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TESTING FOR UNIT ROOTS: MEXICO'S GDP

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Abstract

The document presents an analysis of the stochastic nature of the gross domestic product of Mexico for the period 1900-2001. Several specifications to test for the existence of unit roots are presented. The conventional tests, Dickey Fuller, Augmented Dickey Fuller and Phillips-Perron, indicate that the series is non-stationary and integrated of order 1. This result is robust to the inclusion of exogenously and endogenously determined structural breaks. Interestingly, when structural breaks are determined endogenously, a structural break in 1907 is identified. We interpret this results as suggesting that setting the date of a structural break ex-ante might not be the most efficient procedure when testing for unit roots.

Key words: unit roots tests, structural break, gross domestic product of Mexico.

Clasificación JEL: C10, C22, C52

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Introduction

In recent years, there has been an increasing interest in improving the methodology employed to test for unit roots. Researchers have focused their attention on macroeconomic series which are most likely to have unit roots, and that have relevance at the policy making level, including the gross domestic product (GDP), consumption, exchange rates, and the money supply. Among these series the GDP of the United States, especially after the second world war, has received particular attention among economists. There has been extensive work in trying to pin down the precise nature of the GDP. Some of the most influential papers on this subject include Nelson and Plosser (1982), Stock and Watson (1986), Christiano and Eichenbaum (1990), and Rudebusch (1993). By now, the consensus on the stochastic nature of the US GDP is clear, the data generating process presents a unit root, that is, the series in non-stationary and integrated of order 1.

For the case of Mexico, Perez-Lopez (1995) develops an econometric model to forecast the Mexican GDP. In particular, based on a general equilibrium model of a small open economy, the author shows that the real exchange rate, industrial production and GDP are cointegrated. Likewise, Carstens and Reynoso (1997) estimate Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) unit root tests on the logarithm of the Mexican GDP and find that the series is non-stationary. Although these studies have provided important insights into the nature of Mexico's GDP, it is evident that more research is needed to precisely identify the stochastic nature of the series. In particular, the previously mentioned studies test for unit roots employing the traditional specifications but without considering one that allows for structural breaks. This fact seems surprising

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since visual examination of Mexico's GDP suggest that the series presents at least one structural break in 1982, and, as Perron (1989) shows, test of unit roots that do not allow for structural breaks when the series appear to present them, are biased toward producing results suggesting that the series is non-stationary. Hence, in the case of Mexico's GDP it is necessary to consider at least one specification that allows for structural breaks when testing for unit roots.

In this document we present a novel procedure to evaluate the existence of unit roots in a series that presents structural breaks. Specifically, we implement a methodology suggested by Zivot and Andrews (1992) by which the structural break in a series is determined endogenously. That is, for each period on the series a test for a unit root is conducted considering the possibility of the existence of a structural break. We shall argue that this procedure is a more efficient test of the existence of unit roots compared to the conventional ADF and PP tests when the series examined present structural breaks. Although determining the precise stochastic nature of the series is an interesting exercise in and of itself, the results of the test become important when the GDP series, or any time series for that matter, is to be analyzed in the context of cointegration or vector autoregression models, since the results of these models rely upon the characteristic of the data generating process of the series.

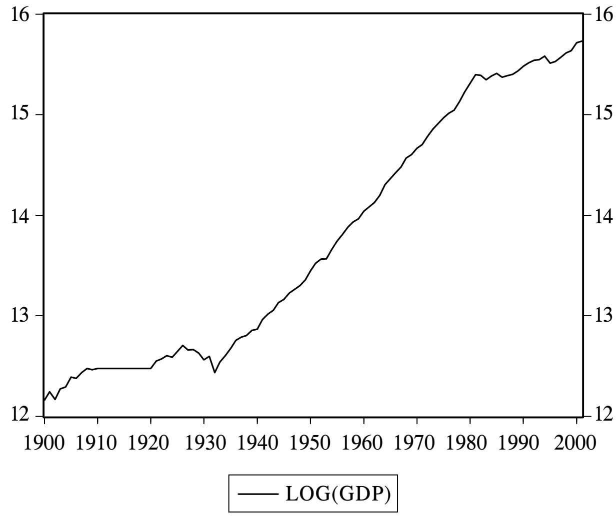
The remaining of the document is organized as follows: Section I presents a description of the data and a brief theoretical argument to justify the presumption that the GDP series presents a unit root. In Section II we perform various unit root tests including the test for endogenously determined structural breaks. Section III concludes.

Section I

I.I Data

The series we consider for the analysis corresponds to the annual Mexico's GDP for the period 1900-2001. We obtained the data primarily from two sources, the system of national accounts of the National Institute of Statistics, Geography and Informatics (INEGI) and the Ministry of Finance (SHCP). The logarithm of the series is presented in Graph 1.

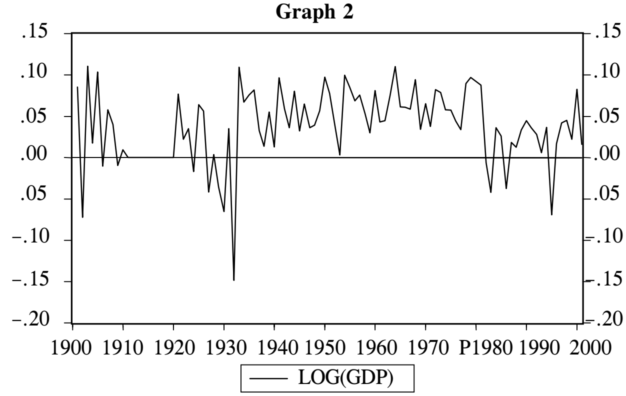
Graph 1



From 1900 to about 1935 the series presents a modest growth, this is primarily due to the fact that during the Revolutionary War the data for the GDP remains constant. After this period, the series shows an upward trend until about 1981 when there appears to be a structural break. From that year onwards the slope of the trend is clearly lower than the slope in previous decades.

The evolution of the GDP can perhaps be more precisely captured with the annual growth series presented in Graph 2.

Graph 2



Notice that, as it was previously mentioned, from 1910 to about 1920 the GDP exhibits zero growth. In 1931 there is a significant decrease in the growth rate and from 1932 to about 1980 the series presents an almost constant growth rate. Not surprisingly, there are several periods of negative growth in the 1980's and 1990's, particularly pronounced is that observed in 1995.

I.II The Theory

From Graph 1, it is evident that the GDP evolves over time following an increasing trend. This observation suggest that the GDP might present a unit root, since the mean of the series appears to be time-dependent. In fact, from a simple growth model it can be shown that innovations to the GDP might have permanent effects, so that the series behaves similar to a random walk. Consider, for instance, a textbook AK model in which there is no technological change and the size of the population is constant. Suppose that the representative agent has preferences given by

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{c_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

with $\sigma > 0$.[1] And the production technology is characterized with

$$y_t = Ak_t$$

where y is production, A is a productivity coefficient and to labor ratio. At each perio t the level of investment is determined by the marginality condition $r_{t+1} = A$ were r is th interest rate. Moreover, the goods market equilibrium is given by

$$c_t + i_t = y_t = Ak_t$$

where i denotes per capita investment. It can be shown that in equilibrium the growth rate of the economy is constant over time an given by

$$\frac{c_{t+1}}{c_t} = 1 + \bar{g}$$

Similarly, under some conditions the equilibrium capital investment is derived as

$$i_t = \frac{\bar{g}}{A} y_t$$

Since $y_t = c_t + i_t$, substituting the previous expressions and solving the model one obtains

$$c_t = \frac{A - \bar{g}}{A} y_t$$

An implication of this result is that a change in the saving rate, for instance, has a permanent effect on the rate of growth of the economy. That is, output will not necessarily revert to a determined trend after experiencing an innovation. In that case, the output series would behave as a random walk.

Section II

II.I Traditional Unit Roots Tests

As a first approximation in the analysis of the stochastic nature of the GDP, in Table 1 we present the autocorrelation for the series in levels and in first difference or twelve periods.

Table 1
Autocorrelation

	1	2	3	4	5	6	7	8	9	10	11	12
Level	0.978	0.955	0.932	0.909	0.886	0.863	0.840	0.815	0.791	0.765	0.739	0.712
First Difference	0.111	0.337	0.152	0.116	0.147	0.112	0.067	0.226	0.082	0.178	0.062	0.086

[1] The notation is standard in the literature, for a detailed derivation of the model see for instance Obstfeld and Rogoff (1997).

The autocorrelation series for the level exhibits a gradual decay, behavior that is characteristic of time series that posses a unit root. Notice that this regularity disappears when the series is differentiated. Together, these two characteristics suggest that the series is non-stationary and integrated of order 1, I(1). In order to confirm this perception formal unit root tests are conducted next.

The simples test for a unit root follows the methodology employed by Dickey Fuller (DF) (1979, 1981) who consider an autoregressive process of order 1, AR(1), as follows:

$$y_t = p y_{t-1} + u_t \quad [1]$$

with the assumption that u_t is an identical independent distributed (IID) sequence of random variables. Under the null hypothesis, $H_0 : \rho = 1$, y_t is a non-stationary random walk variable without drift. In the case of Mexico's GDP, however, it is clear that the series is characterized by a drift and a trend, thus, the specification for the first unit root test is augmented to consider both terms and the estimating equation is specified as follows

$$y_t = \alpha + \beta_t + \rho y_{t-1} + u_t \quad [2]$$

For estimation purposes we consider the reparameterization of (2) subtracting y_{t-1} from both sides

$$\Delta y_t = \alpha + \beta_t + \phi y_{t-1} + u_t \quad [3]$$

where $\phi = \rho - 1$ and the null hypothesis becomes $H_0 : \phi = 0$.

The results obtained by estimating this equation would be valid if the assumption about the distribution of u_t were correct, however, if the error term is not IID, that is, if the data generating process was characterized by serial correlation of an order greater than 1, then the results of the test would be invalid. To address this issue, the DF specification can be augmented (ADF) by adding lagged terms to the right hand side of the equation as follows:

$$y_t = \mu + \gamma y_{t-1} + \sum_{j=1}^{k} \lambda_j \Delta y_{t-j} + \varepsilon_t \quad [4]$$

where k is the terminal number of lags and considers the null hypothesis. $H_0 : \gamma = 0$. Dickey and Fuller (1981) demonstrate that if this regression is run when u_t is an autoregressive process greater than 1, the limiting distribution and critical values are still valid.

Another methodology that allows for the correction of an AR(p) with $p > 1$ is that suggested by Phillips-Perron (PP) (1988). In this case, the correction of the higher order correlation is performed by a non-parametric method. The estimating equation under this methodology is the AR(1) process

$$\Delta y_t = \alpha + \beta y_{t-1} + \varepsilon_t \quad [5]$$

where the null hypothesis is $\beta = 0$.

The previously three methodologies, DF, ADF and PP, are the most commonly used to test for unit roots. In Table 2 we present the results of these tests on the GDP series for Mexico. The number of lags included, 2, was calculated following the Akaike and Schwarz criteria.

Table 2
Unit Root Tests

Series	*DF Test*	*Critical Value**	*ADF Test*	*Critical Value**	*PP Test*	*Critical Value**
log(GDP)	–1.61	–3.45	–1.59	–3.45	–1.65	–3.45
Dlog(GDP)	–9.06	–3.45	–3.83	–3.45	–9.36	–3.45

* at 5%.

The result of the DF, ADF and PP tests indicate that the series is non-stationary and integrated of order 1. These results confirm the evidence provided by the auto-correlogram previously presented. In general, researches conclude their analysis of unit roots with the results form these tests and deduce that the series is non-stationary, however, as noted by Perron (1989) when a series presents structural breaks, as it is the case with Mexico's GDP, the results of the conventional unit roots tests might be biased. In particular, the author finds that when the series exhibits a breaking trend, the unit root hypothesis cannot be rejected even asymptotically. In the case of Mexico's GDP, it is apparent from Graph 1 that there are breaks in the trend in 1932, 1983, and 1995, hence, a unit root test that allows for structural breaks is desirable.

II.II Unit Roots Tests that Allow for Structural Breaks

Perron (1989) suggests the following model to test for unit roots when the series present a structural break

$$y_t = \mu + \alpha t + \beta y_{t-1} + \gamma DU_t + \delta DT_t + \phi D(TB)_t + \sum_{i=1}^{i=k} \psi_i \Delta y_{t-1} + \varepsilon_t \quad [6]$$

Where TB indicates the period of the structural break.

DU_t = 1 if t > TB and 0 otherwise

DT_t = t if t > TB and 0 otherwise

$D(TB)_t$ = 1 at t = TB + 1 and 0 otherwise

Under the unit root hypothesis $\beta = 1$ and $\alpha = \delta = 0$, whereas under the trend stationary hypothesis, $\beta < 1$, α, δ, γ non-zero, and ϕ close to zero. This specification is clearly more efficient than the ADF or the PP test since they do not allow for a structural break.

We estimate (6) for each of the three possible structural breaks in the series, 1932, 1983 and 1995. The results are presented in Table 3.

Notice that the values for the parameter are, in all cases, close to 1 and significant. Thus, the hypothesis of non-stationary cannot be rejected.[2]

Selecting the date of the structural break exogenously as we did in the previous exercise, however, might not

Table 3
Unit Root Tests with Exogenous Structural Break

Structural Break Date	μ	α	β	γ	δ	ϕ
1932	0.150	–0.002*	0.992*	0.014	0.002	0.037
	(0.372)	(0.001)	(0.030)	(0.051)	(0.001)	(0.045)
1983	0.224	0.001*	0.982*	–0.045	0.000	0.031
	(0.183)	(0.000)	(0.016)	(0.193)	(0.002)	(0.046)
1995	0.294	0.001	0.976*	1.293	–0.013	–0.058
	(0.177)	(0.001)	(0.015)	(1.314)	(0.013)	(0.061)

* Significant at 10%.
Standard Errors in Parenthesis.

[2] Formal t tests for the hypothesis $\beta = 1$ were performed, in all cases the hypothesis could not be rejected. The results are not reported for brevity, for the same reason the values for the parameters ψ's are not shown.

be the most efficient methodology. In other words, it is not clear that the dates we choose are precisely those that correspond to the dates of the structural breaks. For example, the following figures present two periods in which the GDP appears to have experienced a structural break.

Figure 1

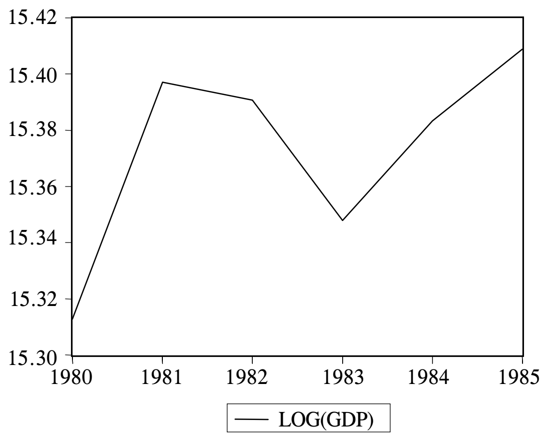
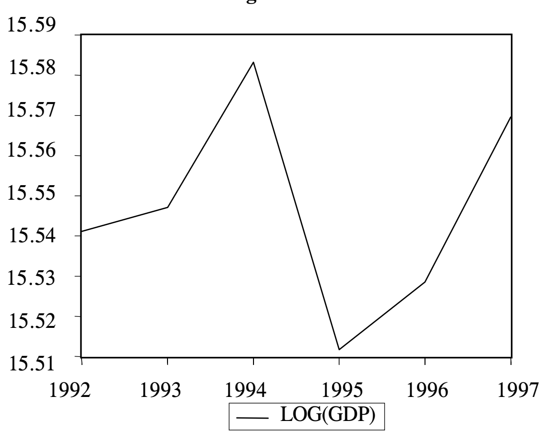


Figure 2



From this evidence some may argue, for instance, that the apparent break in 1983 was not really on that year but in the previous year, 1982, the year of the nationalization of the banking system. Similarly, it could be argued that the structural break in 1995 might not correspond to that year, since the economic crisis began toward the end of 1994. Hence, it might be somewhat imprecise to set the structural break date exogenously.[3] To address this issue, several methodologies have been suggested to allow for the determination of the date of the structural break endogenously, including those advanced by Zivot and Andrews (1992), Banerjee, Lumsdaine and Stock (1992) and Perron (1990).

In this document we implement the methodology suggested in Zivot and Andrews (1992). The authors present a procedure whereby a test statistic is estimated for each period while, simultaneously, allowing for the possibility of a structural break. Specifically, an equation that includes a variable to capture a structural break is estimated in each individual period, a test statistic is obtained and then, the test statistic with the most negative value is compared with the critical values. The tests assumes that the date on which the most negative value appears is the date of the structural break.

The test statistic for this procedure (Z_{value}) is obtained by estimating the following algorithm:

$$Z_{value} = (Sto^2/Stl^2)^{1/2} * t\alpha - 1 * [(Stl^2 - Sto^2)/St^l] * (T * \alpha\alpha/Su)$$

where $\alpha\alpha$ = OLS standard error for $\alpha, t\alpha = \frac{\alpha - 1}{\sigma\alpha}$

$$Stl^2 = \frac{1}{T}\sum_{t=1}^{t=\tau} u_t^{\ 2} + \frac{2}{T}\sum_{t=1}^{l} W_{tl} \sum_{t=\tau-1}^{T} u_t u_{t-\tau} \quad W_{tl} = \frac{1-\tau}{l+1}$$

$$l = 4*(T/100)^{2/9}, Sto^2 = \frac{1}{T}\sum_{t=1}^{\tau} u_t^2$$

$$Su^2 = \sum_{t=1}^{\tau} u_t^2 / T - K$$ and K is the number of parameters.

[3] Notice that in both periods, 1982-1983 and 1994-1995, there are two turning points corresponding to each year in each period. As such, it is not possible, by visual examination, to determine in which of the two years the structural break occurred. It would be equally correct, or wrong, to set the date of the break at either year. This is precisely the problem of trying to set the dates of structural breaks exogenously. It is difficult to justify on statistical grounds the choice of a date. The methodology employed next addresses this shortcoming.

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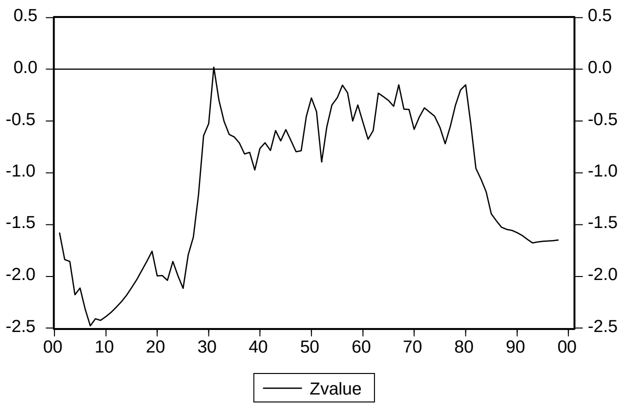
The estimating equation corresponds to equation (3') in Zivot and Andrews (1992) and is defined as follows:[4]

$$y_t = \mu + \theta DU_t(\lambda) + \beta t + \gamma DT_t{*}(\lambda) + \alpha y_{t-1} + \sum_{j=1}^{k} c_j \Delta y_{t-j} + \varepsilon_t \quad [7]$$

where $DU_t(\lambda) = 1$ if $t > T\lambda$, 0 otherwise; $DT_t^*(\lambda) = t - T\lambda$ if $t > T\lambda$, 0if, 0 otherwise; if, 0 otherwise. $T\lambda$ corresponds to the period for which the hypothesis of a structural break is tested.

We implement the test by recursively estimating this equation and calculating the Z_{value} for each period in the entire sample.[5] The results of the test are presented in Graph 3

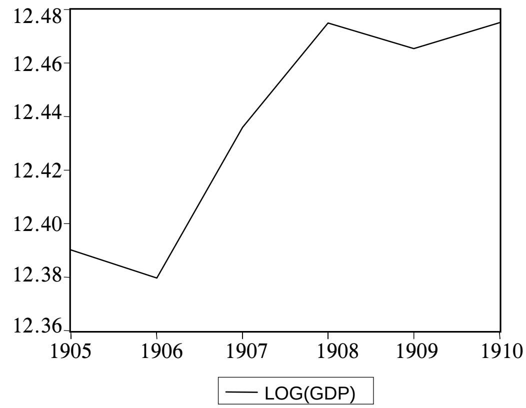
Graph 3



The lowest value is –2.48 and corresponds to the year 1907. The critical values for this specification are –5.08, -5.30 and –5.57 at the 10%, 5% and 1% confidence interval respectively. These values can be found in Table 4 in Zivot and Andrews (1992). Hence, at any significant level the hypothesis of non-stationarity cannot be rejected.

In addition, as it was previously mentioned, this methodology allows for estimating more accurately the date of the structural break, in this case, that date would be 1907.[6] At first sight, this result might seem unreasonable, after all, no significant event occurred in that year. Figure 3 shows the GDP series around that time.

Figure 3



Notice that there are two turning points, one in 1906 and another in 1908. Why would this test determine the date of the structural break on a different period as that apparent in the data? Christiano (1992) provides an answer. In particular, the author suggest that determining the structural break exogenously, as it is evident in the data, might not necessarily be the most efficient methodology, since this choice will most likely be correlated with the data and in that case, the finite sample and asymptotic distributions of the statistics would not be valid. Allowing the data "to talk" on the other hand, eliminates the correlation problem and provides a more robust result. Thus, it might not always be the case that the endogenously determined structural break coincides exactly with the dates apparent in the

[4] Our estimating equation does not include lagged values since we found no evidence of temporal dependence in the disturbances.

[5] The estimation of the Z_{value} for each period is computed with a program developed in the econometric package Eviews.

[6] Notice that for the more recent history the lowest Z_{value} occurs in 1994. Once again, one would expect the date of the structural break to be in 1995, since it was in that year when the GDP experienced a dramatic decrease. Evidently, the endogenously determined structural breaks are not correlated with the data and hence, it can be argued that the determination of the same is more reliable using this methodology.

data.[7] Also, one can argue that if the unit root hypothesis is rejected under such an demanding model (determine the structural break endogenously) it would also be rejected under less rigorous assumptions.

Section III. Conclusion

In contrast with the GDP from the USA, the stochastic nature of Mexico's GDP has hardly been analyzed. Usually, the series is assumed to be non-stationary and integrated of order 1. The assumption follows results that are produced by conventional unit root tests including the ADF and PP tests. In this document, we show that it is important to consider structural breaks in unit root analysis of time series. Using annual data of Mexico's GDP from 1900 to 2001 we find that the non-stationarity result is robust to the inclusion of structural breaks in 1932, 1983 and 1995. Also, we find under a more demanding specification, one that allows for the date of the structural break to be determined endogenously, that the series is in fact I(1). Interestingly, under this last specification the results indicate that the series presents a structural break in 1907, which is a date that does not correspond to any significant event in the history of Mexico. We argue, nonetheless, that this result might be perceived as evidence that determining the date of the structural break endogenously is a more robust methodology relative to the case when said date is set exogenously, since in the first case the date of the structural break is not correlated with the data.

Beyond showing various methodologies for testing for unit roots, there are some important implications of our findings in other contexts. At the research level, for example, the result that the GDP is I(1) can be taken as an input in the econometric analysis of the series including cointegration and the estimation of vector autoregressions. At the policy making level, finding that Mexico's GDP is non-stationary imply that innovations to the series might produce permanent chances in its behavior. Hence, fiscal or monetary shocks might have a greater and more lasting effect on the economy than it is usually thought. This, of course, is an important insight for policy makers, especially in this particular moment in time, when the economy is undergoing a slowdown, since it can be expected that a fiscal reform, for example, will have a significant effect on output.

Evidently, there is ample opportunity to test for the effectiveness of the methodology that allows structural breaks to be identified endogenously in other time series, exercises of this nature will be conducted in future research.

[7] It is worth mentioning that Perron (1989) conducts all the unit roots tests considering only a one time predetermined change. He argues that if the break date is known, (1929 for the Great Crash and 1973 for the oil price shock) the model with exogenous breaks is appropriate.

References

- Balke, Nathan S. and Gordon, Robert J. "The Estimation of Prewar Gross National Product: Methodology and New Evidence". *Journal of Political Economy,* February 1989, 97(1), pp. 38-92.
- Banerjee, A., Lumsdaine, R.L. and Stock, J.H. "Recursive and Sequential Tests of the Unit Root and Trend Break Hypothesis", *Journal of Business and Economic Statistics*, 1992, 10, pp. 271-287.
- Carstens, Agunstin and Alejandro Reynoso, "Alcances de la Política Monetaria: Marco Teórico y Regularidades Empíricas en la Experiencia Mexicana". *Documento de Investigación 9705,* Banco de México, 1997.
- Christiano, L. J. "Searching for Breaks in GNP", *Journal of Business and Economic Statistics,* 1992, 10, pp. 237-250.
- Christiano, L. J., and Eichenbaum, M. "Unit Roots in Real GNP: Do We Know and Do We Care?" *Discussion Paper* 18 Federal Reserve Bank of Minneapolis, 1989.
- Dickey, D. A. and Fuller, W. A. "Distribution of the Estimators for Autoregressive Time Series With a Unit Root", *J. Am. Statist. Assoc.* 1979, 74, pp. 427-31.
- Dickey, D. A. and Fuller, W. A. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root", *Econometrica* 1981, 49, pp. 1057-1072.

- Diebold, Francis X. and Senhadji, Abdelhak S. "The Uncertain Unit Root in Real GNP: Comment", *American Economic Review,* December 1996, 86(5), pp. 1291-98.
- Nelson, Charles R. and Plosser, Charles I. "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications", *Journal of Monetary Economics*, September 1982, 10(2), pp. 139-62.
- Obstfeld, Maurice and Kenneth Rogoff *Foundations of International Macroeconomics,* The MIT Press 1997.
- Pérez, López A. Alejandro, "Un Modelo de Cointegración para Pronosticar el PIB de Mexico", *Documento de Investigación 9504*, Banco de México, 1995.
- Perron, P. "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis", *Econometrica,* 1989, 57, pp 1361-1401.
- Perron P. "Testing for a Unit Root in a Time Series with Changing Mean", *Journal of Business and Economic Statistics,* 1990, 8, pp. 153-162.
- Phillips, Peter C. and Perron Pierre, "Testing for a Unit Root in Time Series Regression", *Biometrika,* 1988, 75 (2), pp. 335-46.
- Rudebusch, Glenn D. "The Uncertain Unit Root in Real GDP", *American Economic Review,* March 1993, 83(1), pp. 264-72.
- Stock, James H. and Mark W. Watson, "Does GNP Have a Unit Root", *Economic Letters,* 1986, 22, pp. 147-151.
- Zivot, E. and Andrews, D.W.K., "Further Evidence on Great Crash, the Oil Price Shock and the Unit Root Hypothesis", *Journal of Business and Economic Statistics,* 1992, 10, pp. 251-270.