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On the zonally-symmetric circulation in two-level quasi-geostrophic models

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RESUMEN

La circulación permanente promediada zonalmente en el modelo de circulación general de Phillips se determina usando los mismos parámetros como en el experimento original.

El estado zonalmente simétrico aproximado se determina por procedimientos espectrales y resolviendo directamente las ecuaciones diferenciales. Los resultados se comparan con una determinación semejante de la misma circulación llevados al cabo por Charney, resultantes en velocidades del viento muy grandes.

Se concluye que los nuevos resultados son más realistas que la circulación extremadamente intensa obtenida por Charney; pero, también, que el estado permanente tiene velocidades promediadas zonalmente de unos pocos cientos de metros por segundo.

Usando una determinación más realista del balance térmico medio de la atmósfera, resultando una forma newtoniana de calentamiento, cuando los términos de Boltzman son linealizados, da resultados finalmente aceptables de acuerdo, en general, con los últimos resultados de Charney.

ABSTRACT

The zonally-averaged steady state circulation in Phillips' general circulation model is determined using the same physical parameters as in the original experiment. The approximate zonally-symmetric state is determined both by spectral procedures and by solving the differential equations directly. The results are compared with a similar determination of the same circulation carried out by Charney resulting in very large wind velocities.

It is concluded that the new results are more realistic than the extremely intensive circulation determined by Charney, but also that the zonal steady state has zonally averaged velocities of a few hundred meters per second.

Employing a more realistic determination of the atmospheric averaged heat budget resulting in a Newtonian form of the heating when the Boltzman terms are linearized gives finally acceptable results in general agreement with Charneys later results.

1. Introduction

This paper is mainly of historical and pedagogical interest. It considers some aspects of early general circulation models and, in particular, the steady state of the zonally averaged temperatures and winds in various models. It has not been customary to calculate these steady states in numerical integrations of the model equations, but the first general circulation model (Phillips, 1956) used the zonally averaged equations to calculate a state that was unstable with respect to small perturbations.

Phillips' (1956) general circulation experiment demonstrated clearly many aspects of the atmospheric general circulation in middle latitudes including the creation of cyclones and an essentially correct determination of most aspects of the atmospheric energetics. The experiment started with a model atmosphere at rest and with a constant temperature. The first part of the experiment was an integration of the zonally-symmetric part of the model equations including a determination of the zonally-averaged winds and the mean-meridional circulation. This part of the experiment was run for 130 days. After this time the heating in the model had created a temperature difference between the southern and northern walls of about 60 degrees and a maximal wind shear in the central part of the region of about 12.5 m per s between the upper and the lower levels in the two-level model.

The remaining part of the experiment started with an introduction of small perturbations on the zonal structure, and the following development of atmospheric eddies that were analysed in detail. In the present note we shall be interested in the first part of the experiment. The main purpose of the initial integration was to create a baroclinically unstable state such that small perturbations would grow to mature waves. We note, in particular, that the zonally averaged steady state was not determined.

In a paper by Charney (1959) he discusses several aspects of Phillips' model and experiment. One of the points of discussion is the use of lateral diffusion in the equations of motion at each of the two pressure levels with the diffusion coefficient equal to $10^5 \mathrm{m}^2\ \mathrm{s}^{-1}$. The same diffusion coefficient is used in the thermodynamic equation. It is one of Charney's main point that the use of lateral diffusion is undesirable because it will create a steady, zonally-averaged state in which the wind velocities will be extremely large (4480 m per s at the upper level and 1490 m per s at the lower level). The vertical wind shear in the proposed steady state would correspond to a north-south temperature gradient of about 1 C per km. Charney finds these values so extreme that he discards horizontal diffusion as a valid physical mechanism in the large-scale flow of the atmosphere and proceeds to design a different two-level quasi-geostrophic model in which the lateral diffusion is replaced by internal dissipation depending on the vertical wind shear.

While one may agree with Charney in his general conclusion it is still a question if the steady state of the zonally averaged structure and circulation in Phillips' model is as extreme as calculated by Charney. Since the author was unable to reproduce the solutions given (without any detailed derivation) by Charney it was decided to recalculate the steady state of the zonally-symmetric state of Phillips' model. The remaining part of the paper deals with this question.

It should be stressed that we are dealing with a quasi-geostrophic model in which the advective velocities are non-divergent. A consequence of the choice of model is that meridional transport of momentum and sensible heat is carried out by the eddies. One may indeed calculate the mean meridional circulation consistent with the model assumptions, but this circulation is so weak that it cannot provide the required transports of momentum and heat. A consequence of the model assumptions is therefore that when no eddies are permitted in the model integrations, we may determine the zonally averaged state from the heating in the model and the dissipation mechanisms introduced in it. This aspect simplifies the determination of the steady state because the major non-linear terms are disregarded from the beginning.

The large-scale eddies created by the instability of the zonal steady state can be studied by a numerical experiment in the way it was carried out by Phillips (1956). Charney (1959) treats the eddies as a perturbation problem, but due to the meridional variation of the basic state he had to use numerical methods to calculate the speed and the amplification rate of the waves as well as the structure of the developing disturbances.

Some aspects of the problems mentioned above may also be discussed using low-order models (Wiin-Nielsen, 1991, 1992)

2. The selected procedure

In the remaining part of the note we shall be dealing with zonally averaged quantities. We shall for convenience use a coordinate system in which the southern boundary is at $y = 0$ and the northern boundary at $y = W$. To be consistent with Phillips' model we shall take $W = 10^7$ m, corresponding to the distance from equator to pole. The relevant equations for the zonally averaged quantities disregarding the contributions from the eddies are given in (2.1) and (2.2).

$$\frac{d}{dt}[\frac{d^2\Psi_*}{dy^2}] = -\frac{\epsilon_4}{2}(\frac{d^2\Psi_*}{dy^2} - 2\frac{d^2\Psi_T}{dy^2}) + A\frac{d^4\Psi_*}{dy^4} \tag{2.1}$$

$$\frac{d}{dt}[\frac{d^2\Psi_T}{dy^2} - q^2\Psi_T] = \frac{\epsilon_4}{2}(\frac{d^2\Psi_*}{dy^2} - 2\frac{d^2\Psi_T}{dy^2}) + A\frac{d^2}{dy^2}[\frac{d^2\Psi_T}{dy^2} - q^2\Psi_T] - \frac{\kappa}{2f_o}q^2Q \tag{2.2}$$

These equations are obtained by adding and subtracting the equations applying at levels 1 and 3 in the standard notation for the levels of the two-level model. We shall use $\epsilon_4 = 4 \times 10^{-6}s^{-1}$ (corresponding to Phillips' k) and $A = 10^5 m^2 s^{-1}$. $\kappa = R/c_p = 0.286, f_o = 10^{-4}s^{-1}$, and $q^2 = 3.0 \times 10^{-12}m^{-2}$ (corresponding to Phillips' $2\lambda^2$). The heating Q is with the present coordinate system $Q = H_o(1 - 2y/W)$ giving a heating of H_o at the southern boundary and a cooling of the same rate at the northern boundary. We use $H_o = 4x10^{-3}Jkg^{-1}s^{-1}$.

In treating the steady state problem corresponding to (2.1) and (2.2) one could proceed by setting the left hand sides to zero and solving the remaining ordinary differential equations as apparently was done by Charney. It is, however, also possible to solve these equations by using the spectral methods. We shall prefer the latter procedure. To satisfy the same boundary conditions as postulated by Phillips we shall use series of cosine functions for the variables Ψ_* and Ψ_T. The series for the heating should be of the same form, but it will naturally be necessary to evaluate the Fourier coefficients separately for the given specification of the heating.

We write therefore:

$$\Psi_* = \sum \Psi_*(n)\cos(n\pi\eta)$$

$$\eta = \frac{y}{W} \tag{2.3}$$

with similar series for Ψ_T and Q. The single problem in using the spectral representation is to find the Fourier coefficients of the forcing. However, it may be determined by integration by parts that these coefficients are:

$$H(n) = \frac{4H_o}{n^2\pi^2}(1 - \cos(n\pi)) \tag{2.4}$$

indicating that all even coefficients are zero. Since linear terms only appear in the equations we may write the steady state equations as stated in (2.5).

$$\alpha(n)\Psi_*(n) - \beta\Psi_T(n) = 0$$

$$-\gamma(n)\Psi_*(n) + \delta(n)\Psi_T(n) = \mu(n)H(n) \tag{2.5}$$

The coefficients are given in (2.6), while the coefficient in the heating term is given in (2.7). The equations in (2.5) are solved for all n included in the calculation.

$$\alpha(n) = \frac{\epsilon_4}{2} + \frac{A}{w^2}n^2\pi^2; \beta = \epsilon_4$$

$$\gamma(n) = \frac{\epsilon_4}{2}\frac{n^2\pi^2}{n^2\pi^2 + q^2W^2}; \delta(n) = n^2\pi^2\frac{\epsilon_4 + \frac{A}{W^2}n^2\pi^2 + Aq^2}{n^2\pi^2 + q^2W^2} \tag{2.6}$$

$$\mu(n) = \frac{\kappa}{2f_o}\frac{q^2W^2}{n^2\pi^2 + q^2W^2} \tag{2.7}$$

When the solution is ready one finds the sum of the Fourier series. It is most convenient for our purposes to determine the values of u_* and u_T by using the Fourier series given in (2.8) for u_* with an analogous series for u_T. It is seen from the expression for $H(n)$ that these coefficients decrease rapidly due to the n^2 appearing in the denominator. It should therefore not be necessary to select n_{max} very large. A value of 10 has been used.

$$u_* = \sum_{n=1}^{n_{max}} \frac{n\pi}{W}\Psi_*(n)\sin(n\pi\eta) \tag{2.8}$$

One may also wonder how many Fourier coefficients are necessary to obtain a good approximation to the linear relation adopted by Phillips (1956). Figure 1 shows that a total of 10 components are a reasonable approximation. It will be noted that the approximation cannot be exact at the two boundary walls because all the cosine function will have a zero derivative at these places, while the linear specification gives slope different from zero. Figure 2, shows the two velocities u_* and u_T in the steady state. They are considerably larger in the middle of the channel than normal climatological values. On the other hand, the largest values are very much smaller than the values obtained by Charney (1959). It would appear that some error has been made in his determination of the zonal circulation. Figure 3 shows the zonally averaged winds in the steady state at 1000 hPa obtained by extrapolation. We note that in agreement with Phillips we obtain weak easterlies everywhere with the numerically largest value being about

1/2 m per s. Figure 4 shows the computed mean meridional circulation at the upper level. One finds, also in agreement with Phillips, a single cell with northward directed winds at the upper level. The mean meridional winds at the lower level are opposite of those at the upper level. We observe therefore a single cell of the Hadley type.

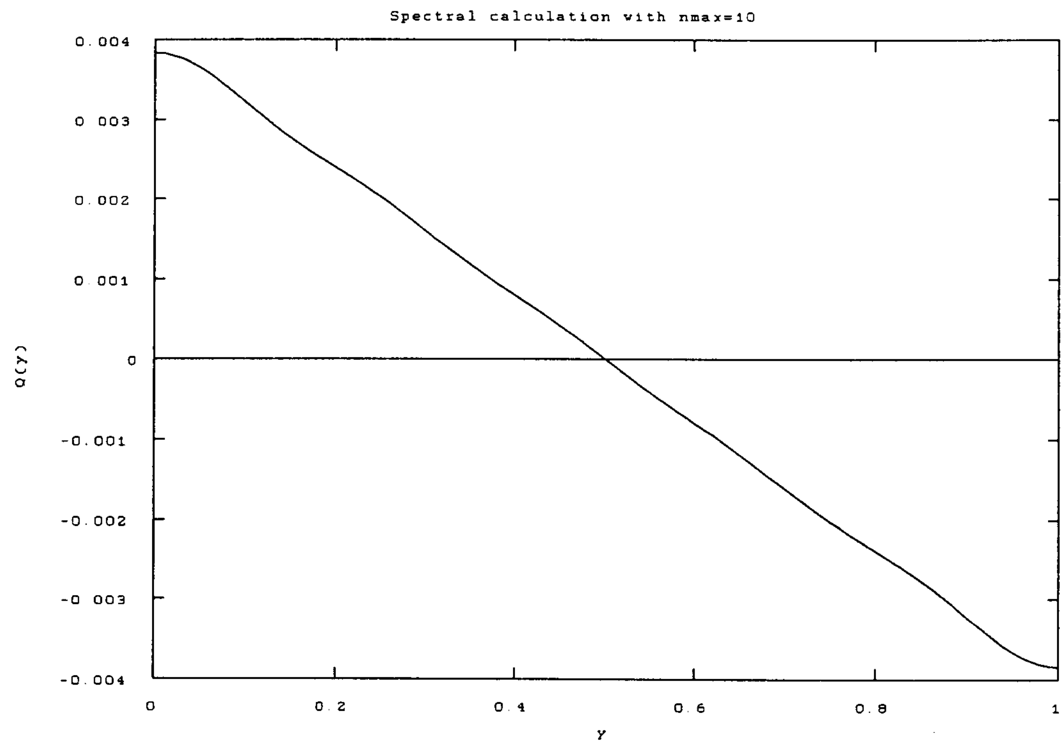

Fig. 1. Heating as a function of the meridional coordinate. The unit of the heating is $J\ kg^{-1}\ s^{-1}$.

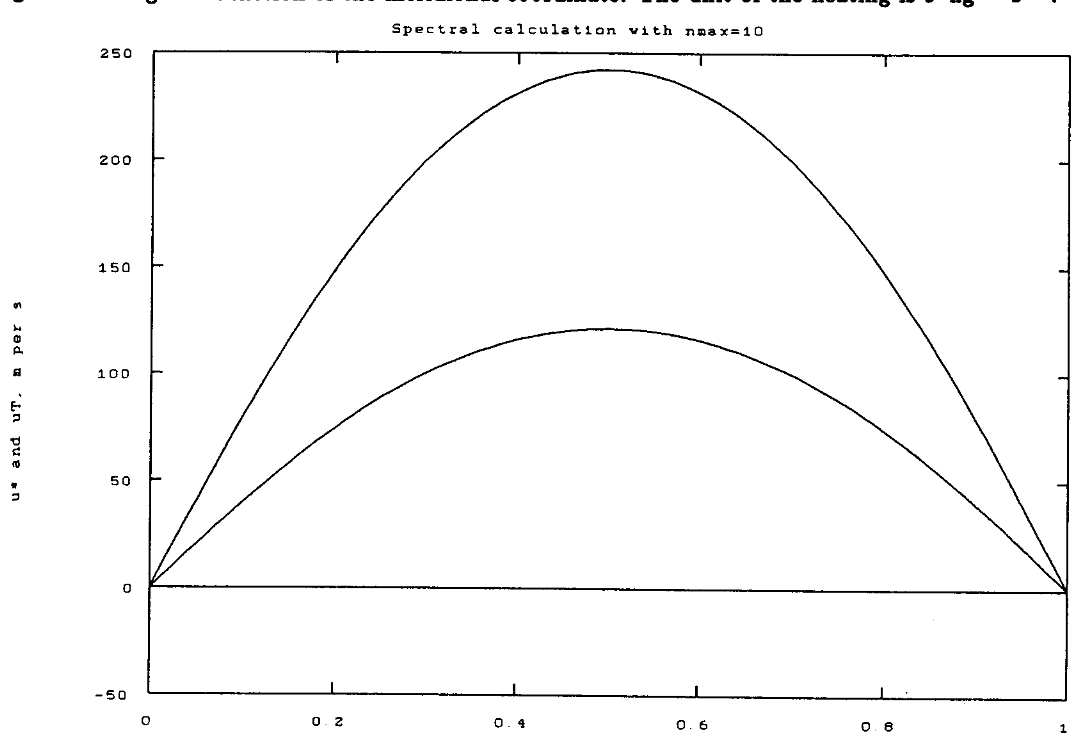

Fig. 2. The zonally averaged wind velocities as a function of the meridional coordinate.

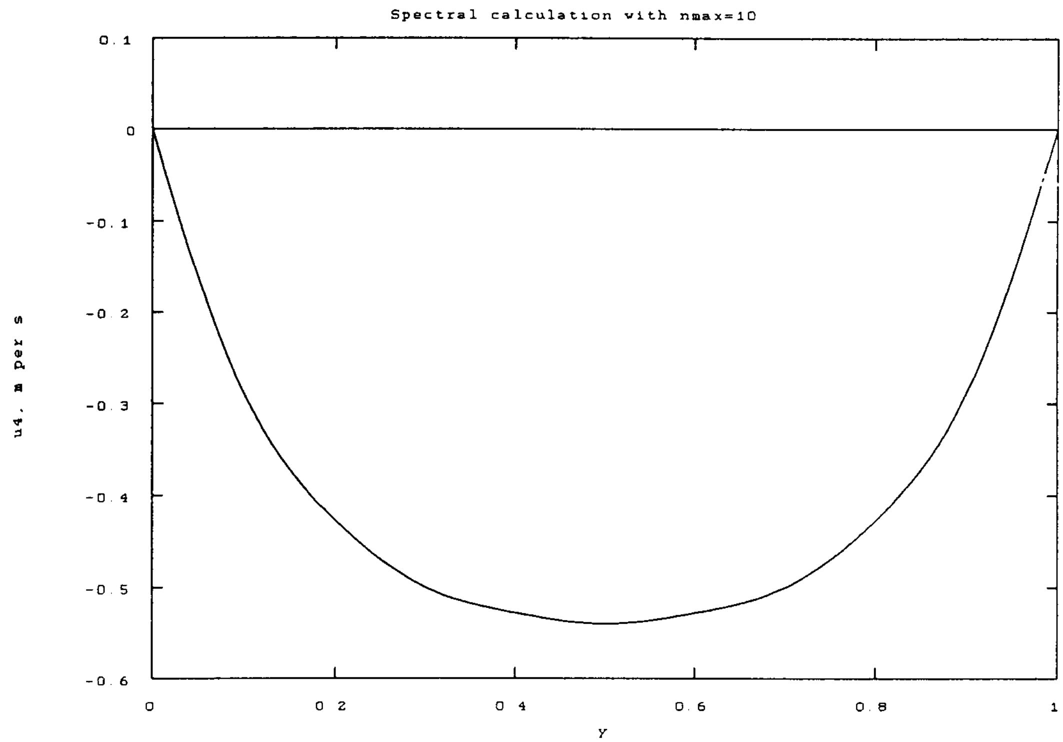

Fig. 3. The zonally averaged wind velocity at level 4 (1000 hPa)

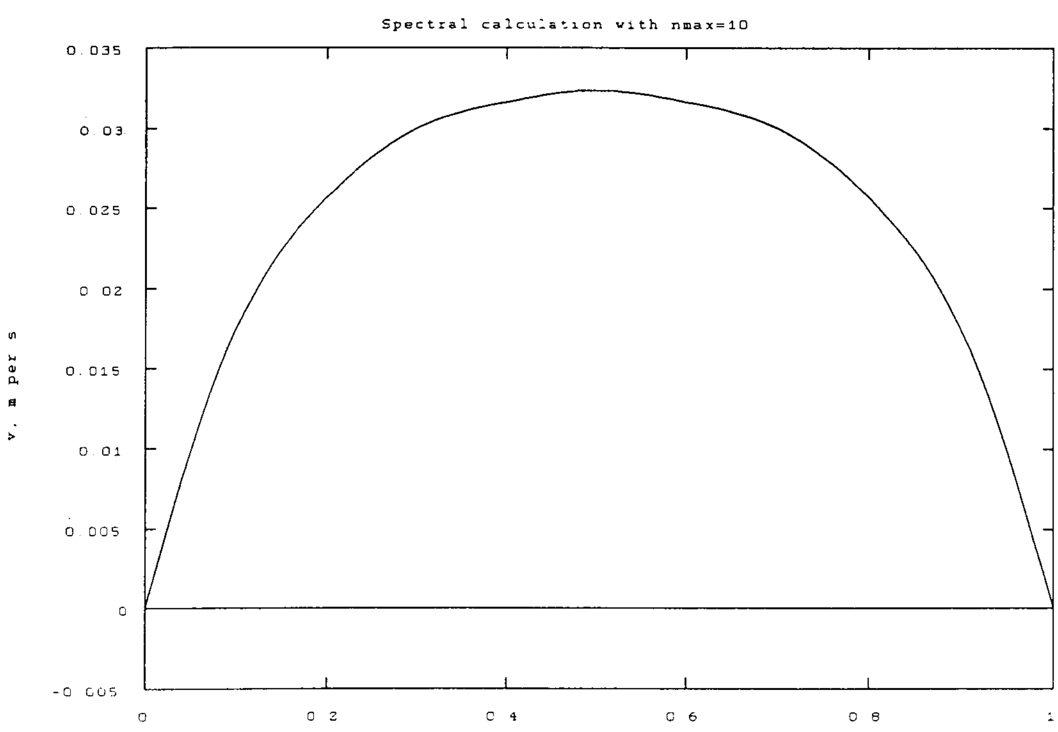

Fig. 4. The mean meridional velocity as a function of the meridional coordinate .

As mentioned above the values of the Fourier coefficients for the heating will decrease rapidly with the meridional wave number. It is thus of interest to make a calculation permitting just one component with $n = 1$. The zonal velocities are shown in Figure 5 which should be compared with Figure 2. It will be noted that only minute differences are present between the curves. This calculation replaces the linear relation of Phillips with a single cosine function.

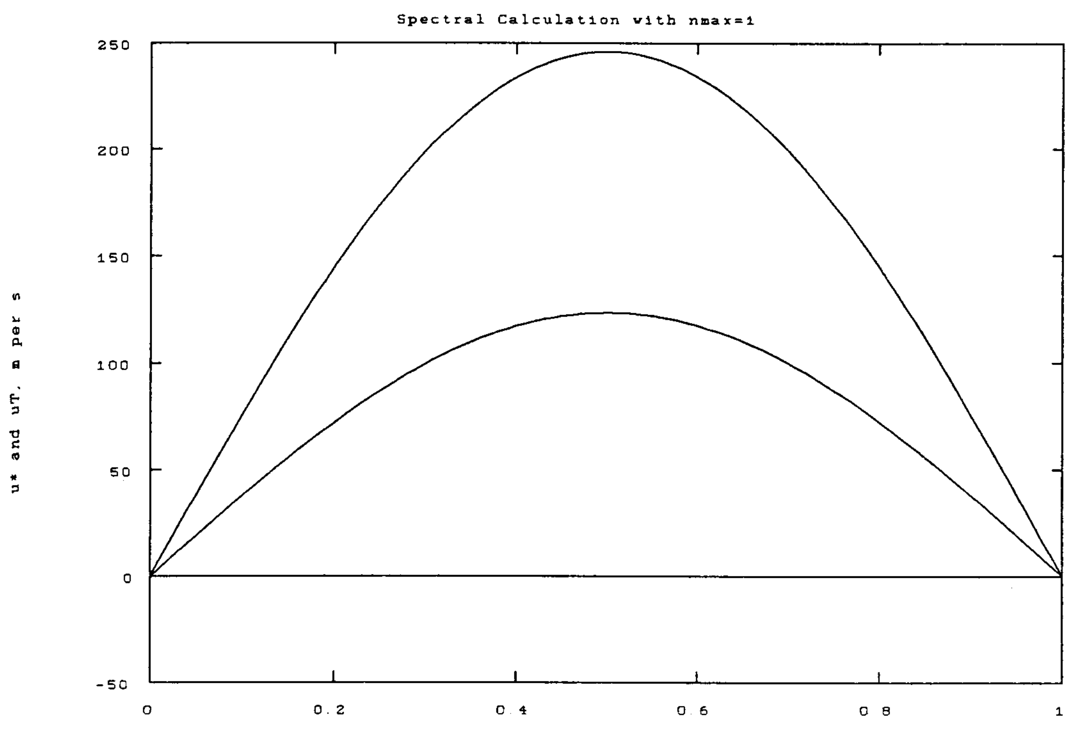

Fig. 5. The zonally averaged velocities as a function of the meridional coordinates for $n = 1$.

It was noticed in the introduction that Phillips integrated the zonal equations for 130 days before the perturbations were introduced. From the values of the zonal velocities in his paper it is obvious that the model did not at this stage make a good approximation to the steady state derived here. His purpose was only to obtain a zonal state where the vertical wind shear (or, equivalently, the horizontal temperature gradient) was sufficiently large to be a state which was unstable for small perturbations. It is, however, of interest to see what the spin-up time is for the model to approximate the derived steady zonal state. To answer this question we used the time-dependent zonal model with a single cosine component to make a long integration. The values of the streamfunctions for the single component are shown in Figure 6. It is seen that the asymptotic level is almost reached after $t = 4.32 \times 10^8$ seconds which is equivalent to 5000 days (13.7 years).

The procedure applying a spectral formulation to calculate the steady state of Phillips' model in the zonal state is relatively easy to use because of the linear nature of the equations. The model treated in this way should certainly arrive at the correct order of magnitude of the zonal parameters, but any spectral integration is restricted by the maximum wave number introduced in the integration. In the next section we shall report on a more accurate, but also much more cumbersome integration of the steady state equations.

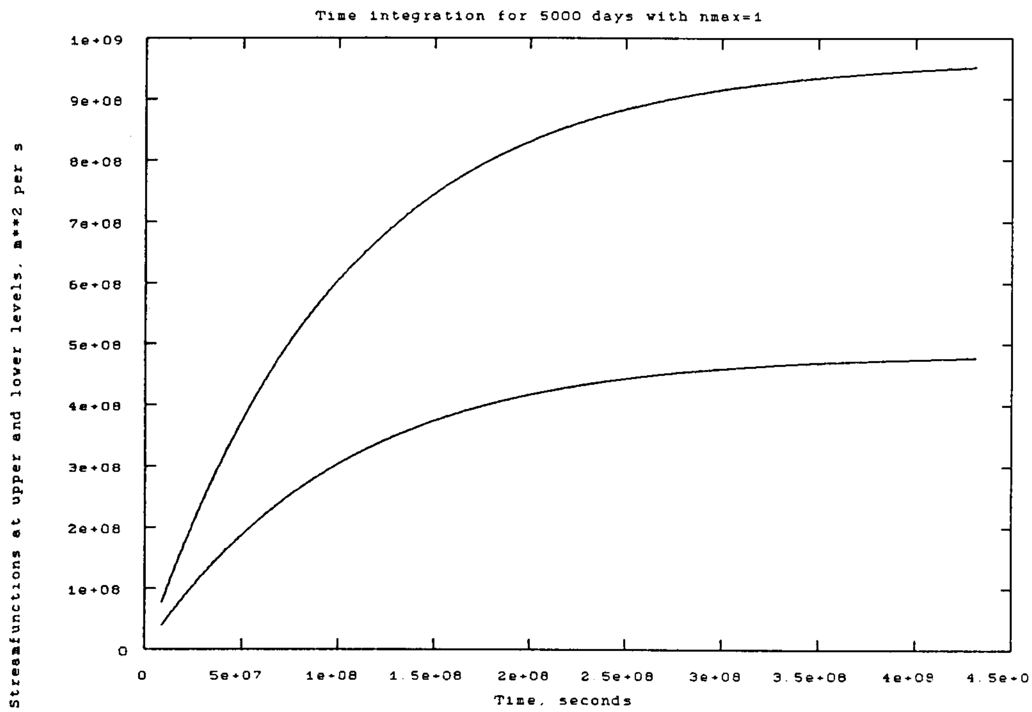

Fig. 6. The streamfunction amplitudes for $n = 1$ as a function of time. Total time for the integration is 5000 days.

3. A direct solution

The basic zonal steady state equations for Phillips' model may be written for the middle level and the thermal layer in the form:

$$A\frac{d^2\varsigma_*}{dy^2} - \frac{\epsilon_4}{2}(\varsigma_* - 2\varsigma_T) = 0$$

$$A\frac{d^2\varsigma_T}{dy^2} + \frac{\epsilon_4}{2}(\varsigma_* - 2\varsigma_T) - \frac{\kappa}{2f_o}q^2Q - Aq^2\varsigma_T = 0 \tag{3.1}$$

It will be seen that the first of the two equations may be solved for the thermal vorticity. When this expression for the thermal vorticity is introduced in the second equation we obtain a single equation of the fourth order in the vorticity at the middle level, see (3.2) in which we have used the independent variable $\eta = y/W$.

$$\frac{A}{\epsilon_4 W^4}\frac{d^4\varsigma_*}{d\eta^4} - [\frac{3}{2W^2} + \frac{q^2A}{\epsilon_4 W^2}]\frac{d^2\varsigma_*}{d\eta^2} + \frac{q^2}{2}\varsigma_* = -\frac{\kappa}{2f_o}\frac{q^2}{A}Q \tag{3.2}$$

Equation (3.2) is an ordinary differential equation with constant coefficients. To find a complete solution we need to solve the characteristic equation and to find a particular solution. The heating in Phillips' model with the selected coordinate system is given in (3.3).

$$Q = H_o(1 - 2\eta) \tag{3.3}$$

By inspection it is then seen that a particular solution can be obtained by guessing a solution of the same form in η, and we find that

$$\varsigma_* = \frac{\kappa H_o}{f_o A}(2\eta - 1) \tag{3.4}$$

The characteristic equation with the unknown R is a special fourth degree equation which may be solved for R^2. Inserting the values of the parameters used by Phillips it turns out that both roots for R^2 are positive leading to four real roots of the characteristic equation which may be denoted $\pm R_1$ and $\pm R_2$. We find $R_1 = 78.761$ and $R_2 = 9.8348$. The total solution may thus be written as stated in (3.5).

We use next the relation between the vorticity and the wind in order to

$$\varsigma_* = \frac{\kappa}{Af_o}H_o(2\eta - 1) + C_1\exp(R_1\eta) + C_2\exp(-R_1\eta)$$
$$+C_3\exp(R_2\eta) + C_4\exp(-R_2\eta) \tag{3.5}$$

convert (3.5) to an equation for u_* with the result given in (3.6).

$$u_* = -\frac{\kappa W H_o}{f_o A}(\eta^2 - \eta) - \frac{W}{R_1}C_1\exp(R_1\eta) + \frac{W}{R_1}C_2\exp(-R_1\eta)$$
$$-\frac{W}{R_2}C_3\exp(R_2\eta) + \frac{W}{R_2}C_4\exp(-R_2\eta) \tag{3.6}$$

To determine the integration constants we make use of the boundary conditions that u_* and $d^2u_*/d\eta^2$ vanish at the two boundaries. The four equations are solved in a straightforward way. The results are given in (3.7).

$$C_1 = K_1(1 - \exp(-R_1)); \quad C_2 = K_1(1 - \exp(R_1))$$
$$C_3 = K_2(1\exp(-R_2)); \quad C_4 = K_2(1 - \exp(R_2)) \tag{3.7}$$

The constants K_1 and K_2 are given in (3.8).

$$K_1 = \frac{\kappa}{f_o A}\frac{R_1}{R_2^2 - R_1^2}\frac{1}{\sin h(R_1)}$$
$$K_2 = \frac{\kappa}{f_o A}\frac{R_2}{R_1^2 - R_2^2}\frac{1}{\sin h(R_2)} \tag{3.8}$$

After these intermediate calculations we give the final solution for u_* in (3.9). The solution for u_T is then obtained from the first of the two equations in (3.1). A check on (3.9) shows that it satisfies the boundary conditions as stated above. These boundary conditions are identical to those employed by Phillips.

$$u_* = \frac{\kappa W H_o}{f_o A}[\eta(1-\eta) + \frac{2}{R_2^2 - R_1^2}(\frac{\sin h(R_1(\eta-1)) - \sin h(R_1\eta)}{\sin h(R_1)} - \frac{\sin h(R_2(\eta-1)) - \sin h(R_2\eta)}{\sin h(R_2)})] \tag{3.9}$$

The solutions from the described procedure are shown in Figure 7. The curves give u_* and u_T as a function of the meridional coordinate. Comparing Figure 7 with Figure 2 it is seen that although the orders of magnitude are the same in the two figures, some differences will be noticed. The maxima in Figure 7 are larger than those in Figure 2. The reason for this can be seen in Figure 1 in which the approximate generation of the straight line for the heating fails to reach the minimum and the maximum of 0.004 J kg^{-1} s^{-1}.

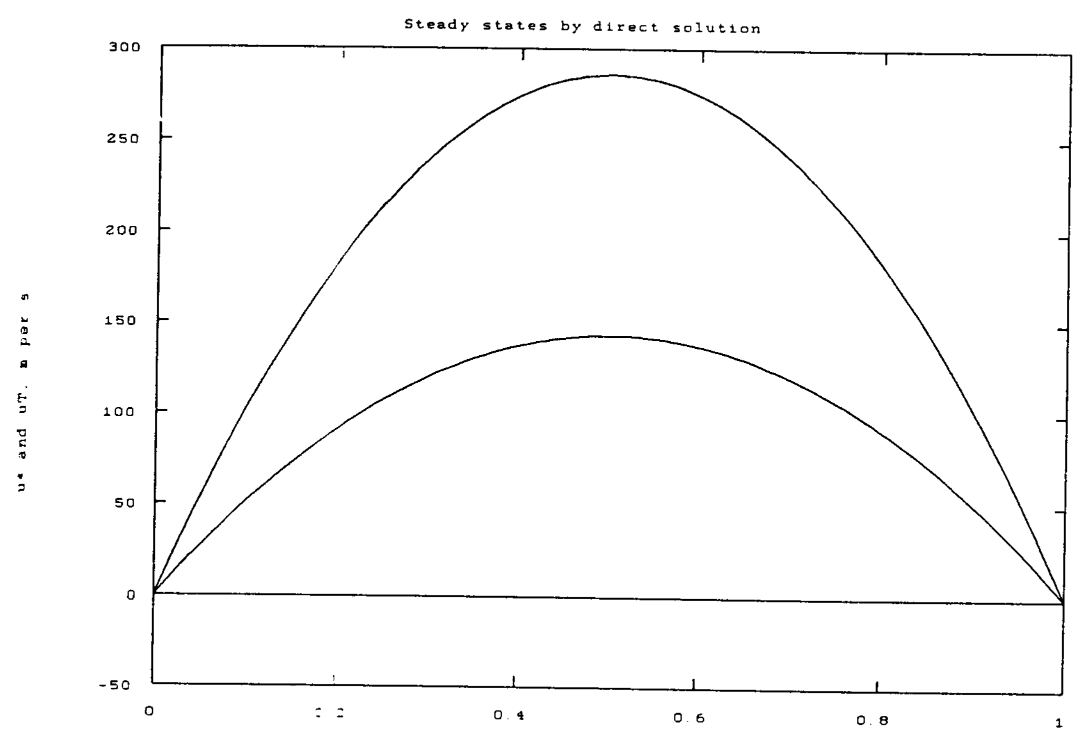

Fig. 7. The analytical solution for the zonally averaged winds.

It will be noticed that the particular solution, i.e. the first term in (3.9), satisfies the boundary condition that the zonal velocities vanish at the southern and northern boundaries. One may therefore wonder how much the first term of the total solution contributes and how much comes from the remaining terms. In Figure 8 we have plotted the total contribution by lines and the contribution from the first term by dots. It is seen that the two solutions are very close to each other. This does not mean that the contribution from the other terms can be disregarded because both terms are necessary to satisfy the second boundary condition requiring that the second derivative should vanish.

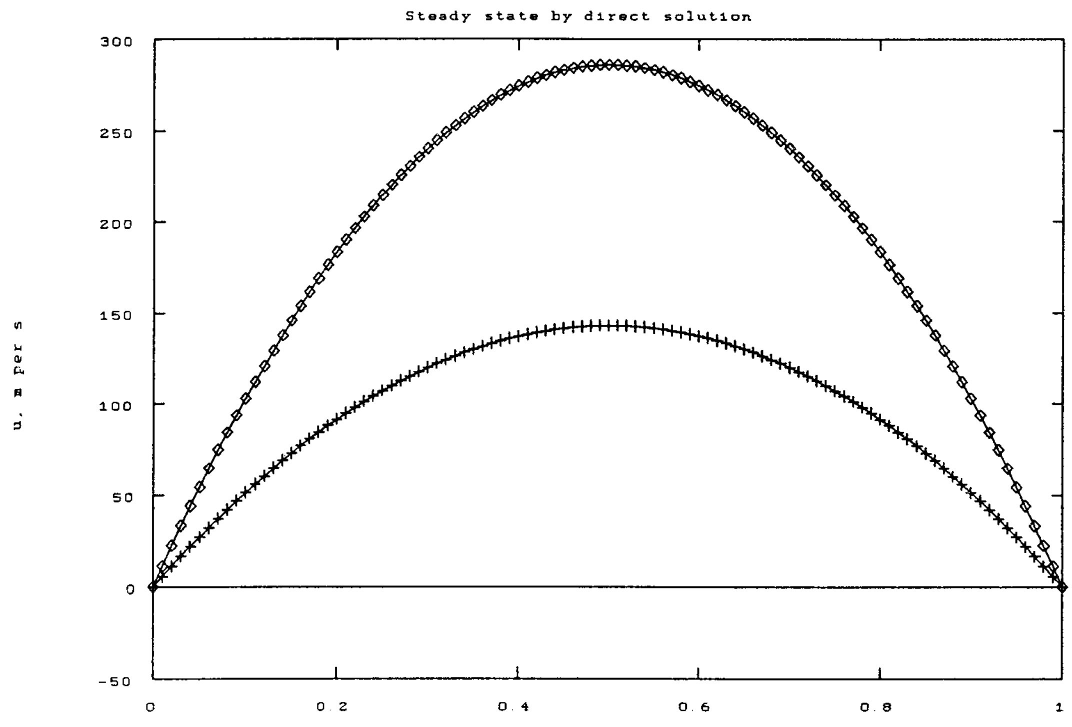

Fig. 8. The same as Figure 7. The solutions for $n = 1$ are indicated by diamonds and crosses.

The maximum value of u_* is essential determined by the value of the first coefficient in (3.9) multiplied by 1/4 which is the value of the particular solution in the middle of the channel. Using Phillips' values of the parameters we find the numerical value of 286 m per s in agreement with the curves for u_* in Figures 7 and 8.

From the exact solution we may make the same conclusion as we did from the approximate solution, i.e. that the steady state zonal velocities in Phillips' model are large compared with climatological means, but small compared with the extreme values obtained by Charney (1959).

4. A modified version of Charney's zonally averaged model

From the discussion by Charney (1959) one may get the impression that when the horizontal diffusion is abandoned and replaced by an internal atmospheric dissipation formulated by considering the vertical wind shear, the problems connected with too large velocities in the zonal steady state will disappear completely. This is, however, not the case. It can easily be demonstrated by computing the zonally-averaged state using the same formulation as earlier of the heating, i.e. a time independent linear function. The new formulation of the internal dissipation requires a frictional coefficient $\epsilon_T (\approx 6 \times 10^{-7} s^{-1})$ as derived by Charney (*loc. cit.*). The steady state equations are given in (4.1)

$$-\frac{\epsilon_4}{2}(\varsigma_* - 2\varsigma_T) = 0$$

$$-\frac{\kappa}{2f_o}q^2 Q - \epsilon_T \varsigma_T + \frac{\epsilon_4}{2}(\varsigma_* - 2\varsigma_T) = 0$$

$$Q = H_o(1 - 2\eta); \quad \eta = y/W \tag{4.1}$$

It is seen that $\varsigma_* = 2\varsigma_T$. Using exactly the same elementary technique as in the previous cases we find a thermal zonally averaged velocity as given in (4.2).

$$u_T = \frac{\kappa}{2f_o}q^2 H_o \eta(1 - \eta) \tag{4.2}$$

With the adopted numerical values of the parameters one finds that the maximum value of u_T is 35.75 m per s giving a maximum value of $u_* = 71.5$ m per s. These values are smaller than those determined with the horizontal diffusion, but still large compared with observed values.

A further modification is introduced by formulating the heating in a Newtonian form. Charney (1959) obtains this formulation by considering the radiation budget of the atmosphere using an extremely simple formulation and by adopting values of the planetary albedo and the absorptivity that would not be considered as realistic today. In addition, he neglects the absorption of the incoming short wave radiation by the water vapor and clouds. In the modified form we shall use the same idea, but attempt to be more realistic with respect to the treatment of the radiation budget. Such a model has been formulated by Christensen and Wiin-Nielsen (1996) based on a detailed description of the radiation budget. The model equations are given in (4.3).

$$b(1 - \alpha)S + c\sigma T_s^4 - 2\sigma T_a^4 = Q$$

$$(1 - b)(1 - \alpha)S - \sigma T_s^4 + \sigma T_a^4 = 0 \tag{4.3}$$

In these budget equations T_a and T_s are the atmospheric and the surface temperatures, respectively, a the planetary albedo, b the absorption coefficient for the incoming short-wave radiation, c the absorption coefficient for the outgoing long-wave radiation, S the solar constant divided by 4, and σ the Boltzman constant. It has been assumed that a balance is maintained at the surface of the Earth. T_s can be eliminated by solving the second equation for this variable and inserting in the first equation. The average atmospheric temperature T_a that would exist for $Q = 0$ is

$$\overline{T}_a = [\frac{(b + c(1 - b))(1 - \alpha)}{(2 - c)\sigma}S]^{1/4} \tag{4.4}$$

For $a = 0.31$, $b = 1/3$, $c = 0.92$ and $S = 344$ W m^{-2} we find that the averaged temperature is 246 K. To express Q in a convenient form we linearize around the averaged temperature, express

S, which really varies as cosine of the latitude at the time of an equinox, as a linear function of the meridional coordinate y and use the first two terms of the series expansion $(1+x)^m$. We obtain finally after some rearrangements the expression for the heating given in (4.5).

$$Q = \frac{1}{4}(2-c)\sigma\overline{T}_a^{\,4}[1 - 2\eta - \frac{4T_a}{\overline{T}_a}] \tag{4.5}$$

in which T_a now means the deviation from the averaged atmospheric temperature. In the model we use the thermal streamfunction and not the temperature itself, but we convert between the two quantities by using $T = (2f_o\Psi_T)/R$ where R is the gas constant. Finally, the steady state, zonally averaged equations become:

$$-\frac{\epsilon_4}{2}(\varsigma_* - 2\varsigma_T) = 0$$

$$-\frac{1}{4}\frac{\kappa}{2f_o}q^2Q_s[1 - 2\eta - \frac{4\Psi_T}{\overline{\Psi}_T}] + \frac{\epsilon_4}{2}(\varsigma_* - 2\varsigma_T) - \epsilon_T\varsigma_T = 0 \tag{4.6}$$

It is seen from the first equation that the middle level vorticity is twice the the thermal vorticity. The second equation can then be solved for the thermal vorticity.

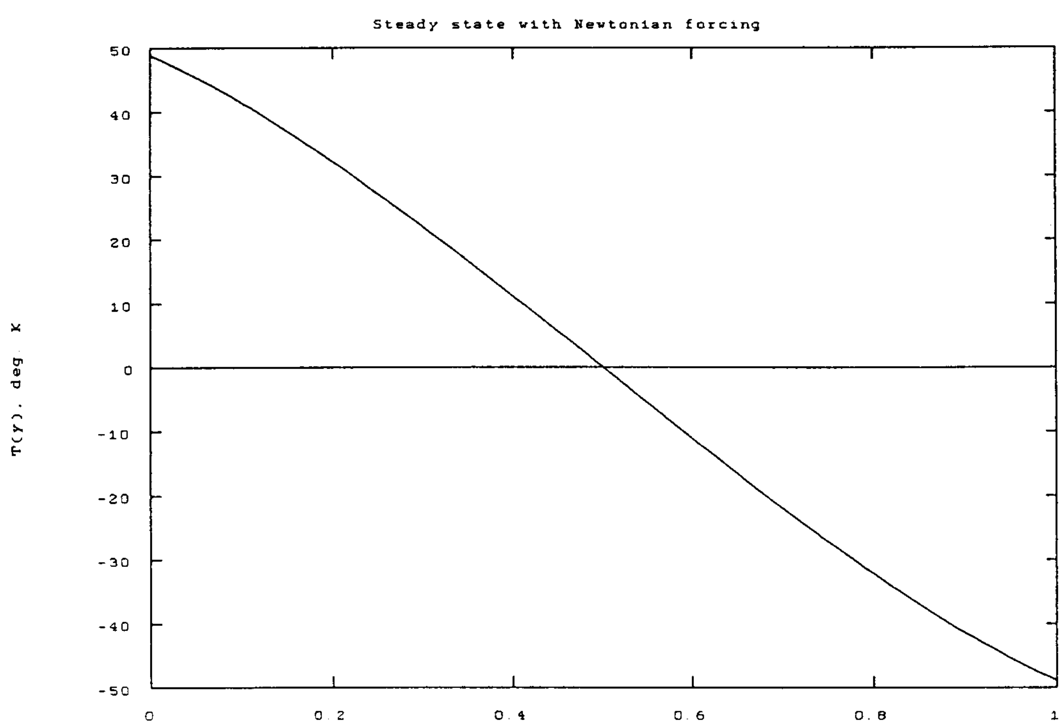

Fig. 9. The solution for the deviation of temperature from the meridional mean with Newtonian heating.

Figure 9 shows the temperature between the southern and the northern walls. The total temperature difference is a little less than 100 K which is large compared to the climatological mean value as it should be because the baroclinic eddies will decrease the difference as they develop.

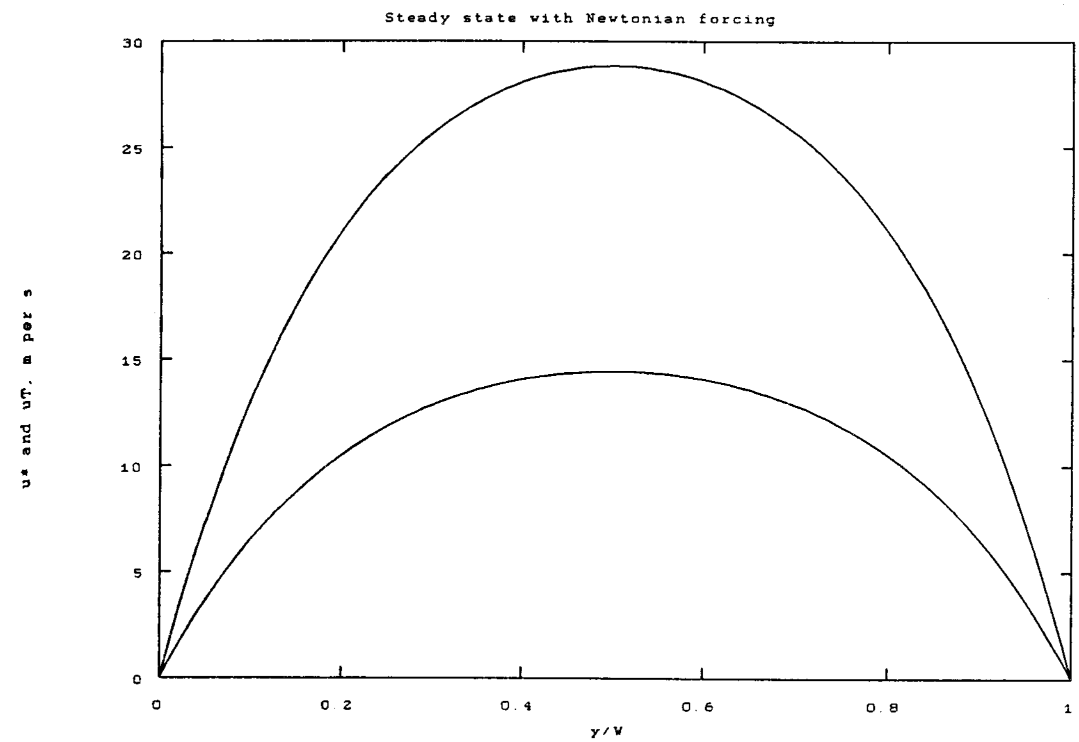

Fig. 10. The solution with Newtonian heating for the two zonally averaged wind velocities.

The zonal winds in the steady state solutions are displayed in Figure 10. They are slightly smaller than those found by Charney (1959), but still large enough to create a baroclinically unstable zonal state.

5. Summary and conclusions

A few two-level quasi-nondivergent models of the zonally averaged state has been investigated. In all cases the magnitude of the zonal winds were determined as a result of a given heating. The heating has in all, but one case been given as a fixed, linear function of the meridional coordinate, but in the last example the heating was formulated in a Newtonian form.

In the first example based on a fixed heating, boundary layer dissipation and horizontal diffusion and thus simulating Phillips' (1956) original model it was shown that the resulting zonal winds became much smaller than the results obtained by Charney (1959), but still quite large giving a zonal state far removed from reality with respect to the magnitude of the zonal winds and the implied meridional temperature gradient. To be sure that no errors should creep into the solutions these were carried out using a spectral model as well as a direct analytical solution of the problem. Excellent agreement was obtained.

The second example disregarded the horizontal diffusion as a dissipation mechanism and introduced as a replacement an internal dissipation proportional to the vertical wind shear. This model was first tested with the time-independent heating. The results show that the zonal winds were reduced compared to the first example, but not in a sufficient way. Reasonable results were finally obtained with a model in which the fixed heating was replaced by a Newtonian forcing derived from a linearized version of the heat budget of the atmosphere.

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