$

In particular, if $V_\alpha = V$ then the Kriging estimate is unbiased, the estimate Z^* is identical to the known value Z_α , and $\sigma = 0$.

3. Shepard's interpolation

Let $f(p_i)$ be a known value of the function f in the point $p_i = (x_i, y_i)$ of a two-dimensional domain $(i = 1,2,\ldots,n)$. The distance between two points p and q is defined as $\varphi(p,q) = \|p - q\|^2$ where $\|\cdot\|$

is the Euclidean norm (Kincaid and Cheney, 1994). It is apparent that

$$\varphi(p,q) = 0 \quad \text{if and only if} \quad p = q \tag{10}$$

and the basic functions

$$u_i(p) = \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{\varphi(p,p_j)}{\varphi(p_i,p_j)} \qquad (1 \le i \le n) \tag{11}$$

are analogous to the interpolating Lagrange polynomials in the one-dimensional case. In addition

$$u_i(p_j) = \delta_{ij} \qquad (1 \le i, \quad j \le n) \tag{12}$$

whilst an approximate value *f(p)* of the function *f* in a point $p = (x,y)$ may be defined through

$$F(p) = \sum_{i=1}^{n} f(p_i)u_i(p) \tag{13}$$

Using (13), (11) and

$$\varphi(p,p_j) = \|p - p_j\|^2 = (x - x_j)^2 + (y - y_j)^2 \tag{14}$$

we then obtain

$$F(x,y) = \sum_{i=1}^{n} f(x_i,y_i) \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x - x_j)^2 + (y - y_j)^2}{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{15}$$

4. Diagnosis of the humidity by Kriging's and Shepard's methods

Daily and monthly mean values of the relative humidity (*RH*, %), absolute humidity (*a*, g/kg) and total water vapor content (*WV*, mm) of the atmospheric column were calculated by using radiosonde information on temperature, dew-point temperature, atmospheric pressure, and the direction and magnitude of the wind velocity, taken by Mexican National Meteorological Service stations between 1992 and 1996. The *RH* fields at 1000, 850, 700 and 500 mb levels and the total vapor content *WV* in the layer from the Earth's surface up to 100 mb were constructed by using Kriging's method (Davydova, 1995, 1997). According to the station data, the highest *WV* values were observed in August, whilst the lowest *WV* values were in March. In the present work, therefore main peculiarities of the *RH* and *WV* fields over Mexico were studied by using information for March and August of 1994, representative of dry and humid seasons. Monthly mean values of *WV* at the aerological stations of the Mexican Republic are tabulated in Table 1. It is seen that the maximal *WV* values are located at such stations as Mazatlán (MZT), Manzanillo (ZLO) and Acapulco (ACA) which lie in a rather narrow band of humid air along the Mexican Pacific coast. This band extends over the Southern part of Mexico to Veracruz, whilst the driest atmospheric zones are to the North-West and North of the country, as well in the central part.

Table 1. Total vapor content of the atmospheric column at the Mexican Republic stations (in mm).

Key	Longitude (°W)	Latitude (°N)	WV_{march}	WV_{august}
Guadalupe Island (GPE)	118.316	29.166	11.5	28.1
Chihuahua (CUU)	106.066	28.633	09.1	24.2
Empalme (EMP)	110.800	27.950	10.0	50.3
Torreón (TRC)	103.450	25.533		29.7
Monterrey (MTY)	100.200	25.866	21.5	40.2
Mazatlán (MZT)	106.416	23.200	24.6	54.4
Guadalajara (GDL)	103.383	20.683	13.5	31.7
Mérida (MID)	110.350	24.166		43.7
Manzanillo (ZLO)	104.333	19.050	21.0	46.4
México (MEX)	99.294	19.427	13.3	22.0
Veracruz (VER)	99.116	19.100	30.0	50.0
Socorro Island (SOC)	111.000	18.833	23.2	45.7
Acapulco (ACA)	99.916	16.833	31.2	47.2

The *WV* fields compare well with those obtained by Jáurequi (1986) with the maximal *WV* values over the Pacific coast of Mexico, and minimal values in the North and center of the country. Certain distinctions between the results of both the works are likely due to:

1. Different number of aerological stations and dissimilar measurements accuracy. In the present work, the *WV* fields have been obtained with higher precision (to 0.1 mm) by using recent devices and currently available radiosonde methods at the aerological stations. In addition, 13 aerological stations have been used here in contrast to 8 stations used by Jáuregui (1986).

2. Different and rather short observation periods. Both periods 1965-1974 and 1992-1996 are likely to be anomalous in mean total water vapor content of the atmospheric column since both comprise El Niño events. For example, monthly mean *WV* values obtained by Jáuregui (1986) are 3-5 mm less than those presented in our work, while the *RH* values obtained for the same 10-year period by Jáuregui's pupil (Ramirez, 1990) are 7-10% less than our results. The higher values of *WV* and *RH* registrated in our research are related with the two periods of "El Niño" observed from 1992 to 1994.

3. Different depths of the considered columnar water vapor. Jáuregui considers only sfc-500 mb column against sfc-300 mb considered here.

Shepard's interpolation has been used to calculate the values of the *WV* and *RH*. In particular, the results obtained above some mountain tops have been taken into account in determining potential locations for the Large Millimetric Telescope (Torres *et al.*, 1996, 1997). Note that for each month, the monthly mean *RH* field at every isobaric level is more homogeneous than the corresponding *WV* field due to an exponential distribution of humidity with altitude (Zuyev and Komarov, 1987).

The *WV* and *RH* fields constructed for the Mexican region allow us to relate the month-to-month behavior of water vapor with characteristic features of atmosphere circulation, in particular, with the influence of anticyclonic conditions that prevail in the mid-lower troposphere in Northern Mexico and the stabilizing effect of the cold California current at the lower levels to the West.

a) *Humidity distribution during the wet season*

The maximal *WV* values in August extend along the Sierra Madre Occidental to the south and southeast of Mexico (Fig.1). Similarly the corresponding minimal *WV* values, they extend from the North to the central parts of Mexico between the Sierra Madre Occidental and Sierra Madre Oriental. This demonstrates the important role of orography in affecting the distribution of the water vapor in the troposphere.

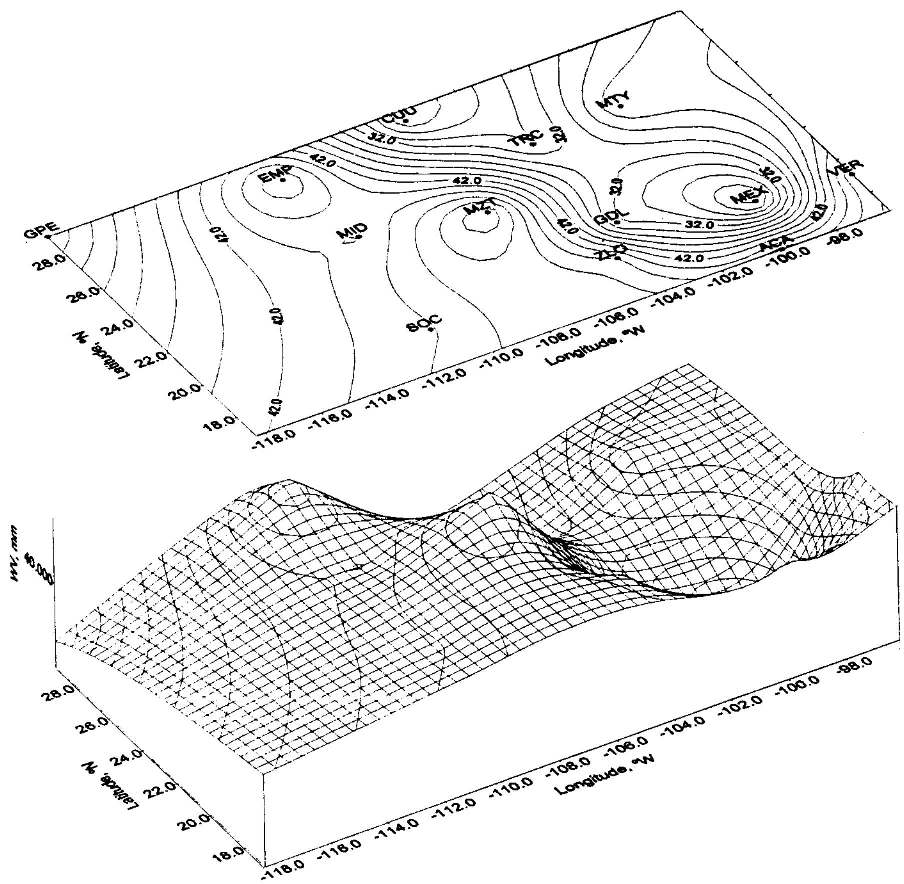

Fig. 1. Distribution of water vapor (WV) over Mexico in August of 1994.

The mean *RH* values in August for the radiosonde network at various reference pressure surfaces are shown in Table 2. The relative humidity field at 1000 mb is reasonably homogeneous, with mean value about 72%, and two maxima observed at Mazatlán (MZT), and Mérida (MID) (77.8% and 77.5%, respectively). In contrast to this, the *RH* field at 850 mb is rather heterogeneous, varying from a minimum near Chihuahua (CUU) and Guadalupe Island (GPE) (28.6%), to the maximal values near Mazatlán (MZT) (76.1%), Monterrey (MTY) (72.6%) and Veracruz (VER) (71.4%). The *RH* field at 700 mb is characterized by mean values of ~ 62.5% , two maxima near Guadalajara (87.6%) and Mexico City (72.0%), and a minimum near the Guadalupe Island (39.6%). It should be noted that the relative humidy registrated at 700 mb by the stations GPE, ZLO and ACA on the Mexican Pacific, and by continental stations CUU and TRC, are relatively high as compared to those at 850 mb (Table 2). This fact is explained by the advective thermal inversion when a warm and humid mass of air from the Pacific comes into contact with a relatively cold continental surface; a phenomenon which results in *WV* accumulation in the layer from 700 mb to 500 mb.

Table 2. Distribution of the relative humidity (in %) in August, 1994 (radiosonde information, National Meteorological Service, Mexico).

Key	RH_{1000}	RH_{850}	RH_{700}	RH_{500}
Guadalupe Island (GPE)	76.3	30.2	39.6	31.3
Chihuahua (CUU)		28.6	50.7	45.1
Empalme (EMP)	68.1	62.2	57.7	55.2
Torreón (TRC)		35.5	59.8	41.9
Monterrey (MTY)		72.6	62.1	40.7
Mazatlán (MZT)	77.8	76.1	65.6	63.2
Guadalajara (GDL)			87.6	73.1
Mérida (MID)	77.6	62.5	56.1	56.7
Manzanillo (ZLO)	64.7	65.2	65.6	60.8
México (MEX)			72.0	69.8
Veracruz (VER)	77.1	71.4	67.9	65.4
Socorro Island (SOC)	64.2	67.1	57.7	56.1
Acapulco (ACA)	69.2	64.0	67.5	63.3

b) *Humidity patterns during the dry season*

In March, the *WV* field is smooth and has rather low values compared to those in August (Fig.2). This is explained by the homogeneity of the atmospheric structure due to dominant anticyclonic circulation over Northern Mexico at mid-lower troposphere. The trade winds, from moderate to strong, generated by such anticyclonic circulation are directed from the North and North-East, introducing dry air from extratropical latitudes. However, the *RH* values registered at 700 and 500 mb at Guadalupe Island (GPE), Chihuahua (CUU), Monterrey (MTY) and Guadalajara (GDL) are higher-than-expected (Table 3). The analysis of the synoptic situation shows that such elevated values are due to the existence of a jet stream at 10,000 m, transporting clouds from the Pacific to the continent in the belt from 20°N to 30°N. At 850 mb, the *RH* data contain two maxima located near Monterrey and Acapulco (Table 3). There is a tendency to low values in the West and high values on the North-East is evident. At 700 mb, the *RH* field is more heterogeneous due to increases in humidity before and above the Sierra Madre Occidental, while its distribution at 500 mb is well correlated to the circulation of the free atmosphere. In particular, a zone of the maximal humidity is located below the jet stream winds.

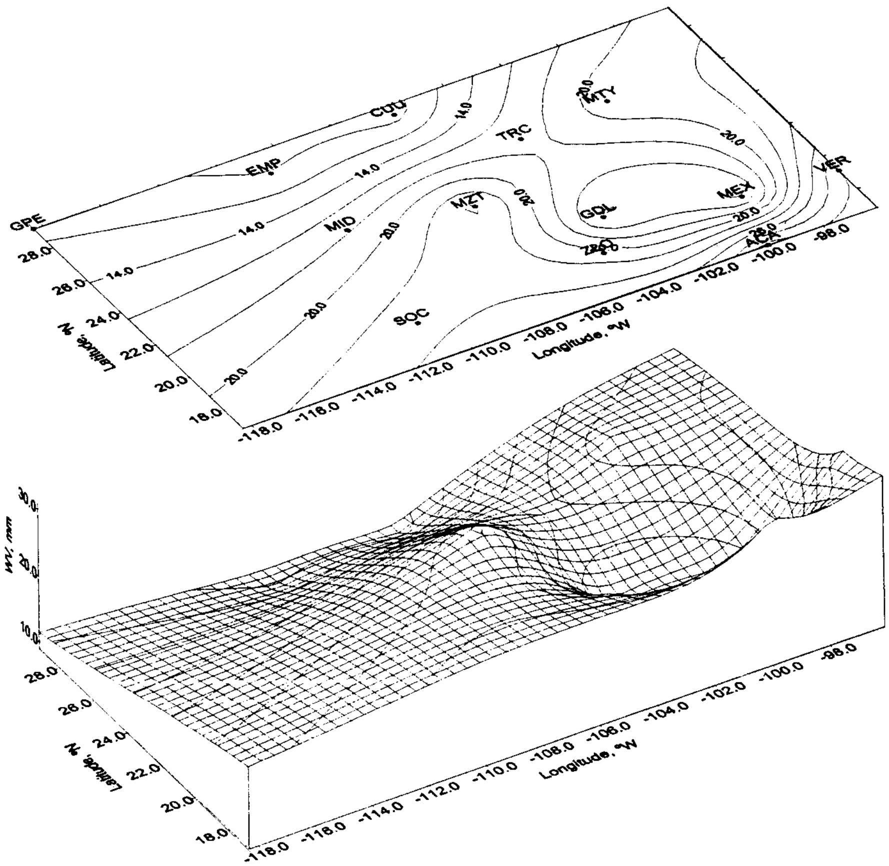

Fig. 2. Distribution of water vapor (WV) over Mexico in March of 1994.

Table 3. Distribution of the relative humidity (in %) in March, 1994 (radiosonde information, National Meteorological Service, Mexico).

Key	RH_{1000}	RH_{850}	RH_{700}	RH_{500}
Guadalupe Island (GPE)	68.8	42.5	06.3	08.8
Chihuahua (CUU)		41.2	41.6	31.9
Empalme (EMP)	37.8	23.8	20.0	32.2
Monterrey (MTY)		53.6	34.3	40.5
Mazatlán (MZT)	82.6	40.0	31.9	32.3
Guadalajara (GDL)			36.6	31.1
Mérida (MID)	41.5	26.5	20.7	19.9
Manzanillo (ZLO)	63.7	29.5	22.2	33.3
México (MEX)			40.5	12.4
Veracruz (VER)	74.5	50.9	36.4	29.9
Socorro Island (SOC)	52.7	25.1	27.6	13.9
Acapulco (ACA)	70.1	46.3	45.5	52.1

5. The specific humidity transport model

Let $P=P_0=300$ mb $= Const$ be the pressure at the upper boundary of a domain D of the humidity transport model (no water vapor in gaseous state exists above this level), and let $P=P_S$ be the surface pressure (Fig. 3). To incorporate a topography in the model we will use the σ-system of coordinates (Phillips, 1957; Haltiner and Williams, 1980):

$$\sigma = \frac{P - P_o}{P_s - P_o}, \qquad \sigma = \sigma(x,\ y,\ P,\ t) \tag{16}$$

where $P(x, y, \sigma, t)$ is the pressure. Thus σ varies from $\sigma = 0$ at $P=P_0$ to $\sigma = 1$ at $P=P_S$. In this frame of reference, the continuity equation takes the form

$$\frac{d}{dt}[\ln \frac{\partial P}{\partial \sigma}] + \nabla_\sigma \cdot \vec{V} + \frac{\partial \dot{\sigma}}{\partial \sigma} = 0 \tag{17}$$

where $\vec{V} = \{u, v\}$ is the horizontal velocity in the σ-system of coordinates. Since $\frac{\partial P}{\partial \sigma} = P_S - P_O$, (17) can be written as

$$\frac{\partial(P_s - P_o)}{\partial t} + \nabla_\sigma \cdot [(P_s - P_o)\vec{V}] + \frac{\partial}{\partial \sigma}[(P_s - P_o)\dot{\sigma}] = 0 \tag{18}$$

where σ is the vertical velocity, and ∇_σ is the horizontal gradient in the σ-system.

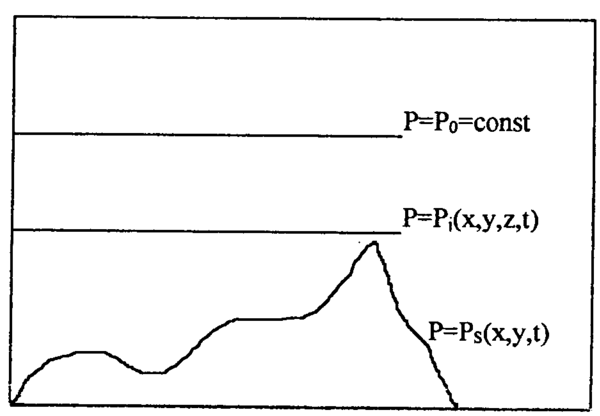

Fig. 3. Topography of the humidity transport problem domain.

The transport of the specific humidity $q(x, y, \sigma, t)$ is described by the equation

$$\frac{\partial q}{\partial t} + \vec{V} \cdot \nabla q + \dot{\sigma}\frac{\partial q}{\partial \sigma} = F + C + E \tag{19}$$

(Marchuk, 1986) where

$$F = \frac{\partial}{\partial x}\mu\frac{\partial q}{\partial x} + \frac{\partial}{\partial y}\mu\frac{\partial q}{\partial y} + \frac{\partial}{\partial \sigma}\nu\frac{\partial q}{\partial \sigma} \tag{20}$$

is the diffusion term, and the terms C and E represent the condensation and evaporation, respectively.

Multiplying (19) by $\pi = P_s - P_o$, and (18) by q, and summing both the results we obtain

$$\frac{\partial \pi q}{\partial t} + \nabla_\sigma \cdot (\pi \vec{V} q) + \frac{\partial}{\partial \sigma}(\pi \dot{\sigma} q) = \pi Q \tag{21}$$

where $Q = F+C+E$. Multiplying now (19) by πq, and (21) by q, and summing both the results we get

$$\frac{\partial}{\partial t}(\pi q^2) + \nabla_\sigma \cdot (\pi \vec{V} q^2) + \frac{\partial}{\partial \sigma}(\pi \dot{\sigma} q^2) = 2\pi q Q \tag{22}$$

Integrating (21) and (22) over domain D and using the conditions $\dot{\sigma} = 0$ at $\sigma = 0$ and $\sigma = 1$ we get the balance equation

$$\frac{\partial}{\partial t}\iiint_D \pi q dD + \iint_S \pi V_n q dS = \iiint_D \pi Q dD \tag{23}$$

for the specific humidity in D, and the equation

$$\frac{\partial}{\partial t}\iiint_D \pi q^2 dD + \iint_S \pi V_n q^2 dS = 2\iiint_D \pi q Q dD \tag{24}$$

for the norm

$$\|q\| = \left(\iiint_D \pi q^2 dD\right)^{1/2}$$

of the solution $q(x, y, \sigma, t)$ where $V_n = \vec{V} \cdot \vec{n}$, $\vec{n}$ is the unit normal to the lateral surface S of domain D, and dS and dD are elementary area and volume, respectively. In particular, if $Q \equiv 0$, and integral fluxes of $\pi V_n q$ and $\pi V_n q^2$ through the boundary S are both zero then (23) and (24) lead to the following conservation laws:

$$\frac{\partial}{\partial t}\iiint_D \pi q dD = 0, \quad \frac{\partial}{\partial t}\|q\| = 0. \tag{25}$$

6. Vertical velocity and pressure tendency equations

Given that the horizontal velocity $\vec{V}$ at an isobaric surface is known from observation, or of some dynamical model, then its values $\vec{V}(x, y, \sigma, t)$ at the σ-levels can be calculated by interpolation. Note that such values can also be obtained as a solution of the shallow-water model (Tan Weiyan, 1992; Marchuk, 1986; Skiba, 1995). Let us show how to determine the humidity $q(x, y, \sigma, t+\delta t)$ at a moment $t+\delta t$ if $q(x, y, \sigma, t)$ is known and δt is small enough. Integrating (18) over σ from $\sigma=0$ to $\sigma=1$, and taking into account the boundary conditions $\dot\sigma = 0$ at $\sigma = 0$ and $\sigma = 1$ one can obtain the pressure tendency equation

$$\frac{dP_s}{dt} = -\int_0^1 \nabla_\sigma \cdot [(P_s - P_o)\vec{V}]d\sigma \tag{26}$$

enabling us to calculate the surface pressure $P_S\ (x, y, \sigma, t+\delta t)$. This step is necessary to adapt the σ-system to the new surface pressure at moment $t+\delta t$. One more integration of (18) over σ from $\sigma=0$ to a value σ leads to the equation

$$(P_s - P_o)\dot{\sigma} = -\sigma\frac{\partial P_s}{\partial t} - \int_o^\sigma \nabla_\sigma \cdot [(P_s - P_o)\vec{V}]d\sigma \tag{27}$$

allowing us to calculate the vertical velocity $\dot\sigma(x, y, \sigma, t)$. The humidity can then be found by solving either (19) or (21) within interval $(t, t+\delta t)$.

7. Boundary conditions of the humidity transport problem

Assume that the humidity transport equation (21) has to be solved in a three-dimensional domain D with a lateral boundary S. Evidently, the humidity flux through S is the possibility. Since, due to advection, the boundary errors in the transport problem propagate rapidly inside domain D, care should be taken in setting physically and mathematically suitable boundary conditions. From mathematical point of view, such conditions must result in the well-posed problem according to Hadamard, where either solution is unique and stable to perturbations (errors) in the initial and boundary conditions. To this end, we divide S into two parts: $S = S^+ + S^-$ (Skiba, 1993; Skiba y Adem, 1995). The part S^+ contains the points in which the normal component V_n of the horizontal velocity is non-negative ($V_n \geq 0$) and the advective humidity flux is directed outside the domain D (point A in Fig.4). The complementary part S^- of the boundary consists of points in which $V_n < 0$ and the advective humidity flux is directed inside D (point B in Fig. 4). We take

$$\mu\frac{\partial \pi q}{\partial n} - \pi q V_n = R(x,\ y,\ \sigma,\ t) \quad \text{at} \quad S^- \tag{28}$$

and

$$\mu\frac{\partial \pi q}{\partial n} = 0 \quad \text{at} \quad S^+ \tag{29}$$

where $\mu\frac{\partial \pi q}{\partial n}$ and $\pi q V_n$ are the horizontal diffusion and advective flux of humidity along the outward

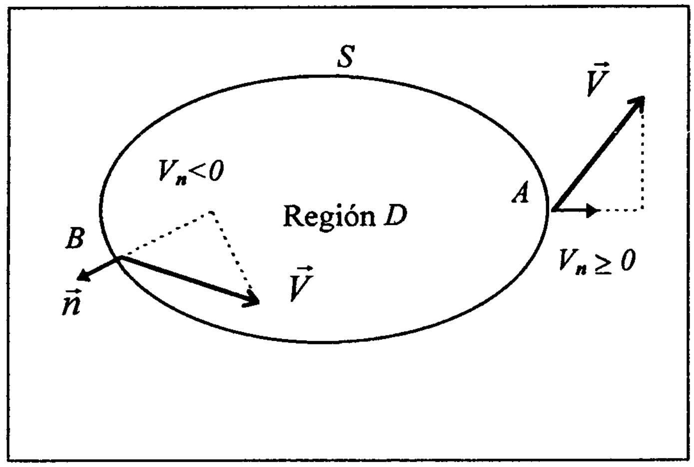

Fig. 4. A section σ = *Const* of the humidity transport problem domain.

normal to S, respectively. Condition (28) means that total (advective plus diffusive) humidity flux is prescribed at the inflow part of the boundary, whereas, by condition (29), the diffusive humidity flux is negligible as compared with the corresponding advective flow $\pi q V_n$ out of domain D. In the limiting non-diffusion case ($\mu = \nu = 0$), (28) is reduced to the Dirichlet condition $q=R/\pi V_n$, while (29) vanishes as it should, since the pure advection problem requires no condition at the outflow boundary (Godunov, 1971). Unfortunately, in practice, the humidity flux R is usually unknown, and as a rough approximation, R is assumed everywhere to be zero. It can be shown that the transport equation with boundary conditions (28), (29) is well-posed according to Hadamard, that is, both of its solutions are unique and stable to initial perturbations (Skiba, 1993; Skiba, 1996a, b; Skiba *et al.*, 1996). Note that the humidity transport equation (21) with condition $\mu \frac{\partial \pi q}{\partial n} = 0$ and $V_n = 0$ at the whole boundary S also represents a well-posed problem according to Hadamard (Marchuk and Skiba, 1978). This condition applies particularly well over oceanic regions of S, where humidity is sufficiently homogeneous, and its gradient is almost zero. It should also be pointed out that equation (21) is linear, and the same equation can also be used for the climatic humidity anomaly, instead of for the humidity itself. Under this circumstances, the assumption $R=0$ implies the absence of anomaly in the climatic humidity flux through boundary S.

8. Concluding remarks

In this work, an Aerology Data Base has been developed for the Mexican Republic which contains information on 14 climate parameters such as the altitude of standard isobaric levels, temperature, the dew-point and virtual temperature, the partial and saturation pressure of water vapor, the absolute, relative and specific humidity, the mixing ratio and saturation mixing ratio, the vapor content and wind velocity. Average daily, monthly and annual values of these parameters have been calculated at the Earth's surface, and 11 standard isobaric levels (1000, 920, 850, 700, 500, 400, 300, 250, 200, 150, 100 mb) employed on the basis of a five-year period (1992-1996) measured by the Mexican National Meteorological Service. Computational programs have been created for automatic processing of a host of aerological information.

A balanced limited area model of humidity transport over Mexico has also been formulated. Due to special boundary conditions, the model is well-posed according to Hadamard, that is, both of its solutions are unique and stable to perturbations in the initial and boundary conditions. In particular, in the absence of humidity sources and sinks (including humidity flux through the boundary), the model has two conservation laws.

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